Integrated Communication and Control Systems: Part I—Analysis[1]

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Computer networking is a reliable and efficient means for communications between disparate and distributed components in complex dynamical processes like advanced aircraft, spacecraft, and autonomous manufacturing plants. The role of Integrated Communication and Control Systems (ICCS) is to coordinate and perform inter-related functions, ranging from real-time multi-loop control to information display and routine maintenance support. In ICCS, a feedback control loop is closed via the common communication channel which multiplexes digital data from the sensor to the controller and from the controller to the actuator along with the data traffic from other loops and management functions. Due to the asynchronous time-division multiplexing of the network protocol, time-varying and possibly stochastic delays are introduced in the control system, which degrade the system dynamic performance and are a source of potential instability. The paper is divided into two parts. In the first part, the delayed control system is represented by a finite-dimensional, time-varying, discrete-time model which is less complex than the existing continuous-time models for time-varying delays; this approach allows for simpler schemes for analysis and simulation of ICCS. The second part of the paper addresses ICCS design considerations and presents simulation results for certain operational scenarios of ICCS.

1 Introduction

Complex dynamical processes like advanced aircraft, spacecraft, and autonomous manufacturing plants require high-speed, reliable communications between system components which perform a set of inter-related functions ranging from active control to information display and routine maintenance support [1]. The system components include a number of computers, intelligent terminals, sensors and actuators, and their functions are executed in real time. The activities of system components can be coordinated by appropriate information exchange via a multiplexed communication network to achieve a better utilization of the resources. However, the network introduces delays in addition to the sampling delay that is prevalent in all digital control systems. The network-induced delays are time-varying and possibly stochastic, and are dependent on the intensity, probability distribution, dynamics of the traffic as well as on mis-synchronization between control system components and noise in the communication medium. The Integrated Communication and Control Systems (ICCS) for these processes must be designed to compensate for these delays. The schematic diagram of an ICCS network in Fig. 1 illustrates how these delays are introduced.

In ICCS, a control loop is closed via the common communication channel which multiplexes digital data from the sensor to the controller and from the controller to the actuator along with the data traffic from other control loops and management functions. Furthermore, the control system components (e.g., the sensor and controller) may not be synchronized. Figure 2 illustrates how the network-induced varying delays Θ_{sc} and Θ_{ca} enter the control system.

In the continuous-time, the control law for a given plant model is derived as a transfer function or using a state-space realization. In the discrete-time, an additional parameter of importance is the sampling time T of the control system [2]. In ICCS, T is essentially the sensor message inter-arrival time and can be considered as a common parameter of the control system and the communication network. From a point of view of control systems design, smaller values of T (with the exception of sensitivity to round-off errors in the controller computer) are desirable as the discrete-time control law more closely approximates its continuous-time design. On the other hand, a smaller T, i.e., a higher sampling frequency, implies a larger network traffic, for a given data transmission rate, which in turn increases the data latency.

Analysis and design of ICCS require interactions between the disciplines of communication systems and control systems engineering. It may be appropriate to bring out the notions of *delay* as it is used in the two disciplines in somewhat different manners. In communication systems, the delay is primarily referred to as *queueing delay* and *data latency* which are associated with only those messages that successfully arrive at the destination terminal [3, 4]; messages that are corrupted by

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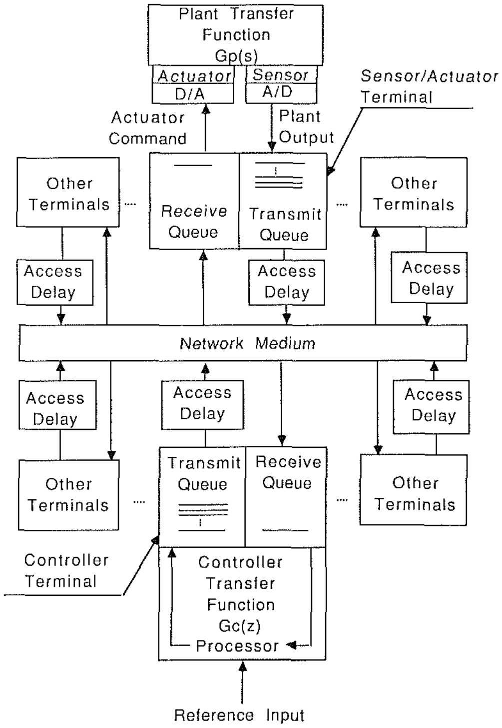

Fig. 1 Schematic diagram for the ICCS

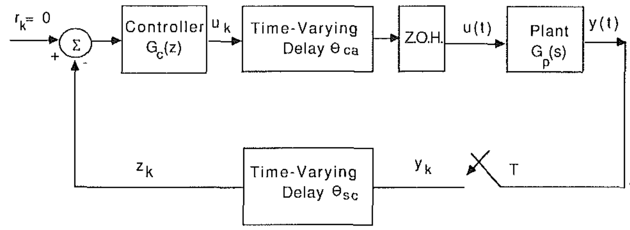

Fig. 2 Delayed control system

noise or deleted due to queue saturation at the transmitter buffer of the source terminal are not considered for this purpose. In control systems, the delay is related to the question: *How old is the data which is currently used?*. When no messages are rejected, the two notions of delay are similar; otherwise they are different.

Although ample research papers in modeling and simulation of communication protocols have been published [5], the significance of network-induced delays relative to the stability of feedback control systems has not been apparently addressed except in a few cases [3, 4].

In our earlier work [4], the basis for selection of the SAE linear token passing bus [6] as the medium access protocol for ICCS networks has been reported along with the simulation results for its performance analysis. We have shown that the bus traffic in an ICCS network is subject to time-varying data latency of messages resulting from asynchronous time-division multiplexing in the communication network. The detrimental effects of data latency on the dynamic performance of ICCS are further aggravated due to possible mis-synchronization between terminals in the control loop as well as to loss of messages resulting from saturation of buffers and noise corruption in the network medium.

The paper is the first of two parts, and presents the results for ICCS analysis focusing on discrete-time control systems which are subject to time-varying delays. The analytical technique developed here is applicable to integrated dynamical systems such as those encountered in advanced aircraft, spacecraft, and real-time control of robots and machine tools via a high-speed network within an autonomous manufacturing environment.

This paper is organized in five sections and one appendix. The current status of research for analysis of time-delayed control systems is summarized in Section 2. The significance of data latency and mis-synchronization between individual system components in ICCS networks is discussed in Section 3 in view of the time-varying delays. A finite-dimensional, discrete-time model for analyzing linear time-invariant feedback control systems with distributed and time-varying delays is derived in Section 4. The summary and conclusions of this paper along with recommendations for future work are presented in Section 5. Appendix A contains a supporting proposition.

Part II is presented as a companion paper [7] which addresses system design considerations. These include the physical significance of the network delay parameters in ICCS design and some of the simulation results.

2 Research Status for Analysis of Delayed Control Systems

Several approaches for analyzing the dynamic performance and stability of delayed control systems have been suggested [8–16]. Most of the literature on delayed systems deals with the case of constant delays, but some results concerning time-varying delays were presented, e.g., Yorke [8], Hirai and Satoh [9], Ikeda and Ashida [10], and Belle Isle [11, 12]. For a network with randomly distributed traffic, the delay in the ICCS loop could be a stochastic process. The use of stochastic Lyapunov functions for stability analysis of systems with randomly varying delays has been suggested by Belle Isle.

The network-induced delays in an ICCS feedback loop are sensor-controller delay and controller-actuator delay as shown in Figs. 1 and 2. Since both these delays are time-varying, they may not be lumped together, in general. However, since the digital control algorithm is time-invariant, the two delays could be lumped together if no message rejection/vacant sampling (defined later in Section 3) occurs. A statement of this property and its proof are given as Proposition A.1 in Appendix A.

Lumping the two delays, Θ_{sc} and Θ_{ca} in Fig. 2, does not necessarily solve the problem of control system design as the lumped delay could still be a time-varying quantity. This makes the system analysis and design difficult because the specifications of feedback control systems are usually given in terms of phase margin and gain margin in the frequency-domain or in terms of smallest decay rate, overshoot, rise time, settling time, etc. in the time-domain. These specifications, the frequency domain data in particular, are stated in view of linear time-invariant systems. Approaches for solving time-varying delay problems are discussed below.

Given a linear finite-dimensional time-invariant system with a lumped time-varying delay placed between the controller and actuator, the closed loop digital control system is approximately represented in continuous-time as

$$d\mathbf{x}(t)/dt = \mathbf{A}\mathbf{x}(t) - \mathbf{B}\mathbf{x}(t - L(t)) \tag{2.1}$$

where $\mathbf{x}$ is the $(n \times 1)$ state vector,

$\mathbf{A}$ and $\mathbf{B}$ are $(n \times n)$ constant matrices, and

$L(t) \geq 0$ for all $t > 0$.

A first order system is considered to illustrate how the complexity of dynamic performance analysis increases with time-varying delays.

$$dx(t)/dt = ax(t) - bx(t-L)(t)) \tag{2.2}$$

where x is a scalar, $a \leq 0$, $b > 0$, and L is a continuous function of t such that $\sup L(t) = q$ and $\inf L(t) \geq 0$.

A sufficient condition for uniformly asymptotic stability of (2.2) has been shown by Yorke [8] to be $q < (1.5)/b$ provided that $a = 0$, i.e., the plant is represented by a pure integrator. It is interesting to note that if $L(t) = q$ for all t, then the above condition can be relaxed to the necessary and sufficient condition of $q < \pi/(2b)$ by use of Nyquist stability criterion. This suggests that replacing $L(t)$ by its supremum may not be the solution. Similar results have been derived by Hirai and Satoh [9] using a different approach.

An overview of methods for analyzing delayed control systems is presented below.

Stochastic Lyapunov Function. The underlying principle relies upon the well known Lyapunov method [13], except for that the definitions of norms are modified to accommodate the time-varying delay argument and the differential operators are defined in a stochastic setting. Bell Isle and Kozin [11, 12] used the concept of a stochastic Lyapunov functional to derive a sufficient condition for almost-sure sample stability of randomly varying delayed systems.

D-Partition Method. It gives the region(s) in the parameter space within which the system is stable. Given a system $dx(t)/dx = ax(t) - bx(t-q)$, the method defines region(s) in the a, b plane within which the system is stable for a given constant delay q. Rekasius [14] adopted this concept to identify stable regions for systems with the constant delay q or as a function of q. Except for the traditional graphical approaches like Nyquist's [17] and Mikhailov's [18], the algebraic approach presented by Rekasius is one of the few methods that gives necessary and sufficient conditions for stability of control systems with constant delays. This method has a potential for application to ICCS design if an equivalent constant-delay system could be identified to replace the time-varying delays.

Method of Steps. In this approach adopted by Hirai and Satoh [9], the time-varying delay is assumed to be of the form: $L(t) = t - nT$ for $nT < t \leq (n+1)T$. Consequently, the term with time-varying delay, $t - L(t)$, in the system equation remains a constant within each time period $(nT, (n+1)T]$, i.e., $\mathbf{x}(t - L(t)) = x(nT)$. Thus the output at the time instant $(n+1)T$ can be recursively expressed in terms of the output at the time instant nT, $x(nT)$ in the form $\mathbf{x}((n+1)T) = \mathbf{F}(.)\,\mathbf{x}(nT)$, where $\mathbf{F}(.)$ is a function of the sampling time and system parameters. The necessary and sufficient condition for the stability of a scalar system is $|F| < 1$.

A common drawback in some of the design methodologies for time-delayed systems, Bell Isle [11, 12] and Mori et al. [15, 16] for example, is that the stability criterion does not involve the exact magnitude of the delay, and its functional characteristics and constraints in the case of time-varying delays. Such results are very conservative since stabilty is guaranteed for a wide range of delays. As a result, many of these criteria have very limited practical significance.

We propose a discrete-time approach for analyzing delayed systems following the concept of the method of steps. If the plant and controller are time-invariant and the control inputs are piecewise constants, the system can be represented by an augmented state vector which consists of past values of the plant input and output in addition to the current state vectors of the plant and controller. Thus the problem of time-varying delays can be treated by a *finite-dimensional* time-varying discrete-time model where the delays are not restricted to be integer multiples of a given time period. From the perspectives of ICCS analysis and design, the proposed method has the following advantages.

- The two time-varying delays in Fig. 2 can be treated separately, i.e., unlike the other methods, these delays are not required to be lumped. Therefore, the proposed method is not limited by the restrictions stated in Proposition A.1 of Appendix A.
- The delays can be a discrete function of time or a discretely sampled sequence of a continuous-time function.

3 Significance of Delays in ICCS Networks

In Section 1 the structure and operating principles of an ICCS network are briefly described. The characteristics of the actual network protocol are not considered and no specific structures for the delays are assumed since these issues have been addressed in our earlier publications [3, 4]. In this section we present how time-varying delays could occur in an ICCS network. To better explain the physical significance of network-induced delays, we make the following assumptions.

1 Traffic is periodic with constant message lengths, i.e., queueing delays and data latencies of all messages are deterministic (but time-dependent).

2 Message lengths for sensor and control signals are identical. This signifies that sensor to controller and controller to actuator data latencies have identical characteristics. Let the infimum and supremum of these data latencies be $\delta_{\min}$ and $\delta_{\max}$, respectively.

3 The sampling intervals of the sensor and controller are identical and equal to T.

4 The time skew Δ_s between the sampling instants of the sensor and controller is a constant ($\Delta_s \in [0, T]$) over a finite time window.

5 The control signal processing delay δ_p is a constant and $\delta_p < T$.

6 The network is not overloaded and the communication medium is error-free, i.e., no message is lost due to saturation of the transmitter buffer at any terminal or by noise contamination. This also implies that $\delta_{\max} < T$.

7 The capacity of the receiver queue at the controller is one. This assumption is consistent with the usual ICCS design practice [3]. (If an observer is used to estimate the delayed sensor data, the queue capacity may have to be increased.)

Let $\mathbf{y}_j$ be the sensor data, generated at the jth sample of the sensor, be immediately stored at the sensor terminal's transmitter buffer at the time $\tau_0 + jT$, and wait to be transmitted as a message via the network medium. Upon transmission, the sensor data is received at the controller at $\tau_1^j + jT$. The sensor data has to wait at the receiver buffer until the next sampling instant of the controller at $\tau_2 + jT$ if $\tau_1^j < \tau_2$ or at $\tau_2 + (j+1)T$ if $\tau_1^j \geq \tau_2$. As the processing is completed at $\tau_3 + jT$, the control signal is put in the controller's transmitter buffer where it waits to be transmitted to the actuator terminal. Finally, at $\tau_4^j + jT$, the control signal arrives at the actuator terminal and immediately acts upon the plant. Referring to the timing diagram for message transactions in Fig. 3, the delays are interpreted as follows.

- Sensor to controller data latency $\delta_{sc}^j = \tau_1^j - \tau_0$
- The interval $\tau_2 - \tau_0$ is the time skew Δ_s between the sensor and controller sampling instants.
- Control signal processing delay $\delta_p = \tau_3 - \tau_2$
- Controller to actuator data latency $\delta_{ca}^j = \tau_4^j - \tau_3$
- Controller to actuator delay $\Theta_{ca}^j = \tau_4^j - \tau_2$, which is essentially the sum of the processing delay δ_p and controller to actuator data latency δ_{ca}^j.

Let $\mathbf{z}_j$ be the delayed sensor data which is used by the controller at its jth sample at time $\tau_2 + jT$. Then the sequences $\{\mathbf{z}_j\}$ and $\{\mathbf{y}_j\}$ are related as

$$\mathbf{z}_j = \mathbf{y}_{j-p(j)} \tag{3.1}$$

where $p(j)$ is a non-negative integer with an upper bound $\mathfrak{p}$. Now we introduce the concepts of the delays involving the data latency and time skew.

Definition 3.1. The modified sensor-to-controller data latency $\nabla_{sc}{}^j$ for a sensor message $\mathbf{y}_j$ at the jth *sensor* sample is defined as the interval between the instant of the sensor data generation and the instant when the controller starts processing these data or it would have been processed if not replaced by any fresh sensor data.

Therefore, $\nabla_{sc}{}^j$ is given by the following relationship in terms of the sensor-controller data latency $\delta_{sc}{}^j$ and time skew Δ_s

$$\nabla_{sc}{}^j = kT + \Delta_s \text{ for } (k-1)T + \Delta_s \le \delta_{sc}{}^j < kT + \Delta_s \qquad (3.2)$$

where k is a non-negative integer and, under the assumptions 1 and 6 stated earlier, k is either 0 or 1.

Lemma 3.1. $p(j) = 0$ or 1 $\forall j$ under the assumptions 1 to 6 stated earlier.

Proof. Since $\nabla_{sc}{}^j$ is time-varying, $0 < \delta_{\min} \le \delta_{sc}{}^j \le \delta_{\max} < T$ $\forall j$ and $0 \le \Delta_s < T$, we have only two possible conditions: (i) $\nabla_{sc}{}^j < \Delta_s$ implying $\mathbf{z}_j = \mathbf{y}_j$ and (ii) $\nabla_{sc}{}^j \ge \Delta_s$ implying that $\mathbf{z}_j = \mathbf{y}_{j-1}$.

Remark 3.1. If $\tau_1{}^j < \tau_2$, i.e., $p(j) = 0$, then $\nabla_{sc}{}^j = \Delta_s$; if $\tau_1{}^j \ge \tau_2$, i.e., $p(j) = 1$, then $\nabla_{sc}{}^j = T + \Delta_s$.

Remark 3.2. The number ν^j, of sensor message arrivals at the controller during its jth sampling period, has an expected value of 1 since the sensor and controller sampling intervals are identical. ν^j assumes exactly one of the values: 0, 1, and 2.

Remark 3.3. $\nu^j = 0$: If $\nabla_{sc}{}^j < \Delta_s$ and $\nabla_{sc}{}^{j+1} \ge \Delta_s$, then $\mathbf{z}_{j+1} = \mathbf{z}_j = \mathbf{y}_j$. This implies that no fresh sensor message arrives at the controller during its jth sampling period, and the old sensor data is used at the $(j+1)$st sampling instant for computing the control signal. This phenomenon is called *vacant sampling* as illustrated in Fig. 4.

Remark 3.4. $\nu^j = 1$: If one of the two conditions occur: (i) $\nabla_{sc}{}^j \ge \Delta_s$ and $\nabla_{sc}{}^{j+1} \ge \Delta_s$ implying that $\mathbf{z}_{j+1} = \mathbf{y}_j$, or (ii) $\nabla_{sc}{}^j < \Delta_s$ and $\nabla_{sc}{}^{j+1} < \Delta_s$ implying that $\mathbf{z}_{j+1} = \mathbf{y}_{j+1}$, then exactly one sensor message arrives at the controller during its jth sampling period, and this message is used to compute the control signal at the $(j+1)$st sampling instant.

Remark 3.5. $\nu^j = 2$: If $\nabla_{sc}{}^j \ge \Delta_s$ and $\nabla_{sc}{}^{j+1} < \Delta_s$, then $\mathbf{z}_{j+1} = \mathbf{y}_{j+1}$. This implies that two sensor messages arrive at the controller during its jth sampling period. The former arrival is discarded and the latter arrival is used for computing the control signal. This phenomenon is called *message rejection* at the controller's receiver as illustrated in Fig. 4.

Definition 3.6. The sensor-controller delay $\Theta_{sc}{}^j$ for the delayed sensor message $\mathbf{z}_j + \mathbf{y}_{j-p(j)}$ is the time interval between the $(j - p(j))$th sampling instant of the sensor and the jth sampling instant of the controller (when $\mathbf{z}_j$ is processed). Therefore, $\Theta_{sc}{}^j$ can be expressed as

$$\Theta_{sc}{}^j = p(j)T + \Delta_s \qquad (3.3)$$

Remark 3.7. The condition $p(j+1) \le p(j) + 1$ $\forall j$ is independent of the stated assumptions. The rationale is that the sensor data, available to the controller at one instant, is also available at the successive instants unless replaced by a fresh data.

Proposition 3.1. Under the assumptions 1 to 6, stated earlier, the sequences $\{\nabla_{sc}{}^j\}$ and $\{\Theta_{sc}{}^j\}$ are identical, i.e., $\nabla_{sc}{}^j = \Theta_{sc}{}^j$ $\forall j$.

Proof. Using Lemma 3.1 and making use of Remarks 3.3, 3.4, and 3.5, we need to consider only two possible cases: (i) $\mathbf{z}_j = \mathbf{y}_j$ implying $\nabla_{sc}{}^j = \Theta_{sc}{}^j = \Delta_s$ and (ii) $\mathbf{z}_{j+1} = \mathbf{y}_j$ implying $\nabla_{sc}{}^j = \Theta_{sc}{}^j = T + \Delta_s$.

The importance of the above proposition is that the control system delay $\Theta_{sc}{}^j$ in Fig. 2 is equal to the communication system delay parameter $\nabla_{sc}{}^j$ which can be readily obtained from (3.2). $\nabla_{sc}{}^j$ can be calculated on the basis of the network traffic characteristics which could be deterministic or random, and Δ_s can be approximately maintained at a desired constant value by periodically broadcasting synchronization signals via the network medium.

Remark 3.8. $\nabla_{sc}{}^j$ and $\Theta_{sc}{}^j$ refer to the same sensor message only when $\nabla_{sc}{}^j = \Delta_s$.

Using the above physical concepts of the network-induced delays, we develop a discrete-time, finite-dimensional, time-varying model of the delayed control system as proposed in Section 2. The delayed system model, to be presented next, is generic and is not restricted to the assumptions that were made in the beginning of this section for physical explanation of the delay phenomena.

4 Development of a Delayed Control System Model

We consider the control system in Fig. 2 where $\mathbf{G}_p(s)$ and $\mathbf{G}_c(z)$ are linear, finite-dimensional, time-invariant models of the continuous-time plant and discrete-time controller, respectively. In Section 3, we described how the sequence $\{\mathbf{z}_k\}$ of the delayed plant output at the controller sampling instants is obtained from the corresponding sequence $\{\mathbf{y}_k\}$ of the measured plant output at the sensor sampling instants. Since the delay Θ_{sc}, in Fig. 2, is time-varying, the controller may use the sensor data generated at the current or earlier samples. The delay Θ_{ca} to which the control input sequence $\{\mathbf{u}_k\}$ is subjected is also time-varying. This implies that even if the controller generates the commands at a constant rate, the interval between their successive arrivals at the actuator terminal may not be a constant. In contrast with the sensor data which may wait at the controller's receiver queue before being processed by the controller, the control input acts upon the plant immediately after arriving at the actuator terminal. This happens because the controller is scheduled to generate signals at constant intervals whereas the actuator operations are essentially asynchronous. The characteristics of the delay Θ_{sc} are different from those of Θ_{ca} in the case where two or more sensor data arrive at the controller during one of its sampling intervals. Since the controller operates in discrete time, unlike the actuator which is essentially a continuous-time device, only the most recent sensor data is accepted. We have referred to this phenomenon as *message rejection* at the controller's

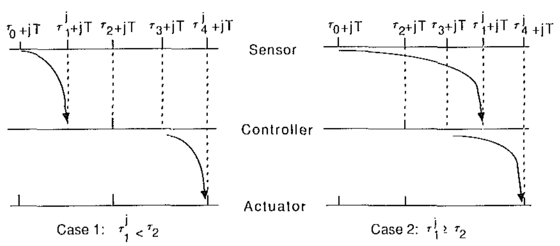

Fig. 3 Timing diagram for message transactions

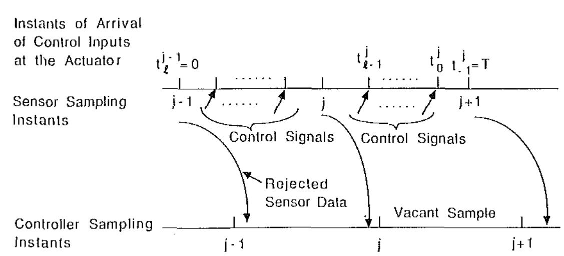

Fig. 4 Timing diagram for sensor and control signals

receiver in Section 3. On the other hand, if no fresh sensor data are available during a sampling interval, the previous sensor data is used to compute the control input, i.e., $\mathbf{z}_{k+1} = \mathbf{z}_k$. We have referred to this phenomenon as *vacant sampling* at the controller. Having this scenario in mind, we present a formal definition of the problem.

The linear, finite-dimensional, time-invariant model $\mathbf{G}_p(s)$ of the plant in Fig. 2 is given in the standard state-variable form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{4.1}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{4.2}$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^m$, $\mathbf{y} \in \mathbf{R}^r$, and the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are of compatible dimensions.

Let the system be sampled every T units of time, and let the input $\mathbf{u}(t)$ to the plant be piecewise constant during each sampling interval. More specifically, let $\mathbf{u}(t)$ assume at most $(l + 1)$ different values in the interval $[kT, (k+1)T)$ where the changes occur at the instants $kT + t_i^k$, $i = 0, 1, 2, \ldots, l$ such that $t_i^k \geq t_{i+1}^k$ with $t_l^k = 0$ and $t_0^k \leq T$. The superscript k indicates that the instants t_i^k may vary from one sampling interval to another.

The solution of the state equation (4.1) is given below.

$$\mathbf{x}((k+1)T) = \exp(\mathbf{A}T)\mathbf{x}(kT) + \int_0^T \exp[\mathbf{A}(T-\tau)]\mathbf{B}\mathbf{u}(\tau)d\tau \tag{4.3}$$

Using the fact that the input is piecewise constant, we have:

$$\mathbf{x}_{k+1} = \mathbf{A}_s\mathbf{x}_k + \sum_{i=0}^{l} \mathbf{B}_i^k \mathbf{u}_{k-i} \tag{4.4}$$

where $\mathbf{x}_k = \mathbf{x}(kT)$, $\mathbf{A}_s = \exp(\mathbf{A}T)$, and $\mathbf{u}_{k-i} = \mathbf{u}(t)$, $t \in [t_i^k, t_{i-1}^k)$ as illustrated in Fig. 4, and

$$\mathbf{B}_i^k = \int_{t_i^k}^{t_{i-1}^k} \exp[\mathbf{A}(T-\tau)]\mathbf{B}d\tau \text{ and } t_{-1}^k = T,\ t_l^k = 0. \tag{4.5}$$

Remark 4.1. Some of the t_i^k's should be set equal to zero and others to T if the actual number of different inputs acting on the plant during one sampling interval is less than $(l + 1)$.

Remark 4.2. The following relation holds regardless of the value of t_i^k:

$$\sum_{i=0}^{l} \mathbf{B}_i^k = \int_0^T \exp(\mathbf{A}\tau)\mathbf{B}d\tau = \text{a constant} \tag{4.6}$$

The right-hand side of (4.6) is the input matrix in case only one constant control input is applied throughout the sampling interval.

Remark 4.3. The sequence $\{t_i^k\}$ satisfies:

$$t_i^{k+1} = 0 \text{ if } t_j^k < T \text{ for } \forall\, i \geq (j+1).$$

To see this, first note that the input $\mathbf{u}_{k-i}$ is the same as $\mathbf{u}_{(k+1)-(i+1)}$. If this input arrives at the actuator before the $(k + 1)$st sensor sampling instant, then all previous inputs have arrived before the $(k + 1)$st instant as well, and their relative arrival time t_i^{k+1} in $[(k+1)T, (k+2)T)$ is zero.

Setting the reference signal $\mathbf{r}_k$ in Fig. 2 to zero, the discrete-time, linear, time-invariant model $\mathbf{G}_c(z)$ of the controller is given in the state-invariable form as:

$$\eta_{k+1} = \mathbf{F}\eta_k - \mathbf{G}\mathbf{z}_k \tag{4.7}$$

$$\mathbf{u}_k = \mathbf{H}\eta_k - \mathbf{J}\mathbf{z}_k \tag{4.8}$$

where $\eta \in \mathbf{R}^q$, $\mathbf{z}_k$ is the last available measurement at the instant when $\mathbf{u}_k$ is processed by the controller, and the matrices $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$, and $\mathbf{J}$ are of compatible dimensions.

We combine (4.4), (4.7), and (4.8) with the expression $\mathbf{z}_j = \mathbf{y}_{j-p(j)}$ for the delayed sensor data in (3.1). Any finite amount of delay is satisfied by having $p(k) = 1, 2, \ldots, \mathrm{p}$ where p is the maximum of $p(k)$ for all k. We obtain an augmented state representation as follows.

$$\mathbf{x}_{k+1} = (\mathbf{A}_s - \mathbf{B}_0^k\mathbf{J}\Upsilon_0^k\mathbf{C})\mathbf{x}_k - \mathbf{B}_0^k\mathbf{J}\sum_{i=1}^{\mathrm{p}} \Upsilon_i^k \mathbf{y}_{k-i} + \mathbf{B}_0^k\mathbf{H}\eta_k + \sum_{j=1}^{l} \mathbf{B}_j^k \mathbf{u}_{k-j}$$

$$\text{where } \Upsilon_i^k = \begin{cases} \mathbf{I}_r & \text{if } i = p(k) \\ 0 & \text{if } i \neq p(k) \end{cases}$$

$$\begin{aligned} &\mathbf{y}_k = \mathbf{C}\mathbf{x}_k \\ &\mathbf{y}_{k-i} = \mathbf{I}_r\mathbf{y}_{k-i},\ i = 1, 2, \ldots, (\mathrm{p}-1), \\ &\eta_{k+1} = -\mathbf{G}\Upsilon_0^k\mathbf{C}\mathbf{x}_k - \sum_{i=1}^{\mathrm{p}} \Upsilon_i^k\mathbf{y}_{k-i} + \mathbf{F}\eta_k \\ &\mathbf{u}_k = -\mathbf{J}\Upsilon_0^k\mathbf{C}\mathbf{x}_k - \mathbf{J}\sum_{i=1}^{\mathrm{p}} \Upsilon_i^k\mathbf{y}_{k-i} + \mathbf{H}\eta_k \\ &\mathbf{u}_{k-i} = \mathbf{I}_m\mathbf{u}_{k-i},\ i = 1, 2, \ldots, (l-1). \end{aligned} \tag{4.9}$$

Equation (4.9) is written in a matrix form as:

$$\mathbf{X}_{k+1} = \Phi_k\mathbf{X}_k \tag{4.10}$$

where $\mathbf{X}_k = [\mathbf{x}_k^T\ \mathbf{y}_{k-1}^T \cdots \mathbf{y}_{k-\mathrm{p}}^T\ \eta_k^T\ \mathbf{u}_{k-1}^T\ \mathbf{u}_{k-l}^T]^T$ is the augmented state vector of dimension $N = n + \mathrm{p}r + q + ml$, and the $(N \times N)$ augmented state transition matrix is given as:

$$\Phi_k = \begin{bmatrix} \mathbf{A}_s & 0 \ldots & & 0 & \mathbf{B}_0^k\mathbf{H} & \mathbf{B}_1^k \ldots & \mathbf{B}_l^k \\ \mathbf{C} & 0 & & & & & 0 \\ 0 & \mathbf{I}_r & & & & & 0 \\ \cdot & \cdot & & & & & \cdot \\ 0 & & \mathbf{I}_r & 0 & & & \cdot \\ 0 & & & 0 & \mathbf{F} & 0 & 0 \\ 0 & & & 0 & \mathbf{H} & 0 & 0 \\ 0 & & & & 0 & \mathbf{I}_m & 0 \\ \cdot & & & & & & \cdot \\ 0 & & & & & 0 & \mathbf{I}_m 0 \end{bmatrix} - \begin{bmatrix} \mathbf{B}_0^k\mathbf{J} \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \mathbf{G} \\ \mathbf{J} \\ 0 \\ \cdot \\ 0 \end{bmatrix} [\Upsilon_0^k\mathbf{C}\ \Upsilon_1^k \ldots \Upsilon_{\mathrm{p}}^k\ 0 \ldots 0] \tag{4.11}$$

Since only one of the Υ_i^k's is nonzero, (4.11) implies that only one matrix column is added to the first term on the right-hand side.

Remark 4.4. In the realization (4.9), the sensor outputs $\mathbf{y}_i$'s can always be replaced by earlier control inputs $\mathbf{u}_i$'s. To see that (4.2) and (4.4) are combined as:

$$\mathbf{y}_{k-1} = \mathbf{C}\mathbf{A}_s^{-1}\left(\mathbf{x}_k - \sum_{i=0}^{l} \mathbf{B}_i^{k-1}\mathbf{u}_{k-i-1}\right) \tag{4.12}$$

which means that $\mathbf{u}_{k-l-1}$ can replace $\mathbf{y}_{k-1}$ as part of the augmented state vector $\mathbf{X}_k$. In that case, the matrices $\mathbf{B}_i^{k-1}$ must be stored as well. Following the same argument, $\mathbf{y}_{k-1}, \ldots, \mathbf{y}_{k-\wp}$ can be replaced by $\mathbf{u}_{k-l-1}, \ldots, \mathbf{u}_{k-l-\wp}$ if the matrices $\mathbf{B}_i^{k-j}$, $j = 1, 2, \ldots, \wp$ are stored.

Similarly, if $\mathbf{F}$ is nonsingular (Note: A_s is always nonsingular), $\mathbf{u}_i$'s may be replaced by earlier $\mathbf{y}_i$'s. To see that (3.1), (4.7) and (4.8) are combined as:

$$\mathbf{u}_{k-1} = \mathbf{H}\mathbf{F}^{-1}(\eta_k + \mathbf{G}\mathbf{y}_{k-1-p(k-1)}) - \mathbf{J}\mathbf{y}_{k-1-p(k-1)} \tag{4.13}$$

and $\mathbf{u}_{k-1}, \ldots, \mathbf{u}_{k-l}$ can be replaced by $\mathbf{y}_{k-\wp-1}, \ldots, \mathbf{y}_{k-l-\wp}$ if the values of $p(k-j)$, $j = 1, 2, \ldots, l$ are stored. The realization (4.10) is minimal if $r = m$, i.e., the number of inputs and outputs are identical. The minimum dimension of the system, assuming $\mathbf{F}$ to be nonsingular, is $n + q + (\wp + l)\psi$ where $\psi = \min(r, m)$. But, for $r \neq m$, this minimal realization would require storage of time-varying parameters, especially if $r > m$.

Equation (4.9) is a linear, finite-dimensional, discrete-time, time-varying model of the system, which is apparently better suited for simulation, analysis, and design than the continuous-time approach proposed by other investigators, Bell Isle [11, 12] for example.

Network traffic is generally random but it can be approximately periodic. As a first step in stability analysis for ICCS, we consider periodic traffic. (The assumption of periodicity needs to be modified to accommodate quasi-periodic traffic; this is a subject of future research.)

For certain applications such as the token bus [6] with periodic traffic, the delay sequences have been shown to be periodic [3, 4]. In that case, there exists a positive integer M such that:

$$p(k+M) = p(k) \text{ and } t_i^k = t_i^{k+M} \text{ for every } i \text{ and } k \tag{4.14}$$

Considering the $(N \times N)$ system matrix Φ_k in (4.11), (4.14) implies that $\Phi_{k+M} = \Phi_k$ for $\forall\, k$. Let us define, for any k,

$$\Psi_k^M = \prod_{j=1}^{M} \Phi_{k+M-j} \tag{4.15}$$

Proposition 4.1. Let the delays in the control system in Fig. 2 be periodic with a period of M. Then the system (4.10) is uniformly asymptotically stable if all eigenvalues λ_i of Ψ_1^M are contained within the unit circle, i.e., $|\lambda_i| < 1$ for $i = 1, 2, \ldots, N$.

Proof of Proposition 4.1: We need the following two lemmas to prove the proposition.

Lemma 4.1. The eigenvalues of Ψ_k^M are identical to those of Ψ_1^M for every k.

Proof of Lemma 4.1: Since $\Phi_{j+M} = \Phi_j\ \forall j$,

$$\Psi_k^M = \left(\prod_{j=M-s+2}^{M} \Phi_{M-j+1}\right)\left(\prod_{j=1}^{M-s+1} \Phi_{M-j+1}\right) \tag{4.16}$$

where $s = k$ modulo M. By Lemma 4.2, it follows that the eigenvalues of Ψ_k^M are identical to those of Ψ_1^M.

Lemma 4.2. If Γ and Ω are two square matrices of same dimensions, the eigenvalues of $\Gamma\Omega$ are identical to those of $\Omega\Gamma$.

Proof of Lemma 4.2: The problem is stated as: If α is an eigenvector of $\Gamma\Omega$ with an eigenvalue μ, i.e., $\Gamma\Omega\alpha = \mu\alpha$, then does there exist a vector β such that $\Omega\Gamma\beta = \mu\beta$. The proof follows by premultiplying the first expression by Ω and setting $\beta = \Omega\alpha$.

Now we present the proof of Proposition 4.1. From (4.10) and (4.15), it follows that $\mathbf{X}_{k+M} = \Psi_k^M\, \mathbf{X}_k$ for any given k. Since this is a linear, autonomous system, $\|\mathbf{X}_{k+iM}\| \to 0$ as $i \to \infty$ iff each eigenvalue of Ψ_k^M lies within the unit circle. By Lemma 4.1, $\|\mathbf{X}_k\| \to 0$ as $k \to \infty$ iff $|\lambda_i| < 1$ for $i = 1, 2, \ldots, N$.

Corollary to Proposition 4.1. The system will have a minimum decay rate (per every M samples) of $\gamma \in (0, 1)$ iff $|\lambda_i| < \gamma$ for $i = 1, 2, \ldots, N$.

Proof of Corollary: The proof directly follows Proposition 4.1.

Remark 4.5. Since $\|\Phi_k\| < \infty$ for every k, there exists a finite $\zeta \in \mathbf{R}_+$ such that, for every $j < M$, $(\|\Psi_k^j\|)^{1/j} \leq \zeta$. For example, one way to choose ζ is

$$\zeta = \max_{1 \leq k \leq M} \|\Phi_k\| \tag{4.17}$$

Remark 4.6. Proposition 4.1 applies to all periodic, finite-dimensional, time-varying systems. In the ICCS, this can be used only if the delays are periodic. This simplification was possible because the infinite-dimensional delayed system could be represented by a linear finite-dimensional model.

5 Summary, Conclusions, and Recommendations for Future Work

The asynchronous time-division multiplexed networking in Integrated Communication and Control Systems (ICCS) introduces time-varying delays between system components. These delays could degrade the system dynamic performance and are a source of potential instability in complex dynamical processes like advanced aircraft, spacecraft, and autonomous manufacturing plants.

The system, under consideration in this paper, consists of a continuous-time plant and a discrete time controller where the sensor and control signals experience time-varying delays; no assumption has been made regarding any specific network topology or protocol. The plant and controller models are finite-dimensional, linear, and time-invariant. In contrast to other investigators' approach of modelling the system by delayed differential equations, we have represented the control system in discrete-time. This yields a finite set of time-varying, linear difference equations. The effects of the network-induced delays, i.e., data latency and mis-synchronization between components, on the system dynamic performance have been taken into account.

No general stability test have yet been established for the time-varying system. However, for periodic delays, which is an idealized case of periodic traffic in a linear token passing bus protocol [3, 6], a necessary and sufficient condition for uniform asymptotic stability has been established. Further research is required for defining stability conditions for nonperiodic delays. The existing techniques for solving time-varying systems need to be investigated for this purpose. The areas of current research are: (1) Development of appropriate observers (or filters) to compensate for network-induced delays, and (2) Construction of an appropriate Lyapunov function for stability analysis of the finite-dimensional time-varying system.

Although the ICCS model is derived in a deterministic setting, it can be extended for random delays without any structural modifications. In that case, the time-varying coefficients in the system difference equations are replaced by stochastic delay parameters [13].

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APPENDIX A

Supporting Proposition

Proposition A.1: The time-varying delays Θ_{sc} and Θ_{ca} in Fig. 2 can be equivalently lumped together as a single delay $\lambda = \lambda(\Theta_{sc}, \Theta_{ca})$ in Fig. A.1 provided that

1. The sensor and controller have identical sampling periods T,
2. No message rejection at the sensor and controller terminals,
3. $\lambda(t) \geq 0$ for $\forall\, t$,
4. $\lambda(kT+\Phi^k) = \Phi^k$, and
5. $\lambda(kT+\tau) \leq \tau \;\forall \tau \in [\Phi^k,\, T+\Phi^{k+1})$

where $\Phi^k \triangleq \Theta_{sc}{}^k + \Theta_{ca}{}^k$.

Proof: The lumped and unlumped delayed systems in Fig. 2 and Fig. A.1 are equivalent with respect to the input/output relation if, for a given $\mathbf{y}(t)$ at anytime $t \geq 0$, $\mathbf{u}(t)$ and $\mathbf{u}^*(t)$ are identical. The function $\mathbf{u}(t)$ in Fig. 2 is piecewise constant and is given by

$$\mathbf{u}(t) = \mathbf{u}_k \text{ if } kT+\Phi^k < t \leq ((k+1)T + \Phi^{k+1}$$

The function $u^*(t)$ in Fig. A.1 is given by

$$\mathbf{u}^*(t) = \mathbf{u}_k \text{ if } kT \leq (t-\lambda(t)) < (k+1)T$$

For $\mathbf{u}(t)$ and $\mathbf{u}^*(t)$ to be equivalent, the time ranges in two equations must be the same. Condition 4 guarantees that

$$\mathbf{u}(kT+\Phi^k) = \mathbf{u}_k$$

Condition 5 assures that this value remains constant as long as $t < (k+1)T + \Phi^{k+1}$. This follows from the fact that

$$t-\lambda(t) = kT+\tau-\lambda(kT+\tau) \geq kT \text{ at the instant } t = kT+\tau$$

Thus the input to the plant is $\mathbf{u}(t) = \mathbf{u}_k$. The proof can also be obtained graphically and is shown in Fig. A.2.

Remark A.1.1: As can be seen from conditions 4 and 5 in Proposition A.1, $\lambda(t)$ is not unique and any admissible function is equally applicable.

Remark A.1.2: If $\lambda(t)$ is a constant or can be approximated by a constant for all t even though Θ_{sc} and Θ_{ca} could be individually time-varying, then the control system can be designed using conventional frequency-domain techniques.

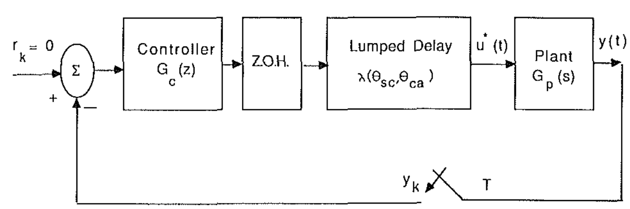
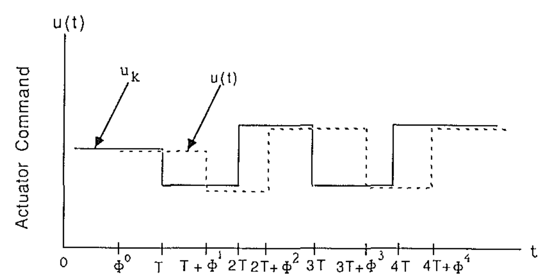
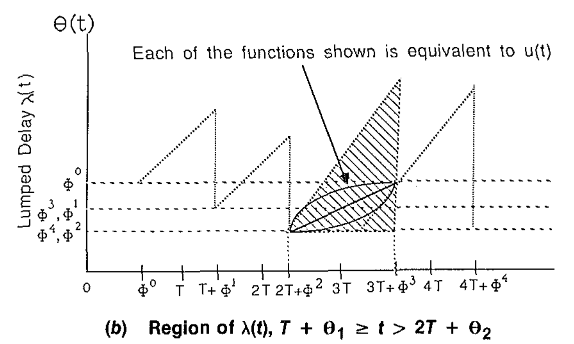


Fig. A.1 Lumped delayed control system



(a) Comparison of delayed and undelayed output



(b) Region of $\lambda(t)$, $T + \Theta_1 \geq t > 2T + \Theta_2$

Fig. A.2 Graphic representation of admissible functions

Each of the functions shown is equivalent to *u(t)*