

# Temperature Measurement

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## Introduction

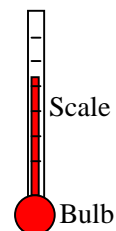
- There are four basic types of temperature measuring devices, each of which uses a different principle:
  - **Mechanical** (liquid-in-glass thermometers, bimetallic strips, etc.).
  - **Thermo-junctive** (thermocouples).
  - **Thermoresistive** (RTDs and thermistors).
  - **Radiative** (infrared and optical pyrometers).
- Each of these is defined and discussed in this learning module, with most of the emphasis placed on thermo-junctive temperature measuring devices – thermocouples.

## Mechanical temperature measuring devices

- **Principle of operation:**
  - A change in temperature causes some kind of mechanical motion, typically due to the fact that **most materials expand with a rise in temperature**. Mechanical thermometers can be constructed that use liquids, solids, or even gases as the temperature-sensitive material.
  - The mechanical motion is read on a physical scale to infer the temperature.

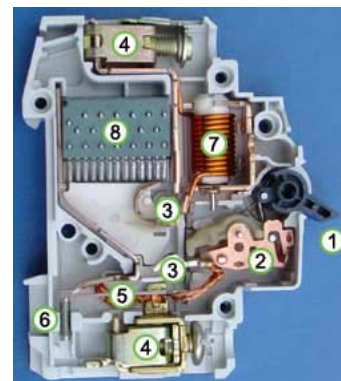
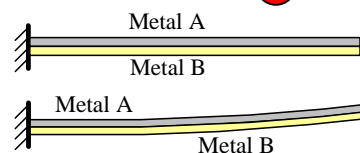
- **Liquid-in-glass thermometer**

- The most common and well-known thermometer is the **liquid-in-glass thermometer**.
- As the temperature rises, the liquid expands, moving up the tube. The scale is calibrated to read temperature directly.
- Usually, mercury or some kind of alcohol is used for the liquid.



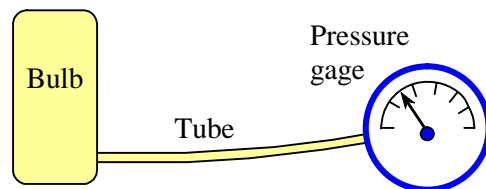
- **Bimetallic strip thermometer**

- Two dissimilar metals are bonded together into what is called a **bimetallic strip**, as sketched to the right.
- Suppose metal A has a smaller coefficient of thermal expansion than does metal B.
- As temperature increases, metal B expands more than does metal A, causing the bimetallic strip to curl upwards as sketched.
- One common application of bimetallic strips is in home thermostats, where a bimetallic strip is used as the arm of a switch between electrical contacts. As the room temperature changes, the bimetallic strip bends as discussed above. When the bimetallic strip bends far enough, it makes contact with electrical leads that turn the heat or air conditioning on or off.
- Another application is in circuit breakers (the bimetallic strip is labeled “5” in the photo to the right). High temperature indicates over-current, which shuts off the circuit [from Wikipedia].
- Another common application is for use as oven, wood burner, or gas grill thermometers. These thermometers consist of a bimetallic strip wound up in a *spiral*, attached to a dial that is calibrated into a temperature scale.



- **Pressure thermometer**

- A **pressure thermometer**, while still considered mechanical, operates by the expansion of a gas instead of a liquid or solid. (Note: There are also pressure thermometers that use a *liquid* instead of a gas.)
- Suppose the gas inside the bulb and tube can be considered an ideal gas. The ideal gas law is  $PV = mRT$ , where  $P$  is the pressure,  $V$  is the volume of the gas,  $m$  is the mass of the gas,  $R$  is the gas constant for the specific gas (not the universal gas constant), and  $T$  is the absolute temperature of the gas.
- Specific gas constant  $R$  is a constant. The bulb and tube are of constant volume, so  $V$  is a constant. Also, the mass  $m$  of gas in the sealed bulb and tube must be constant (conservation of mass). Hence, the ideal gas equation reduces to  $P = \text{constant} \cdot T$ .
- A pressure thermometer therefore measures temperature *indirectly* by measuring pressure.
- The gage is a pressure gage, but is typically calibrated in units of temperature instead.
- A common application of this type of thermometer is measurement of outside temperature from the inside of a building. The bulb is placed outside, with the tube running through the wall into the inside.



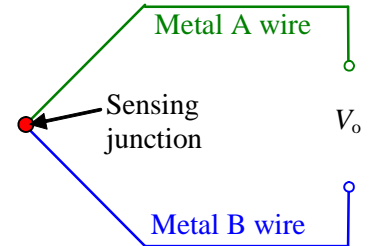
The gage is on the inside. As  $T$  increases outside, the bulb temperature causes a corresponding increase in pressure, which is read as a temperature increase on the gage.

### Thermojunctive temperature measuring devices (thermocouples)

- **Principle of operation:**

- When two dissimilar metals are connected together and current flows, heat is transferred and a small voltage called a **thermojunction voltage** is generated at the junction. This is called the **Peltier effect**.
- Furthermore, if the temperature of the junction changes, the voltage changes too. This is called the **Thompson effect**.
- Both of these effects can be combined to measure temperature. The combined effect is known as the **thermojunction effect** or the **thermoelectric effect** or the **Seebeck effect**.

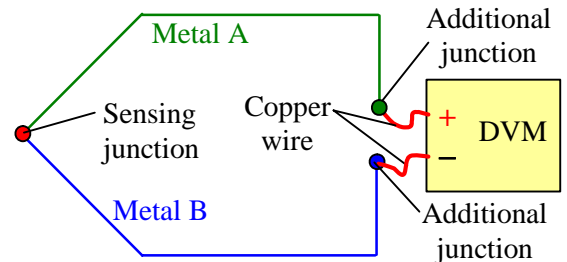
- A **thermocouple** is simply two metal wires joined together at what is called a **thermojunction** or a **sensing junction**, as sketched to the right.



- The voltage is measured to infer the temperature.

- In practical operation, wires A and B are connected to a **digital voltmeter (DVM)**, **digital multimeter (DMM)**, digital data acquisition system, or some other voltage measuring device. If the measuring device has very high input impedance, the voltage produced by the thermojunction can be measured accurately.

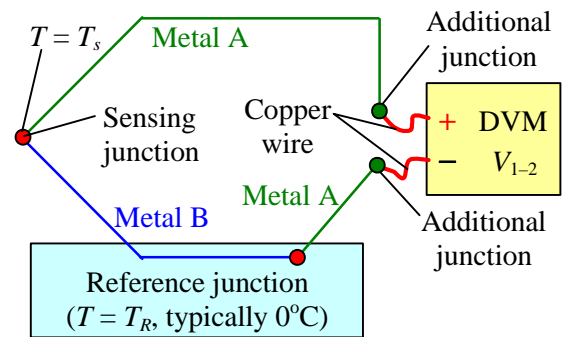
- However, the main problem with thermocouple temperature measurement is that wires A and B must connect to the leads of the voltmeter, which are generally made of copper, as sketched to the right.



- If neither wire A nor wire B is itself copper, connecting to the DVM creates *two more thermojunctions!* (Thermocouple metals are typically not the same as those of the DVM leads.)

- These additional thermojunctions also produce a thermojunctive voltage, which can create an error when trying to measure the voltage from the sensing junction.

- How can this problem be resolved? One simple solution is to add a **fourth** thermojunction, called a **reference junction**, by inserting an additional length of metal A wire into the circuit as sketched to the right. The reference junction consists of metals A and B as indicated on the sketch.



- This modified circuit is analyzed as follows:

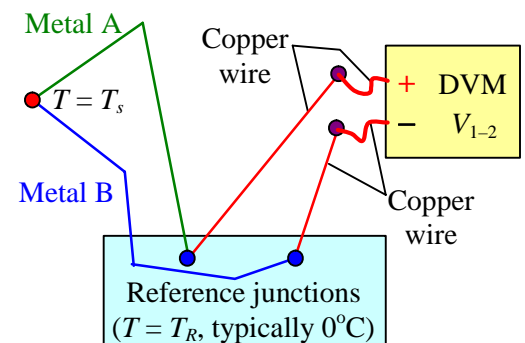
- The two junctions to the DVM are now *both* between metal A and copper.
- These two junctions are placed *close together*, and at the *same temperature*, so that their thermojunction voltages are identical, and cancel each other out.
- Meanwhile, the new reference junction is placed in a location where the **reference temperature**  $T_R$  is known accurately, for example in an ice-water bath with a fixed, known temperature of  $T_R = 0^\circ\text{C}$ .

- In fact, **the standard reference temperature for thermocouples is  $T_R = 0^\circ\text{C}$** .

- If the sensing junction is *also* at  $0^\circ\text{C}$  ( $T_s = 0^\circ\text{C}$ ), the voltage generated by the sensing junction is equal and opposite to that generated by the reference junction. Hence,  $V_o = 0$  when  $T_s = 0^\circ\text{C}$ .

- However, if the sensing junction temperature is *not* equal to  $T_R$ ,  $V_o$  is non-zero.

- There is another way to use reference junctions as sketched to the right.



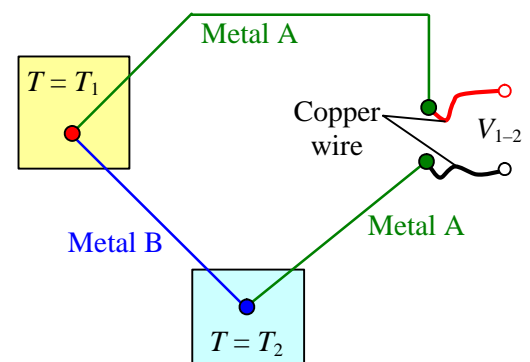
- Here *two* new reference junctions are created, but they are *both* at the reference temperature  $T_R$ .

- The junctions to the DVM are now *both* copper to copper.

- In summary, when set up properly with a known reference temperature (using either of these two circuits with reference junctions, it turns out that  $V_o$  is a unique function of the sensor temperature  $T_s$ , and the two metals used for the thermocouple.
- Thus, for known reference temperature and known thermocouple wire materials, output voltage  $V_o$  can be used to *measure* temperature. This is the fundamental concept of thermocouple usage.
- **Standard thermocouples**
  - Thermocouple manufacturers have standardized the use of certain pairs of metals for thermocouples. Reference books, manufacturer's literature, and websites list these so called *standard thermocouples*, that are commercially available.
  - Each standard thermocouple has been assigned both a letter and a color. For example, type J and type T thermocouples are used in our ME 345 lab.
    - A *type J* thermocouple has the color *black*, and uses *iron* and *constantan* as its component metals. (Constantan is an alloy of  $\approx 45\%$  nickel and  $\approx 55\%$  copper.)
    - A *type T* thermocouple has the color *blue*, and uses *copper* and *constantan* as its component metals.
    - A *type K* thermocouple has the color *yellow*, and uses *chromel* and *alumel* as its component metals. (Chromel is an alloy of  $\approx 10\%$  chromium and  $\approx 90\%$  nickel.) **Type K thermocouples are the most popular variety in use today.** In fact, many digital multimeters (DMMs) can measure temperature by plugging in a type K thermocouple with standard connections.
  - Other thermocouples can be made, even if the pair of metals is not one of the standard varieties. For example, in the lab, iron and copper are used to create a non-standard thermocouple.
  - The voltage produced by a thermocouple varies *almost*, but not exactly, linearly with temperature. Therefore, there are no simple equations to relate thermocouple voltage to temperature. Rather, voltage is tabulated as a function of temperature for the various standard thermocouples.
  - For best accuracy, *thermocouple tables* are used; the tables list output voltage as a function of temperature. Some thermocouple tables are provided on the ME 345 website.
  - **By convention, the reference temperature for thermocouple tables is  $0^\circ\text{C}$ .** Hence, **all thermocouple voltages are given relative to  $0^\circ\text{C}$**  – the voltage at  $0^\circ\text{C}$  is listed as zero volts.
  - **If you wire the thermocouple according to the above diagram with  $T_R = 0^\circ\text{C}$ , the thermocouple voltage at any sensing temperature should match with that listed in the tables.**
  - In addition, some polynomial curve fits have been generated. In computer programs, these curve fit equations are easier to use than tables. Some example polynomial curve fits for several thermocouples can be found in the **Omega Engineering Technical Reference Manual** (see [www.omega.com](http://www.omega.com)).

## Thermocouple Laws

- First some *notation*:
  - Let  $T_1$  be the temperature of bath 1, and  $T_2$  be the temperature of bath 2.
  - Let  $V_{1-R}$  be defined as the voltage produced by a thermocouple at temperature  $T_1$  when a proper reference junction at temperature  $T_R$  is used ( $T_R = \text{reference temperature} = 0^\circ\text{C}$ ). Note that  **$V_{1-R}$  is the voltage listed in the thermocouple tables at temperature  $T_1$ .**
  - Let  $V_{1-2}$  be defined as the difference in voltage between  $V_{1-R}$  and  $V_{2-R}$ , namely,  $V_{1-2} = V_{1-R} - V_{2-R}$ .
- **Sign convention**:
  - Negative sign errors can be problematic when working with these equations if you are not consistent.
  - By convention, the thermocouple tables are constructed such that **higher temperature yields higher thermojuvntive voltage.**
  - In other words, it is always be assumed that the two thermocouple wires (let's call them wire A and wire B) are connected to the voltmeter in such a way that the voltage is *positive* when the temperature being measured is *greater* than the reference temperature.
  - Likewise, the voltage is *negative* when the temperature being measured is *less* than the reference temperature.
  - Since the standard reference temperature for thermocouple tables is  $0^\circ\text{C}$ , **positive temperatures in units of  $^\circ\text{C}$  yield positive thermojuvntive voltages, and negative temperatures in units of  $^\circ\text{C}$  yield negative thermojuvntive voltages.**
  - For all thermocouples, one of the wires (Metal A here) is



connected to the positive terminal of the DMM, and the other is connected to the negative terminal so that the measured voltage increases with increasing temperature.

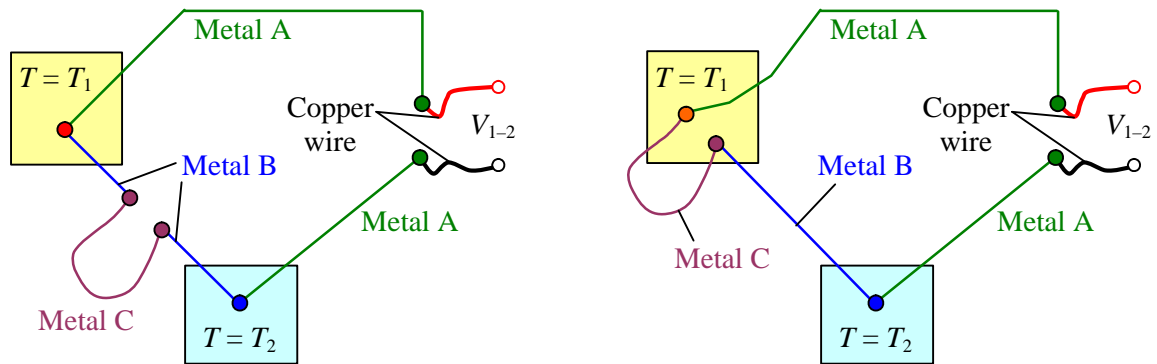
- If the wires were connected the *opposite* way to the voltmeter, the voltages would be of opposite sign.
- There are three laws or rules that apply to thermocouples:

- **Law of intermediate metals**

- **Statement of the law of intermediate metals:**

A third (intermediate) metal wire can be inserted in series with one of the wires without changing the voltage reading (provided that the two new junctions are at the same temperature).

- **Explanation:** Consider the setup sketched above, where a rectangle around a thermojunction indicates a *constant temperature bath* (e.g., a pot of boiling water or an ice-water bath).
- The law of intermediate metals states that the voltage reading  $V_{1-2}$  does not change if a third (intermediate) wire is added in line with any of the wires in the circuit, as sketched below. It is assumed that both of the new junctions (between metal B and metal C in the sketch on the left below, for example) are at the *same temperature*, e.g., ambient temperature,  $T_a$ . Two cases are shown. In the sketch on the left, the intermediate wire (Metal C) is inserted at a break in Metal B.



- You can easily see that the law of intermediate metals must hold here, since whatever voltage is generated at one of the new junctions is exactly canceled by an equal and opposite voltage generated at the other new junction. The output voltage does not change.
- Likewise, metal C can be inserted anywhere else in the circuit without any effect on the output voltage, provided that the two new junctions are at the same temperature.
- In the sketch on the right, for example, the intermediate wire is inserted at a *junction* – even though it disrupts the junction, the output voltage still does not change!

- **Law of intermediate temperatures**

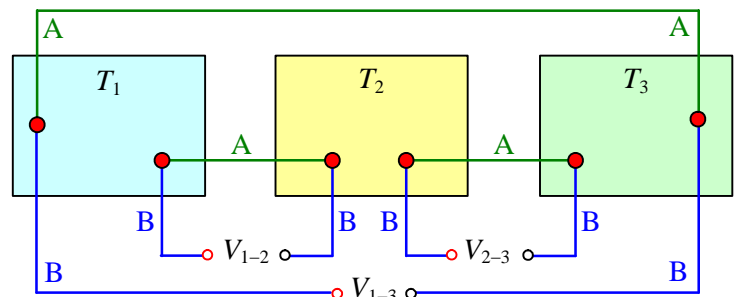
- **Statement of the law of intermediate temperatures:**

If identical thermocouples measure the temperature difference between  $T_1$  and  $T_2$ , and the temperature difference between  $T_2$  and  $T_3$ , then the sum of the corresponding voltages  $V_{1-2} + V_{2-3}$  must equal the voltage  $V_{1-3}$  generated by an identical thermocouple measuring the temperature difference between  $T_1$  and  $T_3$ .

- **Mathematical statement of the law of intermediate temperatures:**

$$V_{1-3} = V_{1-2} + V_{2-3} \text{ for any three temperatures } T_1, T_2, \text{ and } T_3.$$

- **Explanation and Proof:** Consider the setup to the right, where six thermojunctions are shown, two in each constant temperature bath. *Note:* To avoid clutter in the diagram, the copper leads of the DVM are no longer shown. Also, for brevity, letters A and B indicate metal A and metal B, two different types of thermocouple wires.



- By the notation convention adopted here,  $V_{1-3} = V_{1-R} - V_{3-R}$ . We add and subtract  $V_{2-R}$  and rewrite the equation as

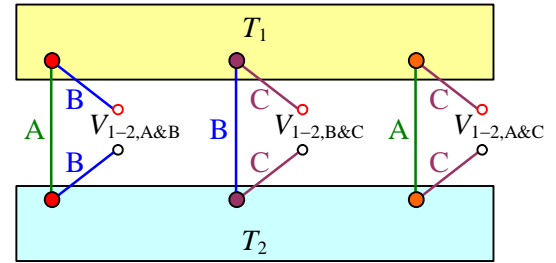
$$V_{1-3} = (V_{1-R} - V_{2-R}) + (V_{2-R} - V_{3-R}).$$

- But since (by definition)  $V_{1-2} = V_{1-R} - V_{2-R}$  and  $V_{2-3} = V_{2-R} - V_{3-R}$ , we see that  $V_{1-3} = V_{1-2} + V_{2-3}$ .

• Law of additive voltages

○ **Statement of the law of additive voltages:**

For a given set of 3 thermocouple wires, A, B, and C, all measuring the same temperature difference  $T_1 - T_2$ , the voltage measured by wires A and C must equal the sum of the voltage measured by wires A and B and the voltage measured by wires B and C.



○ **Explanation:** Consider the setup to the right, where six thermojunctions are shown, three in constant temperature bath  $T_1$ , and three in constant temperature bath  $T_2$ . As above, letters A, B, and C indicate different types of thermocouple wires.

○ The law of additive voltages can be stated mathematically as  $V_{1-2,A\&C} = V_{1-2,A\&B} + V_{1-2,B\&C}$ .

○ Or, rearranging in terms of voltage differences,  $V_{1-2,A\&B} = V_{1-2,A\&C} - V_{1-2,B\&C}$ .

• **Example:**

**Given:** Three constant temperature baths are prepared with  $T_1 = 160^\circ\text{C}$ ,  $T_2 = 100^\circ\text{C}$ , and  $T_3 = 20^\circ\text{C}$ . A type J thermocouple and a digital multimeter are used to measure voltages  $V_{1-2}$ ,  $V_{1-3}$ , and  $V_{2-3}$ .

**To do:** Predict the measured voltages  $V_{1-2}$ ,  $V_{1-3}$ , and  $V_{2-3}$ .

**Solution:**

○ First, the **notation** needs to be clarified:

- $V_{1-R}$  is defined as the difference in thermocouple voltage between temperature  $T_1$  and the *reference* temperature,  $T_R$ , which is  $0^\circ\text{C}$ . The values listed in the thermocouple tables are relative to zero degrees Celsius, and are thus  $V_{1-R}$ ,  $V_{2-R}$ , etc. in the present notation.
- $V_{1-2}$  is defined as the voltage difference between  $V_{1-R}$  and  $V_{2-R}$ , as found in the thermocouple tables.
  - If  $T_1 > T_2$ ,  $V_{1-2}$  is **positive**, since thermocouple voltage increases with temperature.
  - If  $T_1 < T_2$ ,  $V_{1-2}$  is **negative**, since thermocouple voltage increases with temperature.
- As long as consistency with this notation is maintained, you should not encounter negative sign errors, regardless of whether  $T_1$  or  $T_2$  is greater. **To avoid sign errors,  $V_{1-2}$  must be consistently defined as  $V_{1-2} = V_{1-R} - V_{2-R}$ .**

○ From the thermocouple tables for a type J thermocouple,

- $V_{1-R}$  at  $160^\circ\text{C} = 8.560 \text{ mV}$
- $V_{2-R}$  at  $100^\circ\text{C} = 5.268 \text{ mV}$
- $V_{3-R}$  at  $20^\circ\text{C} = 1.019 \text{ mV}$

○ We calculate  $V_{1-2} = V_{1-R} - V_{2-R} = 8.560 \text{ mV} - 5.268 \text{ mV} = 3.292 \text{ mV}$ .  $V_{1-2} = 3.292 \text{ mV}$ .

○ Similarly,  $V_{1-3} = V_{1-R} - V_{3-R} = 8.560 \text{ mV} - 1.019 \text{ mV} = 7.541 \text{ mV}$ .  $V_{1-3} = 7.541 \text{ mV}$ .

○ Likewise,  $V_{2-3} = V_{2-R} - V_{3-R} = 5.268 \text{ mV} - 1.019 \text{ mV} = 4.249 \text{ mV}$ .  $V_{2-3} = 4.249 \text{ mV}$ .

**Discussion:** We verify the law of intermediate temperatures for this example set of three temperatures, namely,  $V_{1-3} = V_{1-2} + V_{2-3} = 3.292 \text{ mV} + 4.249 \text{ mV} = 7.541 \text{ mV}$ , which is the same value calculated above.

• **Example:**

**Given:** The same problem as above, but the labels are changed such that  $T_1 < T_2 < T_3$  rather than  $T_1 > T_2 > T_3$ :

Three constant temperature baths are prepared with  $T_3 = 160^\circ\text{C}$ ,  $T_2 = 100^\circ\text{C}$ , and  $T_1 = 20^\circ\text{C}$ . A type J thermocouple and a digital multimeter are used to measure voltages  $V_{1-2}$ ,  $V_{1-3}$ , and  $V_{2-3}$ .

**To do:** Predict the measured voltages  $V_{1-2}$ ,  $V_{1-3}$ , and  $V_{2-3}$ .

**Solution:**

○ The thermocouple voltages are read from the table as previously,

- $V_{3-R}$  at  $160^\circ\text{C} = 8.560 \text{ mV}$
- $V_{2-R}$  at  $100^\circ\text{C} = 5.268 \text{ mV}$
- $V_{1-R}$  at  $20^\circ\text{C} = 1.019 \text{ mV}$

○ We calculate  $V_{1-2} = V_{1-R} - V_{2-R} = 1.019 \text{ mV} - 5.268 \text{ mV} = -4.249 \text{ mV}$ .  $V_{1-2} = -4.249 \text{ mV}$ .

○ Similarly,  $V_{1-3} = V_{1-R} - V_{3-R} = 1.019 \text{ mV} - 8.560 \text{ mV} = -7.541 \text{ mV}$ .  $V_{1-3} = -7.541 \text{ mV}$ .

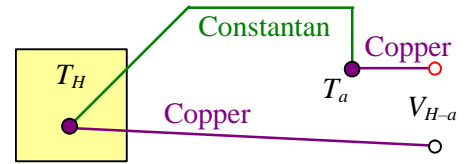
○ Likewise,  $V_{2-3} = V_{2-R} - V_{3-R} = 5.268 \text{ mV} - 8.560 \text{ mV} = -3.292 \text{ mV}$ .  $V_{2-3} = -3.292 \text{ mV}$ .

**Discussion:** We verify the law of intermediate temperatures for this example set of three temperatures, namely,  $V_{1-3} = V_{1-2} + V_{2-3} = -4.249 \text{ mV} - 3.292 \text{ mV} = -7.541 \text{ mV}$ , which is the same value calculated above. **The thermocouple laws work just as well with negative voltages as with positive voltages, provided that we use consistent notation to avoid sign errors.**

• **Example:**

**Given:**

- A simple home-made thermocouple is constructed from constantan and copper wires, and is used to measure some unknown hot temperature  $T_H$ , as sketched to the right.
- The ambient temperature in the room is known,  $T_a = 20^\circ\text{C}$ .
- The voltage reading is  $V_{H-a} = 2.644 \text{ mV}$ . *Note:* Since the reference junction here is *not* at the standard reference temperature of  $0^\circ\text{C}$ , but rather at the local ambient temperature  $T_a$ , the voltage  $V_{H-a}$  is relative to ambient temperature.



**To do:** Calculate  $T_H$ .

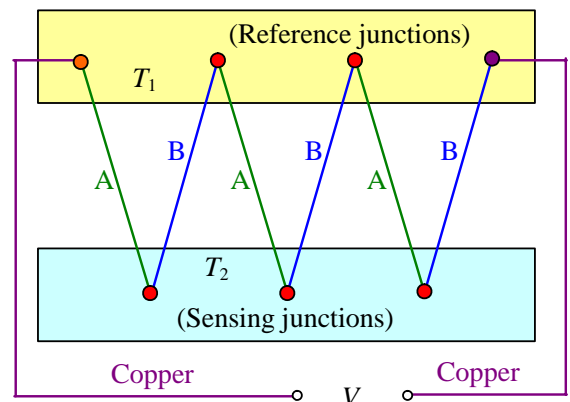
**Solution:**

- First, copper and constantan form a **type T** thermocouple.
- Here is what *not* to do (*incorrect* approach):
  - $V_{H-a} = 2.644 \text{ mV}$ .
  - From the tables for a type T thermocouple (interpolating),  $T_{H-R} = 64.0^\circ\text{C}$ .
  - Since the tabulated values are for a reference of  $0^\circ\text{C}$ , whereas the reference temperature here is  $T_a = 20^\circ\text{C}$ , it is tempting to set  $T_H = T_{H-R} + T_a = 64.0^\circ\text{C} + 20.0^\circ\text{C} = 84.0^\circ\text{C}$ . **This answer is not correct!**
  - The above approach is *incorrect* because the wrong reference temperature is used.
- Now here is the *correct* approach:
  - Let  $T_R = 0^\circ\text{C}$  (the standard reference temperature for thermocouples).
  - Apply the law of intermediate temperatures (using letters  $H$  and  $a$  instead of numbers 1 and 2). We write  $V_{H-a} = V_{H-R} - V_{a-R}$ , where
    - $V_{H-a}$  is the actual measured voltage (hot relative to ambient)
    - $V_{H-R}$  is the voltage that would be measured between the unknown hot temperature and  $0^\circ\text{C}$  if a standard reference junction had been applied.
    - $V_{a-R}$  is the voltage that would be measured between the ambient temperature and  $0^\circ\text{C}$  if a standard reference junction had been applied.
  - Note that  $V_{a-R}$  and  $V_{H-R}$  are the voltages actually listed in the thermocouple reference tables.
  - The above equation is rewritten as  $V_{H-R} = V_{H-a} + V_{a-R} = 2.644 \text{ mV} + 0.789 \text{ mV} = 3.433 \text{ mV}$ , where the value of  $V_{a-R}$  is found from the tables for a type T thermocouple at  $20^\circ\text{C}$ .
  - Now the thermocouple tables are used again to find the temperature  $T_H$  at which the voltage  $V_{H-R} = 3.433 \text{ mV}$ . Interpolation yields  $T_H = 81.6^\circ\text{C}$ , which is the correct answer.  $T_H = 81.6^\circ\text{C}$ .

**Discussion:** The incorrect method would work if thermocouple voltage were *linear*. Unfortunately, it is not, and we have to go through a bit more work to obtain the correct answer.

**Thermopile**

- A **thermopile** is **several thermocouples connected in series**.
- For example, a thermopile with three sensing junctions is shown to the right.
- As  $T_2$  is increased, the output voltage increases significantly.
- **The advantage of a thermopile (as compared to just one sensing junction) is increased sensitivity.**
- Here, the voltage output is three times that which is generated by just one thermocouple under otherwise identical conditions.
- With enough sensing junctions, a thermopile can actually generate a useful voltage. For example, **thermopiles are often used to control shut-off valves in furnaces.**



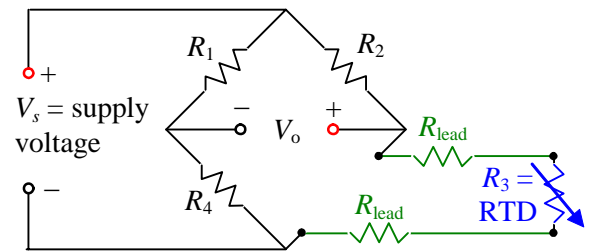
## Thermoresistive temperature measuring devices

- **Principle of operation:**

- A change in temperature causes the electrical resistance of a material to change.
- The resistance change is measured to infer the temperature change.
- There are two types of thermoresistive measuring devices: **resistance temperature detectors** and **thermistors**, both of which are described here.

- **Resistance temperature detectors**

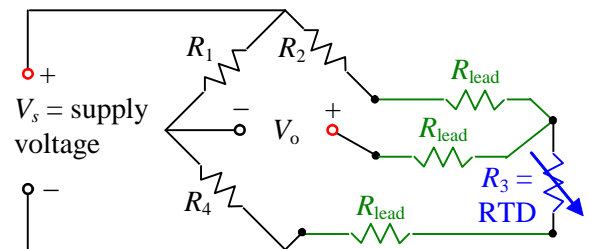
- A **resistance temperature detector** (abbreviated **RTD**) is basically either a long, small diameter metal wire wound in a coil or an etched grid on a substrate, much like a strain gage.
- In fact, some RTDs look very similar to strain gages!
- **The resistance of an RTD increases with increasing temperature**, just as the resistance of a strain gage increases with increasing strain.
- Platinum is a common metal used for RTDs, and **the resistance of the most common RTD is 100 Ω at 0°C**.
- If the temperature changes are large, or if precision is not critical, the RTD resistance can be measured directly to obtain the temperature.



- If the temperature changes are small, and/or high precision is needed, an electrical circuit is built to measure a change in *resistance* of the RTD, which is then used to calculate a change in temperature.
- One simple circuit is the quarter bridge Wheatstone bridge circuit, here called a **two-wire RTD bridge circuit**. It is basically identical to the quarter bridge circuit discussed previously for use with strain gage measurement, and is sketched to the right.

- $R_{\text{lead}}$  represents the resistance of one of the wires (called **lead wires**) that run from the bridge to the RTD itself. Lead resistance is of little concern in strain gage circuits because  $R_{\text{lead}}$  remains constant at all times, and we can simply adjust one of the other resistors to zero the bridge.
- For RTD circuits, however, some portions of the lead wires are exposed to changing temperatures. Since the resistance of metal wire changes with temperature,  $R_{\text{lead}}$  changes with  $T$ , and this can cause errors in the measurement. This error can be non-trivial – **changes in lead resistance may be misinterpreted as changes in RTD resistance, and therefore give a false temperature measurement**.

- Furthermore, there are *two* lead wires in our two-wire RTD bridge circuit – this doubles the error.



- A clever circuit designed to eliminate the lead wire resistance error is called a **three-wire RTD bridge circuit**, as sketched to the right.

- It is still a quarter bridge, since only one of the four bridge resistors has been replaced by the RTD. However, one of the lead wires has been placed on the  $R_2$  leg of the bridge instead of the  $R_3$  leg.

- To analyze this circuit, assume that  $R_1 = R_4$ , and  $R_2 = R_3$  initially, when the bridge is balanced.

- Recall the two equations for a Wheatstone bridge, as discussed in the strain gage learning module,

*Exact*

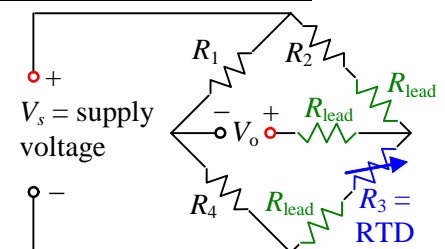
$$V_o = V_s \frac{R_3 R_1 - R_4 R_2}{(R_2 + R_3)(R_1 + R_4)}$$

*Approximate (assumes small  $\delta R$ )*

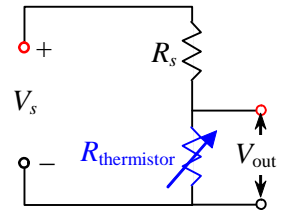
$$\frac{V_o}{V_s} \approx \frac{R_{2,\text{initial}} R_{3,\text{initial}}}{(R_{2,\text{initial}} + R_{3,\text{initial}})^2} \left( \frac{\delta R_1}{R_{1,\text{initial}}} - \frac{\delta R_2}{R_{2,\text{initial}}} + \frac{\delta R_3}{R_{3,\text{initial}}} - \frac{\delta R_4}{R_{4,\text{initial}}} \right)$$

- Notice that  $\delta R_3$  and  $\delta R_2$  have opposite signs in the above equation. So, if the lead wire resistance in leg 2 (top) and that in leg 3 (bottom) are the *same*, **the lead resistances cancel each other out**, with no net effect on the output voltage, thus eliminating the error.

- What about the third lead resistance,  $R_{\text{lead}}$  of the middle wire? Since  $V_o$  is measured using a device with nearly infinite input impedance, **no current flows in the middle lead wire**, so its resistance does not affect anything!



- An equivalent circuit is redrawn above, which may help explain why the lead resistances cancel out. It is clear that if  $R_{\text{lead}}$  changes equally in leg 2 and leg 3 of the bridge, its effect cancels out.
- *Note:* The second Wheatstone bridge equation above is *approximate* – it assumes that  $\delta R \ll R_{\text{initial}}$ . While this is often a good approximation for the lead wires, it is usually *not* a good approximation for the RTD itself, since  $R_{\text{RTD}}$  is a strong function of temperature, and  $\delta R_{\text{RTD}}$  is not small compared to  $R_{\text{RTD,initial}}$ .
- **Thermistors**
  - A **thermistor** is similar to an RTD, but a **semiconductor material** is used instead of a metal. A thermistor is a **solid state device**.
  - A thermistor has larger **sensitivity** than does an RTD, but **the resistance change with temperature is nonlinear**, and therefore temperature must be calibrated with respect to resistance.
  - Unlike RTDs, **the resistance of a thermistor decreases with increasing temperature**.
  - Thermistors are labeled by their resistance at 25°C. For example, two popular thermistors are **type 2252** (2252 Ω at 25°C) and **type 5000** (5000 Ω at 25°C).
  - The upper temperature limit of thermistors is typically lower than that of RTDs. In fact, the maximum temperature of operation is sometimes only 100°C to 200°C for a typical commercial thermistor.
  - A typical thermistor circuit is shown to the right – a simple **voltage divider**, where  $V_s$  is the supply voltage and  $R_s$  is a fixed (supply) resistor.  $R_s$  and  $V_s$  can be adjusted to obtain a desired range of output voltage  $V_{\text{out}}$  for a given range of temperature. If the proper value of  $R_s$  is used, the output voltage is *nearly* (but not exactly) linear with temperature.



### Radiative temperature measuring devices (radiative pyrometry)

- **Principle of operation:**
  - Radiative properties of an object change with temperature.
  - So, radiative properties are measured to infer the temperature of the object.
  - The advantages of radiative pyrometry are:
    - There is no physical contact with the object whose temperature is being measured.
    - Very high temperatures can be measured.
  - The fundamental equation for radiation from a body is the **Stefan-Boltzmann equation**,  $E = \epsilon\sigma T^4$ , where
    - $E$  is the **emissive power radiated per unit area** (units of  $\text{W}/\text{m}^2$ ).
    - $\epsilon$  is the **emissivity**, defined as the fraction of blackbody radiation emitted by an actual surface. Emissivity lies between 0 and 1, and is dimensionless. Its value depends greatly on the type of surface. **A blackbody has an emissivity of exactly 1.**
    - $\sigma$  is the **Stefan-Boltzmann constant**,

$$\sigma = 5.669 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$$

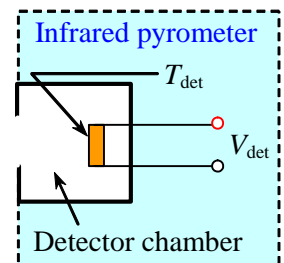
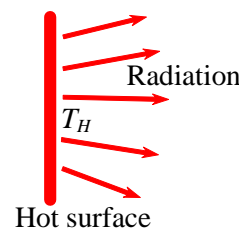
- $T$  is the **absolute temperature of the surface of the object** (units of K).
- The table to the right shows the emissivity of several common surfaces.
- The emissivity of other materials can be found in heat transfer textbooks.

Surface	Emissivity, $\epsilon$
aluminum (anodized)	0.84
aluminum (polished)	0.03
asphalt pavement	0.85 to 0.93
glass	0.62 to 0.95
human skin	approx. 0.95
water (deep)	0.95 to 0.96

- Two types of radiative measuring devices are discussed here: **infrared pyrometers** and **optical pyrometers**.

### **Infrared pyrometer**

- An **infrared pyrometer** infers the temperature of a hot surface by measuring the temperature of a detector inside a **detector chamber** as sketched to the right.
- The detector itself is usually a thermopile. It measures  $T_{\text{det}}$ , the temperature of the detector inside the chamber.
- $T_{\text{ind}}$  is the **indicated** temperature, which is calculated from  $T_{\text{det}}$ , from the known geometry and the radiation equations.  $T_{\text{ind}}$  is calibrated as a function of  $T_H$  for a body



of some assumed emissivity  $\epsilon_{\text{assumed}}$ , typically 0.95 or so.

- The instrument is set up such that  $T_{\text{ind}}$  is a function of the voltage output, and the display typically indicates temperature  $T_{\text{ind}}$  rather than voltage  $V_{\text{det}}$ .
- $T_{\text{ind}}$  can be thought of as an *uncorrected* estimate of  $T_H$ , since the emissivity of the object may not be the same as that assumed by the infrared pyrometer. In other words, if the actual emissivity of the object is not the same as the assumed emissivity,  $T_{\text{ind}}$  is incorrect.

- To correct for the actual emissivity of the object,  $T_H = \left( \frac{\epsilon_{\text{assumed}}}{\epsilon_{\text{actual}}} \right)^{1/4} T_{\text{ind}}$ .

- *Caution:* In all the above equations, **absolute temperatures** must be used!

- The widely used **medical ear thermometer** is a well known example of infrared pyrometry. Since the temperature range inside the ear is very narrow and the emissivity is nearly constant, such ear thermometers are typically quite accurate (less than 1% overall error in temperature).

- **Optical pyrometer**

- An **optical pyrometer** is useful for measuring very high temperatures (even flames).

- **An optical pyrometer works by comparing a glowing wire of known temperature to the glow (optical radiation) from a hot object.**

- When the internal wire and the glow of the object are the same color, the temperatures are assumed to be equal.

- The temperature of the internal wire is controlled and known, and thus the temperature of the object can be inferred.