

Measurement of Mechanical Quantities

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Introduction

- Some examples of *mechanical quantities* that need to be measured include *position* (or *displacement*), *angular velocity* (*rotation rate of a shaft*), *force*, *torque* (moment), and *shaft power*.
- Instruments of various kinds have been invented to measure each of these quantities.
- In this learning module, several of these instruments are discussed; the list here is by no means exhaustive.
- The purpose here is to give you a feel for how mechanical instruments work, and to make you aware of the variety of ways to measure things.

Position and Displacement Measurement

- The length unit in the SI system is the meter (m). For many years, the standard meter was defined as 1,650,763.73 wavelengths of krypton 86 (an orange-colored isotope of the element krypton) in a vacuum.
- The standard meter is now defined as **the distance that light in a vacuum travels in $1/(299,792,458)$ s.**
- Various kinds of instruments have been invented to measure distance, length, or displacement.
- Some of these (especially the mechanical devices) are simple and straightforward, and are discussed first.
- Some more sophisticated measurement devices also exist for measuring displacement, and these are discussed later.

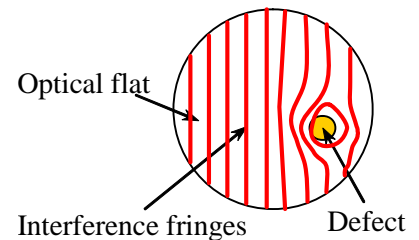
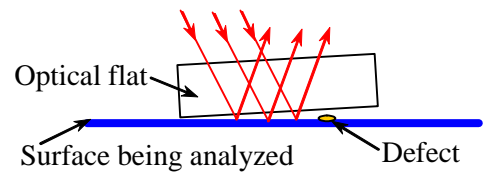
Mechanical devices

- **Principle of operation:**
 - A simple comparison is made between a displacement (or an object's length) and that of a pre-measured displacement or length.
 - **The length or displacement is inferred to be the same as that of the (known) pre-measured length.**
- Examples include:
 - Lengths of various magnitudes are measured with a standard *ruler* or *tape measure*.
 - Small lengths are measured with a more precise device called a *micrometer* (see above picture).
 - Cylindrical or spherical diameters are measured with *vernier calipers*.
 - Various gaps and the inner diameters of small tubes are measured with *gage blocks* (a set of precision ground hardened steel objects of known size).



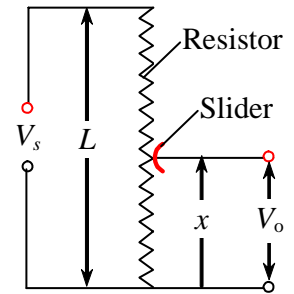
Interferometer

- **Principle of operation:**
 - Interacting light waves produce interference patterns.
 - **Displacement is inferred from the interference patterns produced by light reflecting off a surface.**
- An *interferometer* uses single wavelength light (like that from a laser) for optimum quality.
- A schematic diagram is shown to the right, and the fundamentals of interferometry are analyzed here.
 - The light is passed through a precision ground quartz disk called an *optical flat*. The optical flat is tilted at some small angle, as shown.
 - Incoming light passes through the optical flat, and is reflected as sketched.
 - Interference patterns are visible when viewing the reflected rays of light, since the optical path of individual rays differs. Each dark and light band represents a distance equal to *half of the wavelength of the light*.
 - When the optical flat is viewed from the top, the user sees something like the image sketched to the right.
 - On a perfectly flat portion of the surface, as on the left side of the sketch, the interference fringes are straight and parallel.
 - Where there is a defect, as on the right side of the sketch, the fringe lines get distorted.
 - The elevation of the defect can be inferred since each dark band represents a half wavelength of light.



Potentiometer

- **Principle of operation:**
 - A material's electrical resistance increases with length.
 - The displacement is inferred by measuring the change in resistance as a slider is displaced along the potentiometer.



- In a practical application, a **linear potentiometer** is simply a **voltage divider**, with L being the total length of the resistor, and x being the displacement to be measured, as sketched.
 - As displacement x changes, the **slider** moves along the resistor.

- The equation for output voltage is simply
$$V_o = \frac{x}{L} V_s$$

- Potentiometers are cheap and fairly accurate, but wear out eventually due to the physical contact at the slider.
- The contact point itself can be electrically noisy, which is also undesirable.
- There are also **rotary potentiometers**, which work under the same principle, but measure **angular displacement** rather than **linear displacement**.
- Rotary potentiometers are also used in electronics as **variable resistors** or **pots**.

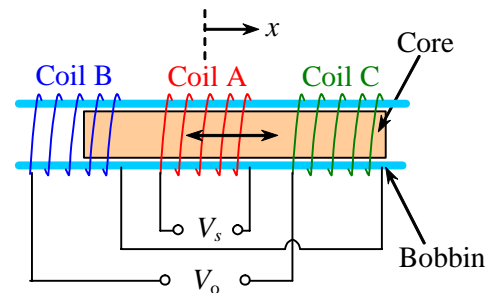


Linear variable displacement transducer

- **Principle of operation:**
 - A **linear variable displacement transducer (LVDT)** works on the same principle as electric motors, electromagnets, etc., namely the link between electricity and magnetism as found by H. A. Lorentz.
 - If a magnetic field moves near an electrical wire, current flows through the wire.
 - Or (vice-versa), if the current flowing through an electrical wire changes, it changes the magnetic field.

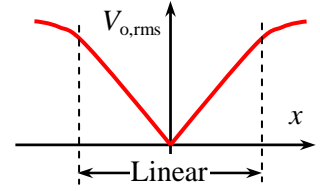


- Some authors call this device a **linear variable differential transformer (LVDT)** (also **LVDT**).
- An LVDT consists of a rod called the **core** (a ferromagnetic material like iron), that slides inside a hollow cylindrical tube called a **bobbin**, that is surrounded by three electrical **coils**:
 - Coil A is the **primary coil**, wrapped around the center of the bobbin (see sketch to the right).
 - Coils B and C are the **secondary coils**, also wrapped around the bobbin as shown.
 - V_s is the supply voltage, an AC supply – **alternating current**.
 - V_o is the output voltage. Note that one lead to V_o comes from the **outside** of coil B, but the other lead comes from the **inside** of coil C.
 - x is the displacement of the core from the center of the bobbin.



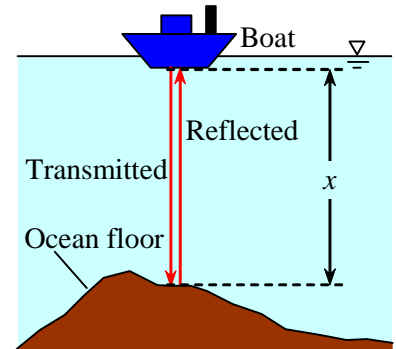
- How does the LVDT work?
 - **AC current** (not DC current) is supplied to the primary coil (coil A, in the center of the bobbin).
 - This causes an **alternating magnetic field** in the core.
 - The magnetic field **induces** an alternating current in coils B and C as well.
 - The electrical connections in coils B and C are **opposite**, as in the sketch, where we notice that coil B is wound from left to right electrically, while coil C is wound from right to left electrically.
 - If the core is exactly at the center of the bobbin, then $V_o = 0$, since the two currents (or voltages) from coils B and C are equal and opposite.
 - However, if the core is displaced closer to one of the secondary coils, V_o is not zero since that coil will have a stronger current. (In the sketch, the core is closer to coil C; it is displaced to the right.)

- Note that the **output voltage V_o is also an alternating (AC) voltage**. Thus, the **rms value** of V_o ($V_{o,rms}$) is of interest rather than V_o itself. In the sketch to the right, $V_{o,rms}$ is plotted as a function of displacement x .
- It turns out that the rms output voltage is **linear** over a certain range of x , and is symmetric with respect to positive and negative x .
- Most LVDTs are designed to operate only within the linear region.
- You can distinguish between positive and negative displacements by looking at the **phase** of the output signal:
 - If the core is displaced to the *left*, towards coil B, the output voltage is *in phase* with the input voltage.
 - If the core is displaced to the *right*, towards coil C, the output voltage is 180° out of phase with the input voltage.
- One clear advantage of the LVDT over the potentiometer is that there is no slider contact to wear out. LVDTs are also typically more accurate and more precise.
- A disadvantage is that they are more expensive than linear potentiometers, and require more sophisticated electronics and signal conditioning.



Sonic and ultrasonic transducers

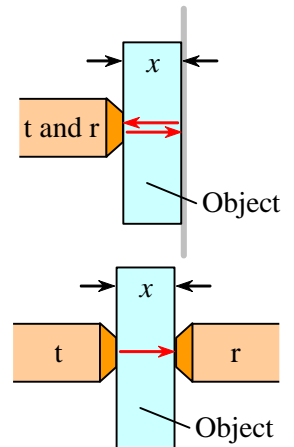
- **Principle of operation:**
 - If the speed of sound through a material is known, the material's thickness can be measured by timing how long it takes for the sound to pass through.
 - Thickness or displacement is inferred by measuring the time it takes for the sound wave to travel through the material.
- An age-old example used by mariners is water depth “sounding” or **sonar** (e.g., Acts 27:28 in the Bible – around 2000 years ago!)
 - Suppose the speed of sound a in water is known. The sonar device on the boat can transmit sound waves and can receive or detect reflecting sound waves.
 - The sonar device transmits a sound wave, and detects its reflection from the bottom, as sketched.
 - The time for the transmitted signal to travel to the bottom, reflect, and return to the ship is measured.
 - The distance (in this case water depth) is calculated as $x = \frac{a\Delta t}{2}$.
 - The factor of two appears because the sound wave must travel *twice* the water depth before being detected by the receiver.
- This same principle is used in the laboratory to measure the thickness of materials or displacement (through air or some other medium), but we typically use sound at frequencies above that of human hearing (**ultrasonic**)



- There are two types of ultrasonic transducers:



- A **pulse-echo ultrasonic transducer** has the transmitter (t) and receiver (r) on the *same* side (as in the boat example above). The gray vertical line represents some hard surface off of which the sound waves reflect.
- For the pulse-echo case, $x = \frac{a\Delta t}{2}$.
- A **through-transmission ultrasonic transducer** has the transmitter (t) and receiver (r) on *opposite* sides. No other surface is needed, but access to *both* sides of the object is required, and this is not always possible.

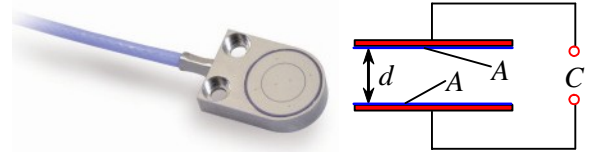


- For the through-transmission case, $x = a\Delta t$. Without the factor of two, the **sensitivity of the through transmission transducer is half that of the pulse-echo transducer**, all else being equal.

Capacitance displacement sensor

Principle of operation:

- The capacitance between two metal plates is a function of the *distance* between the plates and the *overlapping area* of the plates.
- **By measuring the change in capacitance, we infer the displacement.**
- The one shown here is designed for ultra-high precision positioning systems (they claim ±5 nm).

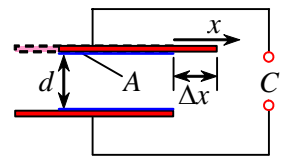
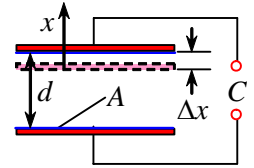


- A schematic diagram is sketched to the right.
- The equation for the capacitance C is $C = K \epsilon_0 A / d$, where

- d is the distance between the plates, as sketched.
- ϵ_0 is a constant called the *permittivity of free space*, $\epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{N \cdot m^2}$.
- K is the *dielectric coefficient* of the material between the plates ($K = 1$ for air).
- A is the *overlapping plate area*. If the two plates are aligned right on top of each other, as in the sketch, A is the entire surface area of one side of a plate.

- There are two ways to move the plates to cause a change in capacitance:

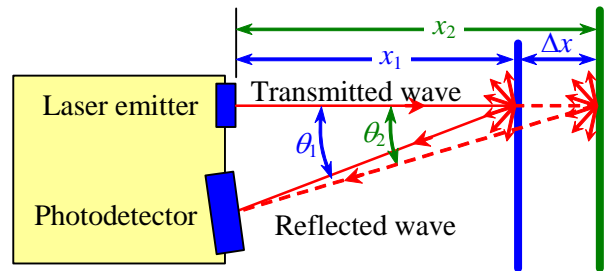
- **Normal displacement** is when the plates move *perpendicularly* away from each other, i.e., d changes linearly with x , but A remains constant. In this case, x is *normal* to the plates. As can be seen from the above equation, C changes inversely with d (C falls off as $1/d$); thus **C changes nonlinearly with x .**
- **Parallel displacement** is when the plates move *laterally* away from each other, i.e., area A changes linearly with x , but d remains constant. In this case, x is *parallel* to the plates. As can be seen from the above equation, C changes linearly with A ; thus **C changes linearly with x ,** at least for some range of x .



Laser displacement meter

Principle of operation:

- **The angle of reflected light from a laser beam changes as the reflecting object is displaced.**
- By measuring the reflected angle, the distance to the object is inferred.
- The diagram to the right illustrates how a laser beam is transmitted to an object and reflects in all directions. One of the reflected waves goes to a photodetector in the instrument. [Notice that since $x_1 < x_2$, $\theta_1 > \theta_2$.]
- Sophisticated optics and electronics in the unit measure angle θ of the reflected beam, and convert to a digital readout of displacement x , or more commonly Δx . [Displacement Δx is inferred by measuring $\Delta \theta$.]
- The main advantage of this measurement technique is obvious – no physical contact with the object is required. This is especially attractive in the electronics industry where physical contact can lead to impurities.
- Laser displacement meters are quite accurate and precise; the one in the M E 345 laboratory can detect displacement changes of ±0.1 mm (±0.004 inch).
- However, the range is limited; the laser displacement meter in the M E 345 laboratory has a nominal operating point of $x = 80.0$ mm, with a measurement range of ±20 mm (±0.78 inch) from this operating point.



Angular velocity or rotation rate measurement

- First some nomenclature and units:
 - The most common unit for rotation rate is *rotations per minute*, or *revolutions per minute*, or *rpm*.
 - Rotation rate is given the symbol N_{rpm} here to emphasize that its units are rotations per minute (rpm).
 - The more mathematically useful measure of rotation rate is *angular velocity* ω , in units of *radians per second*. (ω is not as popular among engineers, but is required in most equations.)

○ The conversion between these two units is
$$\left(N_{rpm} \frac{\text{rotation}}{\text{min}} \right) = \left(\omega \frac{\text{radian}}{\text{s}} \right) \left(\frac{\text{rotation}}{2\pi \text{radian}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right)$$

- Some authors write this more compactly as $N_{\text{rpm}} = \frac{60}{2\pi} \omega$, where it is understood that N_{rpm} is in units of rpm and ω is in units of radians per second. However, this can lead to unit errors if we are not careful.
- To avoid confusion, it is best to always define a **unity conversion factor**, namely, $\boxed{\frac{60 \text{ rpm} \cdot \text{s}}{2\pi \text{ radian}} = 1}$.
- For example, if $N_{\text{rpm}} = 3600$ rpm, the angular velocity is $\omega = (3600 \text{ rpm}) \left(\frac{2\pi \text{ radian}}{60 \text{ rpm} \cdot \text{s}} \right) = 377.0 \frac{\text{radian}}{\text{s}}$.
- In most engineering applications, a **shaft rotation speed** (in either rpm or radians/s) is to be measured.
- The instruments that are used to measure shaft rotation speed are commonly called **tachometers**.
- There are two main categories of tachometers, **contacting** and **noncontacting**.

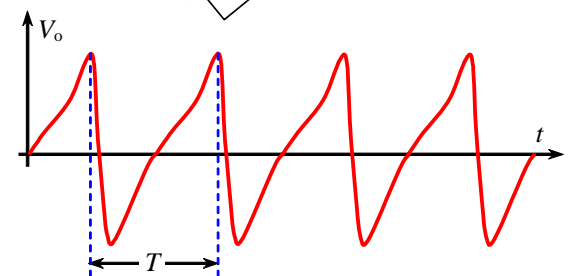
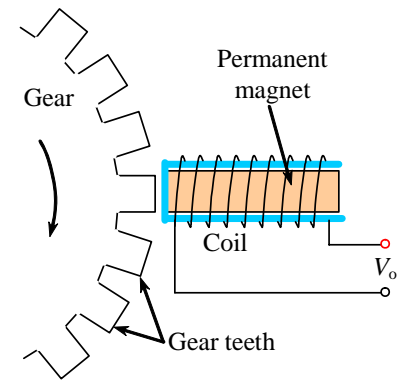
Contacting tachometer

- **Principle of operation:**
 - The instrument is physically attached to the end of a spinning shaft, so that part of the instrument rotates at the same rpm as the shaft.
 - Shaft rpm is measured directly and internally by the instrument.
- The actual measurement of rpm internally in the instrument can be performed by a number of methods including:
 - **Mechanical:** The spinning shaft goes through a gear box that rotates a spring-loaded dial.
 - **Electric generator:** A small DC motor is used “backwards” as a generator. Output voltage is proportional to rpm.
 - **Chronometer:** An electronic clock measures the time between rotations and calculates rpm (like the one shown to the right).



Noncontacting tachometer

- **Principle of operation:**
 - There is no direct physical contact between the instrument and the rotating shaft.
 - Instead, the rpm of the rotating shaft is inferred by timing pulses remotely, either from magnetic or optical signals.
- There are several types of noncontacting tachometers:
- **Magnetic pickup tachometer:** This device, also called a **variable reluctance pickup tachometer**, works on the same principle as the LVDT, i.e., distortion of a magnetic field when ferromagnetic material passes by.
 - A schematic diagram is shown to the right.
 - As ferromagnetic material in a gear tooth passes in front of the device, as shown, the magnetic field is distorted, which causes a pulse in the output voltage.
 - A plot of output voltage as a function of time may look something like that sketched to the right, where T is the period of one pulse (each gear tooth causes one pulse of time duration T)
 - An electronic circuit then either measures T , or counts the number of pulses per second, P . It then converts the time signal into frequency, and then into rpm.
 - The equation to convert from measured number of



pulses per second to rpm is $\boxed{N_{\text{rpm}} = \frac{P}{n_{\text{teeth}}}}$, where

- N_{rpm} = rotation rate (typically in rpm)
- P = measured pulse rate (typically in pulses per second)
- n_{teeth} = number of teeth on the gear (n_{teeth} also equals the **number of pulses per rotation**)
- The number of gear teeth must be known in order to calculate the rpm correctly, and units must be kept straight to avoid errors, as the following example illustrates.

• **Example:**

Given: A magnetic pickup tachometer is used to measure the rotation speed of a gear. Suppose $n_{\text{teeth}} = 30$.

The number of pulses per second P is counted by the electronic circuit.

To do: Calculate the rotation speed of the gear (in rpm) as a function of the measured value of P .

Solution:

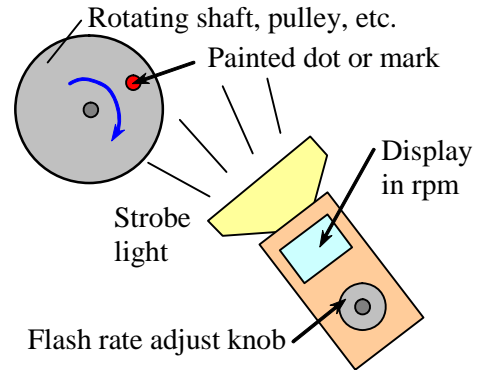
○ We use the equation given above, $N_{\text{rpm}} = \frac{P}{n_{\text{teeth}}} = \frac{\left(\frac{P \text{ pulse}}{\text{s}}\right)}{\left(\frac{30 \text{ pulse}}{\text{rotation}}\right)} \left(\frac{60 \text{ s}}{\text{min}}\right) = 2P \frac{\text{rotation}}{\text{min}}$ or $N_{\text{rpm}} = 2P \text{ rpm}$.

○ Alternatively, in terms of the pulse period, $T = 1/P$, and thus $N_{\text{rpm}} = 2/T \text{ rpm}$.

○ For example, if $P = 750$ pulses per second, $N_{\text{rpm}} = 1500 \text{ rpm}$.

Discussion: Since 30 gear teeth is half of 60, the relationship involves a simple factor of 2. If instead we use a gear with 60 teeth, the relationship would be one-to-one, i.e., $N_{\text{rpm}} = P$, where P is the number of pulses per second and N_{rpm} is the number of shaft rotations per minute.

• **Stroboscopic tachometer:** A strobe light is used to illuminate a spinning shaft, gear, or pulley at precisely timed intervals. **The strobe light frequency is adjusted (by a knob) until the spinning object appears to stop.** Sometimes you need to paint a dot or other mark on the surface of the spinning object in order to see it clearly, as sketched here.



○ You are probably familiar with strobe lights at dance halls and from physics class, where it is often used to illustrate the acceleration of a falling object (see [youtube video](#)). Strobes are also useful to measure rpm.

○ The digital readout of most stroboscopic tachometers is in rpm.

○ **Caution:** Stroboscopic tachometers can suffer from a type of **aliasing error** – the shaft appears to be rotating at some **incorrect rpm**. (We can be fooled by this!)

○ Furthermore, if the flashing rpm is an **integer fraction** of the true shaft rpm, it will still look like the shaft is stationary, and the **wrong rpm** will be inferred!

○ For example, suppose the true shaft speed is 300 rpm, and the strobe is adjusted to 100 rpm, where the mark on the shaft appears to stand still (a “frozen” dot). Although it looks like the correct shaft speed has been determined, the rotation rate would be wrong by a factor of 3! The shaft actually rotates three times around between each flash of the strobe.

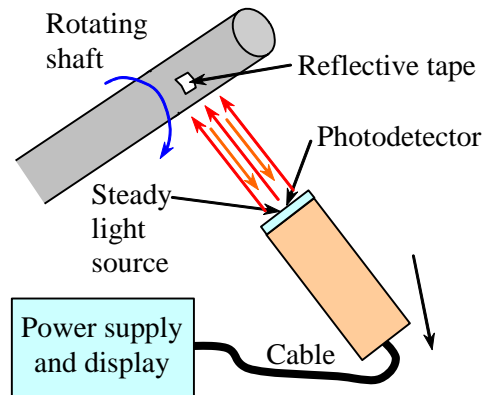
○ To avoid this problem, the user should keep increasing the flashing frequency until the correct rpm is determined – until the shaft can no longer be made to appear to stand still.

○ Although stroboscopic tachometers are widely used, their main disadvantage is that the adjustment knob must be turned manually – there is human interaction involved.

• **Photoelectric tachometer:** This instrument is somewhat similar in principle to the stroboscopic tachometer, except electronics remove the manual component.



○ A **steady light source**, rather than a stroboscopic light source, is transmitted from the device. A **photodetector** (or **photocell**) produces a pulse each time light is reflected from a piece of **reflective tape** on the rotating shaft.



○ A schematic diagram is shown to the right.

○ Since the reflected light produces one pulse per revolution, the shaft rotation rate is easily determined by either counting pulses or measuring the time between pulses.

○ Most modern photoelectric tachometers have a digital readout directly in rpm, like the unit shown to the left.

Torque and Power Measurement (Dynamometers)

- A dynamometer measures the **moment** or **torque** T of a rotating shaft.
- [Do not confuse torque T with temperature T .]
- The word **dynamometer** comes from “**dynamic moment meter**.”
- The angular velocity ω of the shaft is also measured so that **shaft power** \dot{W}_{shaft} can be calculated, since for a

rotating shaft, power = angular velocity \times torque, $\dot{W}_{\text{shaft}} = \omega T$, or $\dot{W}_{\text{shaft}} = \frac{2\pi}{60} N_{\text{rpm}} T$.

- Note that if the angular velocity is given in rpm rather than radians per second, it must first be converted in order to calculate the shaft power, as in the second equation above.
- Although the SI system of units is becoming more popular, many engineers still use **horsepower (hp)** as the unit for power. Below are some useful conversions when dealing with power; first the unit description, and then the unity conversion factor(s):

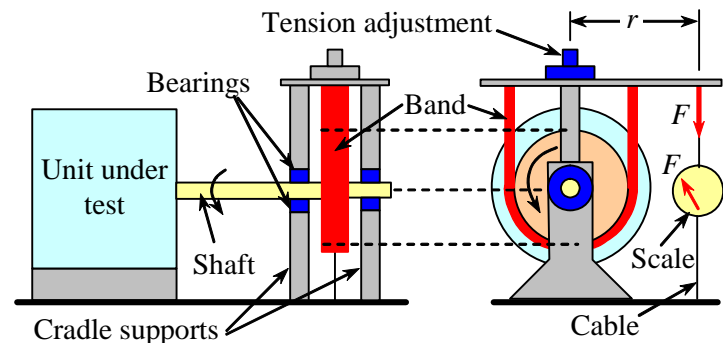
- 1 hp = 550 ft·lbf/s. $\left(\frac{1 \text{ hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}}\right) = 1$.
- 1 hp = 745.7 watts. $\left(\frac{1 \text{ hp}}{745.7 \text{ W}}\right) = 1$ or $\left(\frac{0.7457 \text{ kW}}{1 \text{ hp}}\right) = 1$.
- 1 hp = 42.42 Btu/min. $\left(\frac{1 \text{ hp} \cdot \text{min}}{42.42 \text{ Btu}}\right) = 1$ or $\left(\frac{254.6 \text{ Btu}}{1 \text{ hp} \cdot \text{hr}}\right) = 1$.
- 1 watt = 1 N·m/s. $\left(\frac{1 \text{ W} \cdot \text{s}}{1 \text{ N} \cdot \text{m}}\right) = 1$ or $\left(\frac{1000 \text{ N} \cdot \text{m}}{1 \text{ kW} \cdot \text{s}}\right) = 1$.



- The dynamometer pictured to the right is from DyneSystems, Inc., and is used to measure wind turbine power up to 6000 hp and 31,500 ft·lbf of torque.
- There are several types of dynamometers. Only three are discussed here – the three that are used in the laboratory portion of this course.

Prony brake dynamometer:

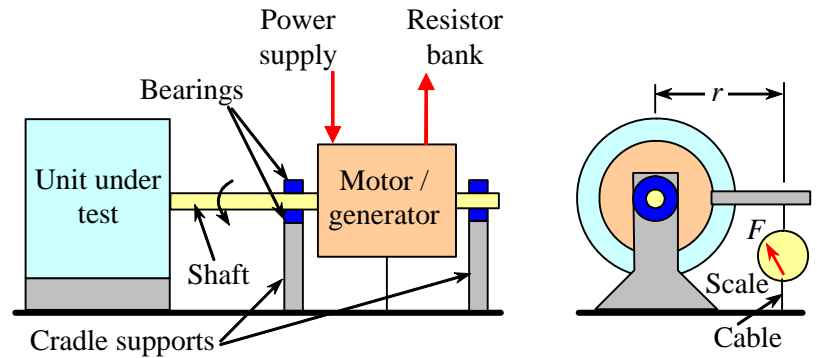
- **Principle of operation:**
 - **Mechanical friction in a braking mechanism absorbs the unit's power.**
 - Torque and rpm are measured to calculate the shaft power.
- Typically, either a **band brake** or a **disk brake** is used. A prony brake dynamometer with a band brake is shown in the schematic diagram to the right.
- The unit under test can be anything with a rotating shaft – e.g., a motor or engine.
- The tension on the brake is adjusted to control the torque (due to friction on the band).
- As torque is applied, the prony brake mechanism tries to rotate (counterclockwise in the sketch above). However, a **cable** keeps the mechanism from rotating. The tension force F in the cable is measured by a **force scale**, as shown.
- Force F is measured at a known moment arm called the **torque arm** r to calculate the torque, $T = Fr$.
- Shaft rotation rate is measured simultaneously.
- Shaft power is then calculated: $\dot{W}_{\text{shaft}} = \omega T = \omega Fr$, or $\dot{W}_{\text{shaft}} = \frac{2\pi}{60} N_{\text{rpm}} T = \frac{2\pi}{60} N_{\text{rpm}} Fr$.
- The advantages of a prony brake dynamometer are that it is simple, small in size, and inexpensive.
- The disadvantages are that limited power can be dissipated with a prony brake, the mechanical brake is sometimes not very stable, and the unit cannot **supply** power – it can only **absorb** power. The band can also get very hot, and this can be dangerous.



Cradled DC motor dynamometer (also called an electric motoring dynamometer)

- **Principle of operation:**
 - **An electric motor, attached directly to the shaft of the unit under test, absorbs the unit's power when it functions as a generator.**

- The load on the motor is varied to control the torque.
- Specifically, the generated power is dissipated (as heat) in a **resistor bank** as shown in the schematic diagram to the right.
- The resistance in the resistor bank is adjusted to control the torque.
- Just as with the prony brake dynamometer, the motor/generator tries to rotate (counterclockwise in the sketch here). However, the cable keeps the mechanism from rotating.
- The torque arm r the force scale, etc. are identical to those of the prony brake dynamometer.
- Shaft rotation rate is measured



simultaneously, and shaft power is calculated with the same equations as for the prony brake, $\dot{W}_{\text{shaft}} = \omega T$, or

$$\dot{W}_{\text{shaft}} = \frac{2\pi}{60} N_{\text{rpm}} T$$

- The power dissipated by the resistor bank can also be calculated easily. Namely, if either the current I or the voltage V across the resistor bank is measured, the dissipated power is $\dot{W}_{\text{dissipated}} = VI$.
- But since $V = IR$ from Ohm's law, $\dot{W}_{\text{dissipated}} = VI = I^2 R = \frac{V^2}{R}$.
- The advantages of a cradled DC motor dynamometer are that it is versatile, and it can **supply power as well as absorb power**, which is sometimes a useful feature.
- The disadvantages are that it is large in size and expensive.

Eddy current dynamometer

- **Principle of operation:**
 - A **fluctuating magnetic field** absorbs the unit's power, and dissipates it as heat.
 - The magnetic field strength is varied to control the torque.
- A schematic diagram is shown here.
- The internal structure of the eddy current dynamometer consists of:
 - A **stator** (the stationary part), which has a **coil** around a ferromagnetic material, powered by a DC supply. The supplied power is adjusted to control the load.
 - A **rotor** (the rotating part), which has **radial lobes**.
- As the rotor turns, the lobes disrupt the magnetic field (same principle as the magnetic pickup device), and cause internal currents known as **eddy currents**.
- The energy of the eddy currents dissipates internally due to electrical resistance.
- Unlike the cradled DC motor dynamometer, there is no resistor bank – the unit itself heats up, and this heat must be dissipated.
- On smaller units, the heat is dissipated into the surrounding air. On larger units, the dissipated energy is carried away by cooling water.
- The torque arm, the force scale, etc. are identical to those of the cradled DC motor dynamometer.
- Shaft power is calculated with the same equations as previously, $\dot{W}_{\text{shaft}} = \omega T$, or $\dot{W}_{\text{shaft}} = \frac{2\pi}{60} N_{\text{rpm}} T$.
- The main advantage of an eddy current dynamometer is it is good for very high power applications, like automobile engines.
- The main disadvantage is that it cannot supply power as can the cradled DC motor dynamometer. An eddy current dynamometer can only **absorb** power.

