# Stochastic Modeling of Fatigue Crack Dynamics for On-Line Failure Prognostics

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*Abstract*— **This paper presents a nonlinear stochastic model of fatigue crack dynamics for real-time computation of the time-dependent damage rate and accumulation in mechanical structures. The model configuration allows construction of a filter for estimation of the current damage state and prediction of the remaining service life based on the underlying principle of extended Kalman filtering instead of solving the Kolmogorov forward equation. This approach is suitable for on-line damage sensing, failure prognosis, life prediction, reliability analysis, decision-making for condition-based maintenance and operation planning, and life extending control in complex dynamical systems. The model results have been verified by comparison with experimentally generated statistical data of time-dependent fatigue cracks in specimens made of 2024-T3 aluminum alloy.**

## I. Introduction

In the post cold war era, under the pressure of global economic competition, increased emphasis is being put on maintenance and life extension of existing plant equipment for continuous-time and discrete processes in an effort to reduce major capital costs and increase productivity. The economic and technological challenges for reliable and cost effective operation and maintenance are met through a combination of damage monitoring, failure prognostics, and remaining service life prediction. Examples are mechanical systems (e.g., rail transport, automotive, and aircraft), power systems (e.g., fossil-fueled power plants), and continuous-time production processes (e.g., chemical and petrochemical plants, and pulp and paper mills) where structural durability and operational reliability are critical. Analogous problems exist in discrete production processes, e.g., manufacture of ceramics, semiconductors, and food products. Although off-line analysis permits assessment of structural durability and service life in the design stage, the current technology does not offer appropriate tools for on-line monitoring of structural damage, failure prognosis, and remaining life as continuous functions of time [1] that are needed for maintenance scheduling and plant operations planning. The major challenge here is to identify both static and dynamic properties of material degradation and then characterize this information in a mathematical form for on-line decision and control systems.

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This paper focuses on monitoring fatigue damage in metallic materials which are commonly encountered in mechanical, power and chemical plants. The analytical results are based on fatigue tests that have been conducted under normal ambient conditions. The objective here is to obtain a clear understanding of the random phenomena in fatigue and fracture and to establish a framework for on-line damage monitoring in the continuous-time setting. It should be noted, however, that the physics of fatigue damage at room temperature in laboratory air is, in most cases, significantly different from that at the elevated temperatures of a plant environment. Nevertheless, the research work presented in this paper is a crucial step toward achieving the final goal of implementing damage monitoring systems that will be functional in the actual environments of plant operations.

The fatigue damage can be characterized at the microscale, mesoscale and macroscale levels [2]. Considerable information is already available in the literature pertaining to the damage mechanisms and states prevalent in different metallic materials and alloys of interest [3]. In general, damage criteria are defined in terms of metallurgical (microstructural) features and parameters, such as inclusions, pores, dislocations, and lattice deformation or crystallographic texture; or physical discontinuities, such as cracks, interfaces and cavities. A focus on the former approach attempts to equate the condition of a specific microstructural feature, or a combination of these features, to a particular damage state, often irrespective of the generation of observable discontinuities. The latter approach, which correlates the initiation and growth of cracks to the state of damage, is adopted in this paper. In this setting, the structural integrity and the associated service life of a plant component are computed as continuous-time functions of degradation in mechanical properties that are directly related to the changes in the microstructural or physical features being monitored.

Two major difficulties arise in the correlation of quantitative measures of specific features or material conditions to the damage state. First, the rates of damage accumulation may vary as time progresses even under isothermal and constant amplitude cyclic loading; the situation is far more complex under transient thermomechanical conditions. Drastic changes in the sensor signal characteristics may occur at different stages as the process of damage accumulation continues. Therefore, more than one approach to the basic mechanism of fatigue crack measurements is necessary over the entire range starting from crack incubation to the final stage of crack propagation. Second, many of the characterization or inspection techniques that offer good potential for real-time, in-situ applications provide cumulative effects of damage that are highly dependent on knowledge of the material properties as well as on the initial state, i.e., prior to exposure to the damaging environment or service loads. These issues are subjects of current research and, therefore, are not fully addressed in this paper.

In view of its application to mechanical, power, chemical, and manufacturing industries, the potential benefits of the research work reported in this paper are envisioned as follows:

- Enhanced safety, reliability, and availability due to timely prediction of impending failures.
- Reduction of the life cycle cost by optimization of the plant operation and maintenance schedule.
- Enhancement of the plant performance and structural durability to achieve the operational objectives.

From the above perspectives, this paper complements the efforts of other investigators on 1) development and implementation of software systems for creep and fatigue life assessment [4] and 2) construction of new sensing elements and sensor hardware such as strain gauges that can be operated at elevated temperatures [5].

## II. Modeling of Fatigue Crack Dynamics

Modeling of fatigue damage has been a topic of intensive research for several decades. Many researchers cited in [6] have proposed empirical and semi-empirical models based on observed experimental data and attempted to provide a physical interpretation to these models. Modeling of fatigue damage dynamics via nonlinear stochastic differential equations is a relatively new area of research, and an extensive list of the literature representing the state of the art is cited by Sobczyk and Spencer [7]. One approach is to randomize an established deterministic model of fatigue damage dynamics to generate the necessary stochastic information.

### A. Deterministic Damage Modeling of Fatigue Crack Growth

The deterministic fatigue crack growth model is based on the concept of the short crack growth model [8] and [9]. While the Paris model [10] is valid in the macro-crack range, the Newman model represents the fatigue crack growth process down to microcracks in the order of 10 m. The Newman model for crack growth is of the form

$$\left.\begin{aligned} \frac{d\mu}{dN} &= C_1 \times (\Delta K_{eff}^{C_2}) \times \left[\frac{1-\left(\dfrac{\Delta K_o}{\Delta K_{eff}}\right)^2}{1-\left(\dfrac{K_{\max}}{C_5}\right)^2}\right]; \\ \mu(N_o) &= \mu_o > 0 \\ \Delta K_o &= C_3\left(1 - C_4\frac{S_o}{S_{\max}}\right); \\ K_{\max} &= S_{\max}\sqrt{\pi\,\mu}\,F; \\ \Delta K_{eff} &= (S_{\max} - S_o)\sqrt{\pi\,\mu}\,F \end{aligned}\right\} \quad (1)$$

where $\mu$ is the mean crack length, $N$ is the number of cycles, the crack growth rate $d\mu/dN$ is the so-called derivative of $\mu$ with respect to $N$ as commonly used in the fracture mechanics literature [11], so is the crack opening stress, $F$ is the correction factor for finite width of the specimen, and $S_{\max}$ is the maximum applied remote stress; and $C_i$, $i = 1, \cdots, 5$ are material parameters. As an alternative to the functional form of (1), a look-up table form can determine $d\mu/dN$ in terms of $\Delta K_{eff}$. In this setting, the crack growth rate is expressed as

$$\frac{d\mu}{dN} = \exp\left[m \ln(\Delta K_{eff}) + b\right]; \quad \mu(N_o) = \mu_o > 0 \qquad (2)$$

where $m$ is the slope and $b$ is the intercept of the linear interpolation of the (log scale) $\Delta K_{eff} - d\mu/dN$ look-up table. Details of this method are reported by Newman *et al.* [9]. It should be noted, however, that any other deterministic model of fatigue crack growth can be used with the only criterion being that the observed experimental crack growth profile is accurately represented. For example, several researchers [12]–[14] have used a cubic polynomial fit in $\ln(\Delta K_{eff})$ to determine crack growth rate. Following [15], the crack growth equation is expressed in the continuous-time setting as

$$\frac{d\mu}{dt} = \frac{\left(\dfrac{\partial\Phi}{\partial S}\right)\left(\dfrac{dS}{dt}\right)}{\dfrac{1-\partial\Phi}{\partial\mu}}$$
$$\mu(t_o) = \mu_o > 0 \qquad (3)$$

where (1) or (2) is represented as $d\mu/dN = \Phi(\mu, S, S_o)$. A common practice is to define the time-dependent fatigue crack damage as the ratio of the mean crack length and the critical value of the crack length which is a function of the material, component geometry, and the operating condition [16].

### B. Stochastic Damage Modeling of Fatigue Crack Growth

It has been shown by several researchers, including Bolotin [16] and Spencer and Tang [12] and [13], that the process of fatigue crack growth can be modeled by nonlinear stochastic differential equations satisfying the Itô conditions [17]. The information generated from these models can be used to formulate algorithms for damage prediction and risk analysis. Specifically, Kolmogorov forward and backward diffusion equations, which require solutions of nonlinear partial differential equations, have been proposed by several researchers, cited in [7], to generate the statistical information required for risk analysis of aircraft structures. These nonlinear partial differential equations can only be solved numerically; the numerical procedures, however, are extremely computationally intensive as they rely on a fine-mesh model using finite-element or combined finite-difference and finite-element methods [14]. Therefore, although this approach might be useful for making off-line maintenance decisions, it is not sufficiently fast for on-line damage monitoring, failure prognosis, and remaining service life prediction. To enhance the numerical efficiency, we propose to adopt the underlying principle of extended Kalman filtering [18] in which the first two moments of the stochastic damage state are computed on-line by constructing the stochastic differential equations in the Wiener form as opposed to the Itô form. This can be achieved following the two-state model of Spencer and Tang [12] provided that the shaping filter is constructed with additive white Gaussian noise. The underlying principle of extended Kalman filtering can be used with or without any sensor(s). The absence of any useful sensor data is equivalent to having the inverse of the intensity of measurement noise covariance tend to zero; in that case, the filter gain approaches zero. Consequently, the conditional density function generated by the filter becomes identical to the prior density function whose evolution is governed by the Kolmogorov forward equation [18].

The stochastic differential equation for the crack growth process $c(\omega, t)$ is expressed in terms of the deterministic damage dynamics as

$$\frac{dc(\omega, t)}{dt} = \exp\left[z(\omega, t) - \frac{\sigma_z^2(t)}{2}\right]\frac{d\mu}{dt} \quad \forall t \geq t_o$$

given

$$E[c(\omega, t_o)] = \mu_o$$

and

$$\mathrm{cov}\,[c(\omega, t_o)] = P_o \tag{4}$$

where $\omega$ and $t$ represent the sample point and time of the stochastic process, respectively. The auxiliary process $z(\omega, t)$ is assumed to be stationary Gauss–Markov of variance $\sigma_z^2(t)$ implying that the crack growth rate is a lognormal-distributed Markov process. The rationale for this assumption is as follows: 1) fatigue crack growth rates in many metallic materials are observed from experimental data to be approximately lognormal-distributed and highly correlated over a short period of time [7] and 2) the crack propagation phenomenon has been modeled as a Markov process by including history-dependent crack opening stresses or reference stresses [15] as an additional input. Yang and Manning [19] recommended the lognormal distribution model of fatigue crack propagation statistics in aluminum alloys for scheduling of aircraft maintenance because 1) this approach is moderately conservative; 2) it is mathematically tractable and can be easily implemented; 3) a small number of replicate fatigue tests are adequate to calibrate the model; and 4) reasonable crack growth predictions for structural details have been obtained previously from this model.

It follows from (4) that $E[dc(\omega, t)/dt] = d\mu/dt$ due to the Gaussian distribution of $z(\omega, t)$. This is in agreement with the governing equation of mean crack growth in (3). The correlated stationary Gauss–Markov process $z(\omega, t)$ is modeled by using a first-order linear shaping filter which is driven by the additive zero-mean white Gaussian noise $w(\omega, t)$ of intensity $Q_o$ as

$$\frac{dz(\omega, t)}{dt} = -\xi\, z(\omega, t) + w(\omega, t) \quad \forall t \geq t_o$$

given

$$E[z(\omega, t_o)] = z_o$$

and

$$\mathrm{cov}\,[z(\omega, t_o)] = \frac{Q_o}{2\xi} \tag{5}$$

where the filter parameter $\xi$ is a measure of "coloredness" of the auxiliary random process $z(\omega, t)$. As $\xi$ is increased, $z(\omega, t)$ becomes more uncorrelated. Specifically, if $\xi$ is set equal to zero, then $z(\omega, t)$ becomes a Brownian motion process which is perfectly correlated. In the extreme case, as $\xi$ tends to infinity, $z(\omega, t)$ approaches zero implying no uncertainties in the fatigue crack growth process. Experimental data show that $z(\omega, t)$ remains highly correlated over a long period and eventually may become uncorrelated [20]. This phenomenon is realized if $0 < \xi \ll 1$.

The constant parameter $z_o$ in (5) is determined from the environmental condition under which the fatigue crack growth takes place. Specifically, $z_o$ is set to zero at the nominal environmental condition (e.g., laboratory air at room temperature) under which experiments are conducted to determine the fatigue damage parameters in (3). In a more hostile environment (e.g., humid and salty air at the ocean surface), however, the increased damage rate can be realized by setting $z_o$ to a positive value. Similarly, in a more benign environment (e.g., dry air), $z_o$ should be set to a negative value to realize the relatively slower growth of fatigue cracks.

The stochastic differential equations (4) and (5) are now combined to yield the augmented stochastic vector $\chi = [c \quad z]^T$ as a vector diffusion Markov process with additive noise, which can be interpreted in the Wiener sense instead of the Itô sense. Then, linearization of the augmented state equation yields the local state transition matrix which would lead to computation of the first two moments of $\chi(\omega, t)$ conditioned on the history of the stress and the mean and variance of $\chi(\omega, t_o)$. This provides full statistical characterization of the crack length at each instant of time based on the assumption of the lognormal distribution which is described by the first two moments.

The measurements, generated from any available sensor(s) (e.g., strain gages and acoustic emission transducers) at the instant $t$, is modeled as an algebraic function of the damage state vector $\chi$ and additive sensor noise $v$

$$y(\omega, t) = g\,[\chi(\omega, t)] + v(\omega, t) \tag{6}$$

where the deterministic function $g(\bullet)$ represents the sensor model; and $v(\omega, t)$ is zero-mean white noise of known intensity representing measurement noise. Since $v(\omega, t)$ is a consequence of electronic and acoustic noise, it is assumed to be independent of the material uncertainties $w(\omega, t)$, i.e., $E[w(\omega, t) \quad v(\omega, \tau)^T] = 0 \;\forall t, \tau \geq t_o$. Conditional mean and variance of the crack length $c(\omega, t)$ can be obtained in real time via (4)–(6).

## III. Failure Prognosis, Risk Analysis, and Decision-Making for Maintenance

Failure prognosis and risk analysis are envisioned to be an augmentation of failure diagnosis to predict and locate impending failures of critical plant components. Traditionally, the risk index and residual service life [16] of mechanical structures and machine components are calculated off-line on the basis of statistical models of the mechanical properties of the material, operating conditions, major disruptions (e.g., emergency shutdown of the plant from full load), and accidents (e.g., seismic and large atmospheric disturbances). The service life of a plant component is finite as the damage accumulation is monotone with time. On-line monitoring of the accumulated damage allows refinements of the risk index and residual service life prediction [16], [21] as time progresses. Specifically, prediction of the accumulated damage and the current damage rate will assist the plant engineer in making dynamic decisions regarding safety procedures, mission accomplishments, and the time interval between major maintenance actions. The concept of life prediction based on the current plant and damage states is outlined below.

We define $M$ hypotheses as disjoint regions of normalized crack length, i.e., $c/c^*$, where $c^*$ is the critical crack length beyond which the crack growth rate becomes very large rapidly leading to complete rupture. Therefore if the initial crack length is $c_o$, then the first $(M-1)$ hypotheses become

$$\left(c_o, \quad c_o + \frac{(c^* - c_o)}{M-1}\right] \quad \left(c_o + \frac{(c^* - c_o)}{M-1}, \quad c_o + 2\,\frac{(c^* - c_o)}{M-1}\right] \quad \cdots, \left(c_o + (M-2)\,\frac{(c^* - c_o)}{M-1}, \quad c^*\right]. \tag{7}$$

The last (i.e., $M$th) hypothesis is chosen as the region $[c^*, \infty)$ which is popularly known as the unstable crack region in the fracture mechanics literature [11]. Each of these $M$ hypotheses represents a distinct range in the entire space of normalized crack lengths from an initial value of $c_o$ till the rupture occurs, and together, they form an exhaustive set of mutually exclusive regions in the damage state space. The probability that the $j$th hypothesis, $H_j$, holds at any given instant of time, $t$, is computed via the instantaneous probability distribution function (PDF) $F_c(\bullet\, t)$ of the crack length $c(t)$

$$\begin{aligned} P_j(t) &= F_c\left[c_o + j\,\frac{(c^* - c_o}{M-1}; t\right] \\ &\quad - F_c\left[c_o + (j-1)\,\frac{(c^* - c_o)}{M-1}; t\right] \\ &\quad j = 1, 2, \cdots, M-1 \\ P_M(t) &= 1 - F_c(c^*; t). \end{aligned} \tag{8}$$

If the crack length is assumed to be lognormal-distributed [7], then the instantaneous PDF, $F_c(\bullet; t)$, of the stochastic process $c(\omega, t)$ can be determined from the first two moments of crack length that are generated as continuous functions of time by the extended Kalman filter. Therefore, the probability that a given hypothesis is valid can be computed on- line as a function of time. Using the PDF, $F_c$, we can compute the remaining service life $T[t; Y_d(t); \varepsilon]$ at any specified time instant, $t$, based on a desired plant output profile $Y_d(t) = \{y(\tau) : \tau \geq t\}$ and a confidence level $\varepsilon$. This implies that if the plant operation is scheduled to yield the desired output, then $T[t; Y_d(t); \varepsilon]$ is the maximum time of operation such that the probability that the crack length $c(\omega, t+T)$ does not exceed $c^*$ is greater than $\varepsilon$. This is mathematically represented as

$$T[t; Y_d(t); \varepsilon] = \sup\{\tau \in [0, \infty) : P[c\omega, (t+\tau) \leq c^*] > \varepsilon\}. \tag{9}$$

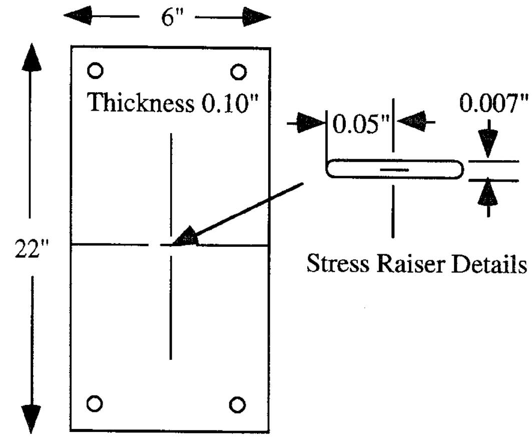


Fig. 1. Test specimen for the Virkler experiments.

This is accomplished by using the same extended Kalman filter formulation described above. Given the expected value and variance of crack length at time $t$, the extended Kalman filter can be simulated using these values (as initial conditions) and the desired load/output profile to find the maximum remaining time of operation $T$, such that $P[c(\omega, t+T) \leq c^*] > \varepsilon$.

The above procedure can be used to provide valuable information to the higher level of decision making or a human operator about the confidence with which the plant operations can be planned for a specified amount of time. This information is also of considerable importance to maintenance scheduling or in making decisions on how to reschedule the plant operations to avoid an untimely shutdown.

## IV. Model Verification and Results

We present, in this section, an example of how to tune and verify the stochastic damage model with statistical data of fatigue tests. In this example we used the data of Virkler *et al.* [22] in which the tests were conducted in an environment of dry air under a constant load amplitude of 4200 lb with a peak load of 5250 lb. Each test specimen was made of center-cracked 2024-T3 aluminum alloy panels 22 in long, 6 in wide, and 0.10 in thick as shown in Fig. 1. A stress raiser of half length 0.05 in was machined in the center of the specimen. The tests were conducted on 68 specimens under very tight quality control to ensure that each test was identical including environmental conditions. Fig. 2 shows that, even under these tight conditions and constant amplitude loading, the fatigue life of the specimens had a large variance. It should be noted, however, that the data in Fig. 2 correspond to macrocracks. Statistical data of short cracks [9] need to be collected for complete validation of the crack growth model.

Variations in the fatigue crack growth profile of the apparently identical specimens are due to the uncertainties inherent in the crack growth process and material characterization. The parameters of the damage model in (3) were identified based on experiments conducted in laboratory air. On the other hand, the Virkler data which are used in this paper for model validation were generated in the more benign environment of dry air. Therefore, to compensate for this

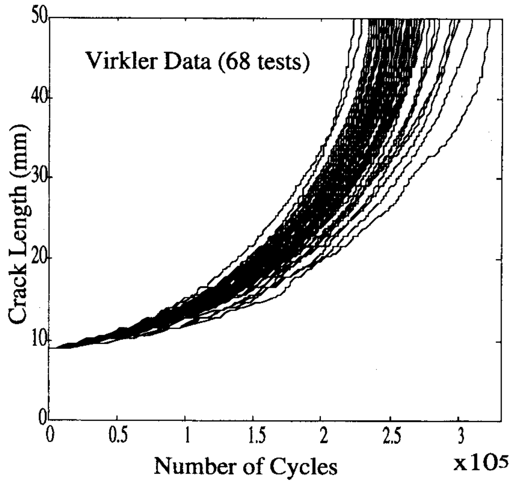


Fig. 2. Statistical data of fatigue crack growth.

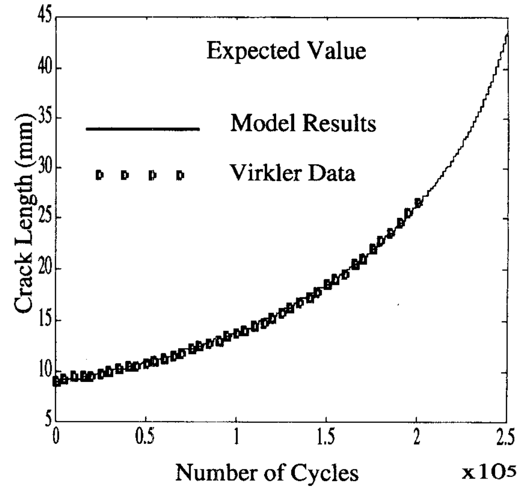


Fig. 3. First moment of crack length.

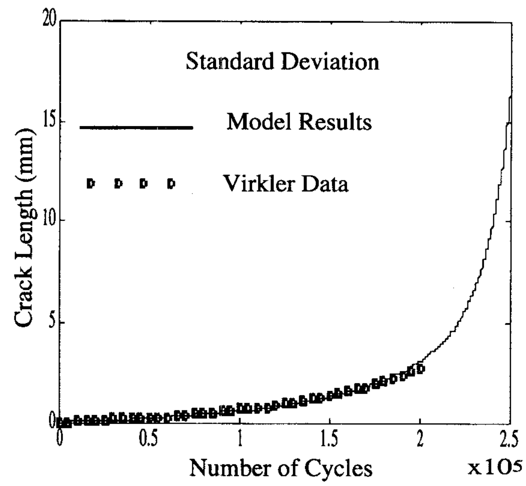


Fig. 4. Standard deviation of crack length.

change in environmental condition, the parameter $z_o$ in (5) was tuned to $-0.471$. That is, (3) is corrected by a multiplicative factor of $\exp(-0.471) = 0.6244$.

Fig. 3 compares the mean crack length profile generated from the test data with the expected value derived from the extended Kalman filter. The deterministic fatigue crack growth model used in the extended Kalman filter formulation uses a table lookup to compute the crack growth rate as a function of $\Delta K_{eff}$ following the procedure outlined by Newman *et al.* [9]. Fig. 4 compares the profile of the standard deviation of crack length generated from the experimental data with that derived from the model. The model results are in very close agreement with the experimental data for the first two moments of the crack length. as seen in Figs. 3 and 4. The rapid rise in the standard deviation after 11 500 s (i.e., 230 000 cycles) in Fig. 4 is due to the fact that unstable crack growth begins here and the specimen is about to break. These results of damage model validation have been generated with no sensor data. That is, the inverse of the intensity of the measurement noise $v$ in (6) has been taken to be zero.

The results in Figs. 3 and 4 are comparable with those reported by Spencer and Tang [12], [13] and Enneking [14] who numerically solved the Kolmogorov diffusion to obtain the statistics of crack length. The solution of the partial differential of Kolmogorov is computationally two orders of magnitude more expensive than that of the ordinary differential encountered in the extended Kalman filter (EKF) which required less than 2 s of CPU time on a Pentium-90 computer for the entire period of 12 500 s, i.e., 250 000 cycles. It should be noted that the Kolmogorov yield the conditional density as a function of time while the extended Kalman filter formulation yields a time series of only the first two moments of the crack length. The CPU time requirement is virtually unchanged, however, if the two parameters of the lognormal distribution are also computed based on the first two moments provided by the Kalman filter. Since Kalman filtering provides a recursive solution, the memory requirements are also significantly lower. This fact coupled with the accuracy of the results make the proposed method ideally suited to real-time sensing of fatigue damage.

As mentioned in the previous section, the instantaneous PDF of the crack length $c(\omega, t)$ is determined from the first two moments of crack length assuming that the distribution of crack length is lognormal. Based on these statistics of $c(\omega, t)$, the conditional probability of any one of the $(M)$ mutually exclusive and exhaustive hypotheses is computed as a function of time. To elucidate the concept of hypothesis testing for remaining life prediction, the damage state space is partitioned into $M$ segments starting from the initial crack length $c_o$. Probabilities of the $M$ hypotheses having the random crack length $c(\omega, t)$ at a given time $t$ located in one and only one of these segments is computed on-line. Given that $c_o$ = 9.0 mm with probability 1 and the critical crack length $c^* = 49.8$ mm, the 11 hypotheses ($M = 11$) are depicted in Table I. The time evolution of probability of the hypotheses for the simulation of the Virkler experiment described above is shown in Fig. 5 where each successive hypothesis starts at time $t = 0$ s on the left and ends at time 12 500 s (i.e., 250 000

TABLE I
DAMAGE PREDICTION IN THE PROBABILISTIC SETTING

| Description | Range |
|---|---|
| Hypothesis $H_1$ | 9.00 mm ≤c(t) <13.08 mm |
| Hypothesis $H_2$ | 13.08 mm ≤c(t)< 17.16 mm |
| • | • |
| • | • |
| • | • |
| Hypothesis $H_{10}$ | 45.72 mm ≤c(t) <49.80 mm |
| Hypothesis $H_{11}$ | 49.80 mm ≤c(t) (Unstable Crack Growth) |

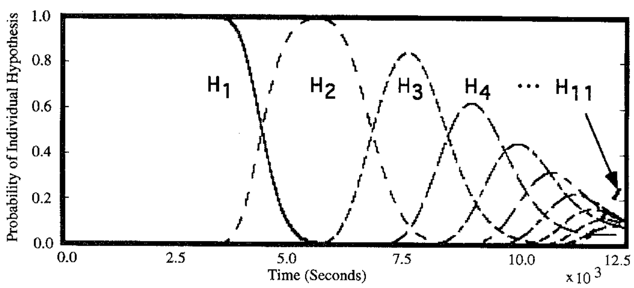


Fig. 5. Probabilities of the individual hypotheses as functions of time.

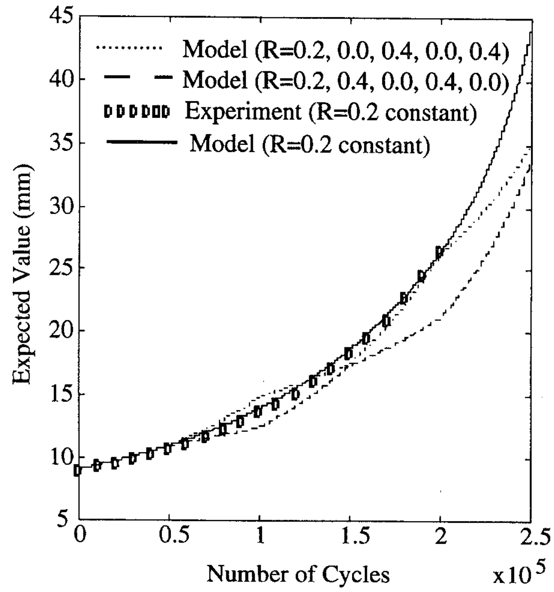


Fig. 6. Expected value of crack length.

cycles) on the right. The plot of $H_1$ begins with a probability close to one and later diminishes as the crack grows with time (i.e., number of cycles). The probability of each of the remaining hypotheses $H_2$–$H_{10}$ is initially close to zero and then increases to a maximum and subsequently decreases as the crack growth process progresses with time. The probability of the last hypothesis $H_{11}$ (on the extreme right in Fig. 5) of unstable crack growth beyond the critical crack length initially remains at zero and increases rapidly only when the specimen is close to rupture. At this stage, the probability of each of the remaining hypotheses, $H_2$–$H_{10}$, diminishes. The plots in Fig. 5 also show the effects of the increasing variance of crack length on the probability of a hypothesis. The higher variance of crack length after about 7500 s (i.e., 150 000 cycles), as seen in Fig. 4, is manifested in the spreading of the probability over several hypotheses. This information can be used in a higher level decision making module (possibly a discrete-event supervisor [23]) to make decisions of failure prognosis, life extending control, and condition-based maintenance, or generate warnings and alerts.

In its current form, the deterministic damage model is valid only for constant amplitude loads. Additionally, the stochastic damage model is tuned for a particular constant amplitude load and due to the lack of experimental data; the parameter $\xi$ for stress amplitudes other than that used in the Virkler experiment could not be determined. These restrictions do not allow the computation of the remaining life following (9) for arbitrary load profiles. Since the deterministic damage model is valid for constant amplitude loads, it can be used to predict fatigue crack growth accurately for step changes in the load amplitude, i.e., a change in the load applied to the structure from an amplitude of 5250 lb to say 5500 lb from one cycle to another. This does not effect the accuracy of the deterministic damage model since the structure can be assumed to be freshly loaded with the previous value of crack length as an initial condition. Preliminary work reported by Tang and Spencer [13] has shown that the parameter $\xi$ is dependent on the maximum stress applied to the structure; it was conjectured that a constant value of $\xi$ can be used for small changes in the stress ratio of the loading applied to the structure without causing large errors. Experimental data at different load levels must be available for testing this conjecture.

Figs. 6 and 7 show the profiles of expected value and standard deviation of crack length, respectively, for the original loading of the Virkler experiment along with two additional load profiles which are constructed by changing the stress ratio (i.e., the ratio of minimum applied stress to the maximum applied stress) after 50 000, 100 000, 150 000, and 200 000 load cycles. The effects of changing the stress ratio, $R$, for a given peak stress were investigated. A smaller $R$ causes a higher stress amplitude and hence a larger crack growth rate (due to increased $\Delta K_{eff}$) even though the mean stress is decreased. Similarly, a larger $R$ results in a smaller crack growth rate due to a reduced stress amplitude. The sequence effect of fatigue crack growth [6] is clearly evident whereby the fatigue damage accumulation is dependent on the order in which different stress amplitudes are applied to the structure. It is interesting to note that the amount of fatigue damage accumulation during the test period of 250 000 cycles is less in both cases of variable amplitude loading than that for the original constant amplitude case. This also results in a lower standard deviation of the crack length.

Fig. 8 shows the probability of two hypotheses, $H_2$ and $H_6$, out of the eleven hypotheses defined above as functions of time for the Virkler and two additional load profiles. Fig. 9 shows the effects of changing the stress ratio on the remaining life under three different scenarios. The first scenario, represented by the dashed line, predicts the remaining life when the stress ratio $R$ in the anticipated load profile is held constant at 0.2 until 200 000 cycles are applied and then changed to

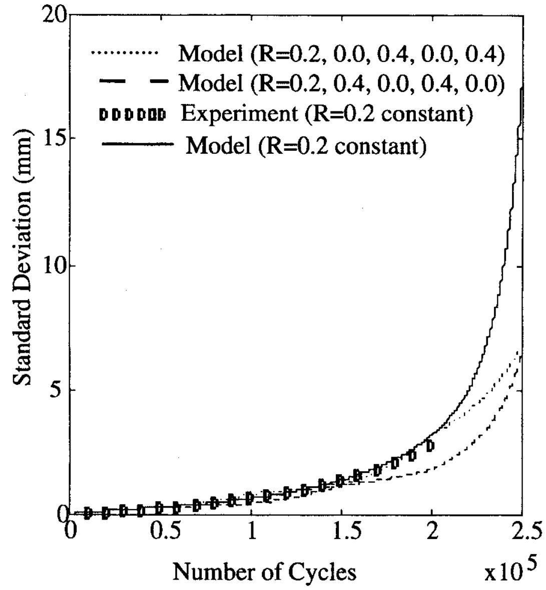


Fig. 7. Standard deviation of crack length under varying amplitude loading under varying amplitude loading.

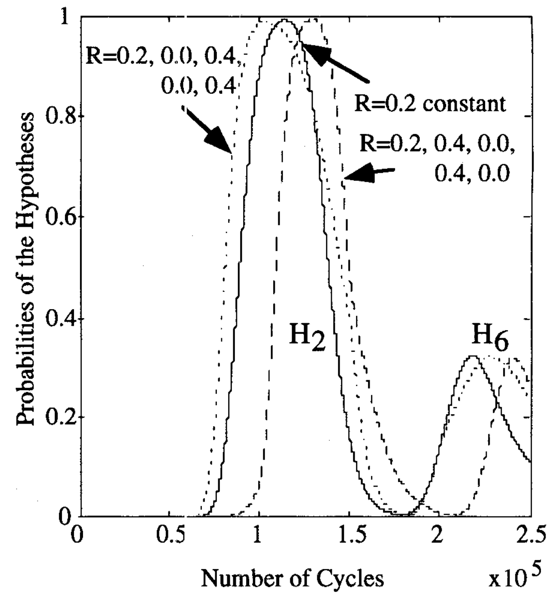


Fig. 8. Probabilities of hypotheses $H_2$ and $H_6$ under varying amplitude loading.

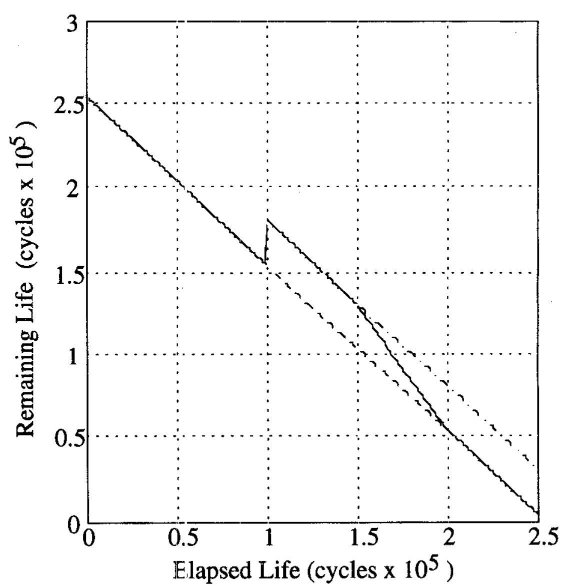


Fig. 9. Predicted remaining life of a specimen under varying amplitude loading under varying amplitude loading.

0.5 for the remaining 50 000 cycles up to the final stage at 250 000 cycles. This is a straight line of slope $-1$ since the remaining life prediction is based on the anticipated load profile which is identical to the actual load applied to the specimen for the entire period of 250 000 cycles. The second scenario represented by the dash-dot line is identical to that for the dashed line up to 100 000 cycles and then differs in the following sense. After 100 000 cycles, the anticipated stress ratio $R$ in the scenario of the dash-dot line is changed to 0.5 at 150 000 load cycles and held constant until 250 000 load cycles have elapsed. This causes the remaining life prediction to show a step increase at 100 000 cycles due to the lower applied stress for part of the entire load sequence. This change is actually implemented at 150 000 cycles and hence the dash-dot line remains at a constant slope after the initial step change at 100 000 cycles. The third scenario, represented by the solid line, is generated by anticipating, at the instant of 100 000 cycles, the same change in value of $R$ to take place at 150 000 cycles but, unlike the second scenario (i.e., the dash-dot line), this anticipated change does not actually happen. Since the fatigue damage rate is higher due to the larger stress amplitude of the applied load, the predicted remaining life starts ramping down at 150 000 cycles and ending at 200 000 cycles when the stress level is known. The solid line merges with the dashed line at 200 000 cycles since, from that moment onward, the applied load and anticipated load are identical for the two scenarios.

## V. Summary, Conclusions, and Recommendations for Future Research

The stochastic model of fatigue damage dynamics presented in this paper is suitable for on-line damage sensing, failure prognosis, life prediction, risk analysis, decision-making for condition-based maintenance and operation planning, and life extending control in complex dynamical systems. Potential application areas are mechanical systems (e.g., rail transport, automotive, and aircraft), power systems (e.g., fossil-fueled power plants), continuous-time production processes (e.g., chemical and petrochemical plants, and pulp and paper mills), and discrete production processes (e.g., manufacture of ceramics, semiconductors, and food products) where structural durability of plant components and operational reliability are critical. The dynamic model of fatigue crack growth model is built upon the following concepts:

- The deterministic part of the model follows the short crack growth model [8], [9]. Any other fatigue damage

model such as [15], however, can also be adopted as long as the predicted values of mean crack length are within the allowable limits of accuracy.
- The stochastic part of the model is based on the underlying principle of extended Kalman filtering in the Wiener integral setting as opposed to the Kolmogorov diffusion equation in the Itô integral setting.

The numerical procedure for the stochastic part is computationally much faster than that based on Kolmogorov equations [7] and yields results of comparable accuracy that have been validated with experimental data of fatigue crack growth statistics. The time-dependent distribution of the random crack growth process was generated from the first two moments obtained the extended Kalman filter based on the assumption of lognormal distributed crack length; this information suffices to determine, at a given level of statistical confidence, the time-dependent remaining service life of mechanical structures that are subjected to cyclic loads. Any distribution that fits the crack length statistics and depends only on the first two moments of crack length can be used for this purpose. If this is not possible, however, further research is needed to generate a fast algorithm for identification of the probability distribution function of crack growth based on the time series of the first two moments and physical characterization of the crack growth phenomenon.

The research work reported in this paper is a crucial step toward achieving the final goal of implementing damage monitoring systems in the actual environments of plant operations. Potential benefits of this damage monitoring concept are envisioned as follows:

- Enhanced safety, reliability, and availability due to timely prediction of impending failures.
- Enhancement of the plant performance and structural durability to achieve the operational objectives.
- Reduction of the life cycle cost by optimization of the plant operation and maintenance schedule.

Further research is recommended in the following areas from the above perspectives:

- *Development of basic sensing elements for real-time non-destructive measurements of fatigue cracks at different stages of crack initiation and propagation.* It is highly unlikely that a specific type of sensing element will be uniformly accurate over the entire span of the service life. Therefore, different techniques of crack damage sensing such as strain measurements [24] and acoustic emission [25] need to be developed.
- *Verification of the fatigue crack growth model at different levels of stress amplitude including overload effects under varying amplitude stresses.* This requires collection of statistical test data for fatigue growth at different stress levels, starting from different initial crack lengths as well as for different materials.
- *Correlation of the stochastic parameters of the fatigue growth model.* This may lead to a classification of the model parameters as material-dependent and stress-dependent. The research work in this area should be directed in two parallel and mutually complementary directions: 1) metallurgical (microstructural) features and parameters, such as inclusions, pores, dislocations, and lattice deformation or crystallographic texture; and 2) physical discontinuities, such as cracks, interfaces and cavities.
- *Extension of the fatigue crack growth model for operation at elevated temperatures and actual plant operating environment.* This research is necessary because the physics of fatigue damage at room temperature in laboratory air is, in most cases, significantly different from that at the elevated temperatures of a plant environment.

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