# ME 597A Thermodynamic Formalism of Dynamical Systems

## Take-Home Exam 04

For the problems, given below, let us follow the notations used in the derivations of Schrödinger equation and Heisenberg uncertainty principle.

**Problem 1**: Let $A$ and $\alpha$ be positive real constants. Let a particle of mass $m$ be represented by the wave function

$$\Psi(x,t) = A\ exp\Big[-\alpha\Big(\frac{mx^2}{\grave{h}}\Big) + i\ t\Big]$$

(i) Find the normalization constant $A$ and an appropriate potential energy function $V(x,t)$ such that $\Psi$ satisfies Schrödinger equation.

(ii) Calculate the expectation values of $x$, $x^2$, $p$ and $p^2$.

(iii) Find $\sigma_x$ and $\sigma_p$. Is their product consistent with Heisenberg uncertainty principle?

**Problem 2**: Let a particle be represented, at time $t = 0$, by the wave function

$$\Psi(x,0) = \begin{cases} A(\alpha^2 - x^2) & \text{if} \quad -\alpha \leq x \leq +\alpha \\ 0 & \text{otherwise} \end{cases}$$

(i) Determine the normalization constant $A$.

(ii) Determine the expectation value of $x$, $x^2$, $p$ and $p^2$ at time $t = 0$. Is it possible in this problem to find expectation value of $p$ from $m\frac{d<x>}{dt}$?

(iii) Determine $\sigma_x$ and $\sigma_p$ at time $t = 0$. Is their product consistent with Heisenberg uncertainty principle?

**Problem 3**: Let $P_{ab}(t)$ be the probability of finding a particle in the range $x(t) \in (a,b)$ at time $t$. Let us define the probability current density $J$ as:

$$J(x,t) \triangleq \frac{i\grave{h}}{2m}\left(\Psi\frac{\partial\Psi^\star}{\partial x} - \Psi^\star\frac{\partial\Psi}{\partial x}\right)$$

(i) Show that

$$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t)$$

(ii) Find the probability current for the wave function in Problem 1.

**Problem 4**: Let $\Psi_1$ and $\Psi_2$ be two (normalizable) solutions to Schrödinger equation. Show that

$$\frac{d}{dt}\int_{\mathbb{R}} dx\ \Psi_1^\star\ \Psi_2 = 0$$