

# ME 597A Thermodynamic Formalism of Dynamical Systems

## Take-Home Exam 01

**Problem 1:** The following ten equations are proposed to be fundamental equations of various thermodynamic systems. However, some of them are inconsistent with one or more postulates II, III, and IV, and consequently are not physically meaningful. In the expressions, given below,  $v_0$ ,  $\theta$ , and  $r$  are positive constants, and only the positive root is taken in all cases where fractional exponents appear.

(a) Find and justify those that are correct and also those that are not.

$$S = \left(\frac{R^2}{v_0\theta}\right)^{1/3} (NVU)^{1/3} \quad (1)$$

$$S = \left(\frac{R}{\theta^2}\right)^{1/3} \left(\frac{NU}{V}\right)^{2/3} \quad (2)$$

$$S = \left(\frac{R}{\theta}\right)^{1/2} \left(NU - \frac{R\theta V^2}{v_0^2}\right)^{1/2} \quad (3)$$

$$S = \left(\frac{R^2\theta}{v_0^3}\right) \left(\frac{V^3}{NU}\right) \quad (4)$$

$$S = \left(\frac{R^3}{v_0\theta^2}\right)^{1/5} (N^2VU^2)^{1/5} \quad (5)$$

$$S = (NR) \ln\left(\frac{UV}{N^2R\theta v_0}\right) \quad (6)$$

$$S = \left(\frac{R}{\theta}\right)^{1/2} (NU)^{1/2} \exp\left(\frac{-V^2}{2N^2v_0^2}\right) \quad (7)$$

$$S = \left(\frac{R}{\theta}\right)^{1/2} (NU)^{1/2} \exp\left(\frac{-UV}{NR\theta v_0}\right) \quad (8)$$

$$U = \left(\frac{v_0\theta}{R}\right) \left(\frac{S^2}{V}\right) \exp\left(\frac{S}{NR}\right) \quad (9)$$

$$U = \left(\frac{R\theta}{v_0}\right) (NV) \left(1.0 + \frac{S}{NR}\right) \exp\left(-\frac{S}{NR}\right) \quad (10)$$

(b) For each of the above physically acceptable fundamental equations, find  $U$  as a function of  $S$ ,  $V$ , and  $N$ .

**Problem 2:** Let a composite system consist of two isolated simple subsystems  $A$  and  $B$ , separated by a constraining internal wall, whose entropies are given as:

$$S_A = \left(\frac{R^2}{v_0\theta}\right)^{1/3} (N_A V_A U_A)^{1/3} \text{ and } S_B = \left(\frac{R^2}{v_0\theta}\right)^{1/3} (N_B V_B U_B)^{1/3}$$

After completely removing the constraint, let the unconstrained system be denoted as  $A\blacklozenge B$ .

- (a) Find  $U_{A\blacklozenge B}$  and  $S_{A\blacklozenge B}$ .
- (b) Is  $S_{A\blacklozenge B}$  greater than or equal to or less than  $(S_A + S_B)$ ? Is  $U_{A\blacklozenge B}$  greater than or equal to or less than  $(U_A + U_B)$ ? Justify your answer.

**Problem 3:** Let the internal wall between the subsystems  $A$  and  $B$  in Problem 2 be diathermal, rigid, and impermeable. In equilibrium, let  $V_A = 9$ ,  $N_A = 3$ ,  $V_B = 4$ , and  $N_B = 2$ ; let the total energy of the composite system  $A\blacklozenge B$  be 20.

- (a) What are internal energies of the subsystems  $A$  and  $B$  in equilibrium?
- (b) Assuming the energy transfer across the internal wall is quasi-static, plot the entropy  $S_A$  against the fraction  $\frac{U_A}{(U_A+U_B)}$  in the subsystem  $A$ .