

ME 597A Thermodynamic Formalism of Dynamical Systems

Take-Home Exam 02

Let us denote the following thermodynamic parameters as:

$$C_P = T \left(\frac{\partial s}{\partial T} \right)_P; \quad C_v = T \left(\frac{\partial s}{\partial T} \right)_v; \quad \alpha = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P; \quad \kappa_T = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T$$

The general ideal gas law states that $Pv = RT$, where R is the gas constant.

For all problems, stated below, let us assume that the mole number N is constant.

Problem 1: Show that

$$C_P = C_v + \frac{Tv\alpha^2}{\kappa_T}$$

Evaluate $C_P - C_v$ for general ideal gases.

Problem 2: The adiabatic bulk modulus is defined as $\beta_s = -v \left(\frac{\partial P}{\partial v} \right)_s$. Show that

$$\beta_s = \frac{C_P}{C_v \kappa_T}$$

Evaluate β_s for general ideal gases.

Problem 3: Show that, for isothermal compression (i.e., constant T),

$$ds = -\alpha v dP; \quad \text{and} \quad du = (-T\alpha + P\kappa_T)v dP$$

Evaluate the above relationships for general ideal gases.

Is this process quasi-static? Is this process reversible?

Problem 4: Show that, for free expansion (i.e., constant internal specific energy u),

$$\left(\frac{\partial T}{\partial v} \right)_u = \left(\frac{1}{C_v} \right) \left(P - \frac{T\alpha}{\kappa_T} \right)$$

Is this process $dT = \left(\frac{\partial T}{\partial v} \right)_u dv$ quasi-static? Is this process reversible?

Problem 5: Express the derivative $\left(\frac{\partial T}{\partial v} \right)_h$, where h is the specific enthalpy potential, in terms of c_v , α , and κ_T . Evaluate $\left(\frac{\partial T}{\partial v} \right)_h$ for general ideal gases.