where, the independent noise term $w_3(t)$ is taken as zero mean Gaussian with variance of "0.01." Two cases for the disturbance input being considered are as follows.

Case I: A square wave disturbance of magnitude 2.0 and a period of 32 samples. The state estimates for this case are as shown in Figs. 1–5, where the solid line designates the estimates of the (UIKF), the dashed line is for the (DEKF), and the small circle designates the estimates of the (DNKF). The disturbance estimate of the (DEKF) is shown by dashed line in Fig. 6 against the true disturbance. The (DNKF) represents the ideal conditions, and therefore, it is taken as the reference of comparison for the other two filters. Table 1, gives the root mean square errors (RMSE) of the (UIKF) and the (DEKF) state estimates using the state estimates of the (DNKF) as reference.

Case II: A two sinusoids disturbance as follows:

$$d(t)=2.0\sin(wt)+1.5\sin(2wt)$$

with a fundamental period of 32 samples. The root mean square estimation errors for this case are as given in Table 2.

It is clear from the results, shown in the figures and tables, that the (UIKF) provides faster and more accurate tracking of the system states. In comparison, the estimates of the (DEKF), although sometimes smoother, they are much delayed and are orders of magnitude less accurate than those of the (UIKF). Moreover, comparison of the RMSE, show the (UIKF) to be less sensitive to the type of disturbance acting on the system than the (DEKF). The estimation of the unknown disturbance input depends on the system structure, the influence of the disturbance on the states, and the measurement scheme. Therefore, the detectability of the disturbance is function of the triplet $\{A,B,C\}$.

5 Conclusion

The state estimation problem of linear dynamic systems influenced by both unknown deterministic disturbance inputs, as well as random noise is treated. A new filter is developed which provides full state estimation and does not require the estimation of the unknown inputs. The developed filter provides faster and more accurate tracking of the system states than the augmented Kalman filter which requires the estimation of the disturbance input. Also, the estimation accuracy of the developed filter is less sensitive to the type of disturbance acting on the system than the disturbance estimating Kalman filter. Moreover, the developed filter has computational advantages as it does not rely on estimating the disturbance inputs.

Acknowledgment

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Output Feedback Linear Parameter Varying (LPV) L_2-gain Control

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This brief paper synthesizes an output feedback L_2-gain Control law for linear parameter varying (LPV) systems. The control law is embedded with an observer that does not require on-line measurements of the scheduling parameter variation rate. Results of simulation experiments are presented to evaluate the control law on a simulation experiments on a two-degree-of-freedom mass-spring-damper system. [DOI: 10.1115/1.1591805]

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Introduction

Dynamical systems often involve transients at different time scales. For control synthesis, the plant dynamics can be modeled by superposition of fast-time motions over the slow-time motions. Furthermore, the two-time scale dynamics can often be decomposed into fast-time perturbation over a quasi-steady equilibrium trajectory (Tan et al., 2000 [1]; Giannelli and Primbs, 2000 [2]). The Linear Parameter Varying (LPV) approach is suitable for two-time scale processes under wide range operation where control of the fast-time scale dynamics is gain-scheduled as a function of the slow-time scale parameters (Packard, 1994 [3]; Hong et al., 2000 [4]). From the numerical perspectives using Linear Matrix Inequalities (LMIs), the LPV synthesis can be classified into two broad categories: *Algebraic* and *differential*. A brief discussion on the present status of LPV control in these two categories follows.

Traditional gain-scheduling approximates each scheduling variable as a series of steps within the operating range and for each step a corresponding control/observing law is synthesized via those LMI approaches which are usually for linear time invariant systems. This approach theoretically allows infinitesimally small parameter variation rates under wide-range operation. An alternative algebraic approach for LMI-based LPV synthesis makes use of the Linear Fractional Transformation (LFT) representation with an internally time-varying coupling Δ-feedback-connected to the nominal plant. For such an LPV plant, the controller has the LFT structure with a Δ-dependent feedback to the nominal controller (Tan et al., 2000 [5]; Apkarian et al., 2000 [6], for example). Another approach to algebraic LMI-based LPV synthesis, which is suitable for affine-parameter-dependent systems, uses a convex hull to contain the operation domain (Gahinet et al., 1994 [7], Bara et al., 2000 [8]). A sufficient condition for robust perfor-

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mance is achieved within the operation domain by considering algebraic LMI solutions at all vertices of the convex hull. These approaches to LPV synthesis allow infinite variation rate of scheduling parameters within a narrow operating range.

Differential LMI-based LPV synthesis have been reported by several investigators including Wu et al. (1996) [9], Wu (2000) [10], and Tan and Grigoriadis (2000) [5], as extensions of the standard LMI synthesis procedure (Gahinet and Apkarian, 1994 [11], for example). In contrast to the algebraic LMI-based LPV synthesis, the differential LMI-based LPV synthesis allows a finite parameter variation rate for wide-range operation.

This brief paper presents an observer-embedded LPV L_2-gain control law following the LPV synthesis reported by Hong et al. (2000) [4]. The proposed control law allows parameterization of feasible state feedback and state estimation in an observer-based control setting. Compared to other types of LPV synthesis addressed above, the main motivation of the work reported in this paper is to develop an LPV L_2-gain control law that provides plant state estimation and does not require on-line information of parameter variation rates.

Observer-Embedded LPV L2-Gain Control Synthesis

Let the generalized model of a linear parameter varying (LPV) plant be represented as:

$$\begin{aligned}\dot{x}&=A(p)x+B_1(p)w+B_2(p)u\\ z&=C_1(P)x+D_{11}(p)w+D_{12}(p)u\\ y&=C_2(p)x+D_{21}(p)w+D_{22}(p)u\end{aligned}\tag{1}$$

with the L_2-gain performance specification:

$$\int_0^T\|z\|^2dt<\gamma^2\int_0^T\|w\|^2dt\quad\forall T>0\forall w;\ \ x(0)=0\tag{2}$$

Without loss of generality, the following simplifying assumptions are made to communicate the main theme of the work reported in this paper:

- The scheduling parameter p is one-dimensional;
- $\gamma=1,\ D_{11}=0,\ C_1^TD_{12}=0,\ D_{12}^TD_{12}=I,\ D_{21}B_1^T=0,$ $D_{21}D_{21}^T=I,\ D_{22}=0.$

The first assumption is extendable to a (multidimensional) gain scheduling parameter vector at the expense of additional numerical complexity. The second set of assumptions can be removed by a series of transformations among state, inputs and outputs.

Let us start with full state feedback control and define a Hamiltonian function H as:

$$H(x,u,w,p)=\dot{V}+\|z\|^2-\|w\|^2\tag{3}$$

where $V(x,p)$ is a positive-definite function that stores the information of the current state. If the following two conditions on the Hamiltonian H, for $\partial^2H/\partial w\partial u=0$, hold:

$$\frac{\partial H}{\partial u}(u=u^*)=0;\ \ \frac{\partial^2H}{\partial^2u}(u=u^*)>0;\tag{4}$$

$$\frac{\partial H}{\partial w}(w=w^*)=0;\ \ \frac{\partial^2H}{\partial^2w}(w=w^*)<0,\tag{5}$$

with the *minimum control* u^* and the *maximum disturbance* w^*, then

$$H(x,u^*,w^*,p)<0,\tag{6}$$

implies that $H(x,u^*,w,p)<0\ \forall w$. This renders the following inequality:

$$\begin{aligned}\int_0^T\|z\|^2dt&<\int_0^T\|w\|^2dt+V(x(0))-V(x(T))\\ &<\int_0^T\|w\|^2dt\quad\forall T>0\forall w,\end{aligned}$$

that is identical to the performance specification in Eq. (2).

If the storage function $V(x,p)$ is structured to be positive quadratic as:

$$V(x,p)=x^TX(p)x;\ \ X=X^T>0\ \ \forall p,\tag{7}$$

then the Hamiltonian function $H(x,u,w,p)$ becomes

$$\begin{aligned}H(x,u,w)&=x^T\dot{p}\frac{\partial X}{\partial p}x+2x^TX(Ax+B_1w+B_2u)+x^TC_1^TC_1x\\ &\quad+u^Tu-w^Tw.\end{aligned}\tag{8}$$

Combining Eqs. (4) and (5) with Eq. (8) yields:

$$u^*=-B_2^TXx;\ \ w^*=B_1^TXx\tag{9}$$

and then substituting Eq. (9) into Eq. (8) yields

$$H(x,u^*,w^*,p)=x^TQ_Xx,$$

$$Q_X\equiv\dot{p}\frac{\partial X}{\partial p}+A^TX+XA+C_1^TC_1-XB_2B_2^TX+XB_1B_1^TX,\tag{10}$$

If $Q_X<0\ \forall p$, then Eq. (6) is satisfied and so is the performance specification in Eq. (2).

Now let us consider the synthesis of the output feedback control with an embedded observer designed as:

$$\dot{\hat{x}}=A\hat{x}+B_1\hat{w}^*+B_2u+ZC_2^T(y-C_2\hat{x})\tag{11}$$

that is structurally similar to the Kalman filter where the matrix $Z(p)$ is yet to be determined. The calibration for maximum disturbance $\hat{w}^*$ is chosen as:

$$\hat{w}^*=B_1^TX\hat{x}\tag{12}$$

based on Eq. (9). Defining the state error vector $\tilde{x}\equiv x-\hat{x}$, Eq. (1) is subtracted from Eq. (11) to yield:

$$\dot{\tilde{x}}=(A+B_1B_1^TX-ZC_2^TC_2)\tilde{x}+(B_1-ZC_2^TD_{21})w-B_1B_1^TXx.\tag{13}$$

Following Eq. (3), a Hamiltonian function H for the output feedback is defined as:

$$H(x,\tilde{x},u,w,p)=\frac{dV(x,\hat{x})}{dt}+\|z\|^2-\|w\|^2\tag{14}$$

By separating the storage function $V(x,p)$ into two parts as:

$$V(x,\tilde{x},p)=x^TX(p)x+\tilde{x}^TZ^{-1}(p)\tilde{x};\ \ X=X^T>0;\ \ Z=Z^T>0\ \ \forall p,\tag{15}$$

and using Eqs. (1) and (13), the Hamiltonian function $H(x,\tilde{x},u,w,p)$ becomes:

$$\begin{aligned}H(x,\tilde{x},u,w)&=x^T\dot{p}\frac{\partial X}{\partial p}x+\tilde{x}^T\frac{\partial Z^{-1}}{\partial p}\tilde{x}+2x^TX(Ax+B_1w+B_2u)\\ &\quad+x^TC_1^TC_1x+u^Tu-w^Tw+2\tilde{x}^TZ^{-1}[(A+B_1B_1^TX\\ &\quad-ZC_2^TC_2)\tilde{x}+(B_1-ZC_2^TD_{21})w-B_1B_1^TXx]\end{aligned}\tag{16}$$

Equations (4) and (5) are now extended for output feedback control as:

$$\frac{\partial H(x,\tilde{x},u,w,p)}{\partial u}(u=u^*)=0;\ \ \frac{\partial^2H(x,\tilde{x},u,w,p)}{\partial^2u}(u=u^*)>0,\tag{17}$$

$$\frac{\partial H(x,\tilde{x},u,w,p)}{\partial w}(w=w^*)=0;\ \ \frac{\partial^2 H(x,\tilde{x},u,w,p)}{\partial^2 w}(w=w^*)<0, \tag{18}$$

and then Eqs. (16)–(18) yield to

$$w^*=B_1^TXx+(B_1-ZC_2^TD_{21})^TZ^{-1}\tilde{x}, \tag{19}$$

However, unavailability of the full information on the current state x prevents the minimum control from being chosen as: $u^*=-B_2^TXx$. Instead, we choose $u^*=-B_2^TX\hat{x}$ as the best approximation, using the available information of the estimated state $\hat{x}$. That is

$$u^*=-B_2^TX(x-\tilde{x}) \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (16) and several algebraic manipulations yield:

$$H(x,\tilde{x},u^*,w^*,p)=x^TQ_Xx+\tilde{x}^TQ_z\tilde{x} \tag{21}$$

$$Q_X\equiv\dot{p}\frac{\partial X}{\partial p}+A^TX+XA+C_1^TC_1-XB_2B_2^TX+XB_1B_1^TX \tag{22}$$

$$Q_Z=\dot{p}\frac{\partial Z^{-1}}{\partial p}+Z^{-1}(A+B_1B_1^TX)+(A+B_1B_1^TX)^TZ^{-1}$$
$$-C_2^TC_2+XB_2B_2^TX+Z^{-1}B_1B_1^TZ^{-1} \tag{23}$$

Based on Eq. (21), we have:

$$Q_X<0;\ \ Q_Z<0;\ \ X=X^T>0;\ \text{and}\ \ Z=Z^T>0 \tag{24}$$

that guarantee $H(x,u^*,w,p)\leqslant 0\ \forall w$, which is equivalent to:

$$\int_0^T\|z\|^2dt<\int_0^T\|w\|^2dt\ \ \ \forall T>0\forall w,$$

and is identical to the performance specification in Eq. (2).

Construction of Feasible Control and Estimation Laws

The addition of Eqs. (22) and (23) yields:

$$Q_X+Q_Z=\dot{p}\frac{\partial X}{\partial p}+\dot{p}\frac{\partial Z^{-1}}{\partial p}+(Z^{-1}+X)A+A^T(Z^{-1}+X)$$
$$+(Z^{-1}+X)B_1B_1^T(Z^{-1}+X)-C_2^TC_2+C_1C_1^T. \tag{25}$$

Let a p-dependent matrix Y be introduced and defined as:

$$Z^{-1}=Y^{-1}-X, \tag{26}$$

Denoting $Q_Y\equiv Q_X+Q_Z$, we have

$$Q_Y\equiv\dot{p}\frac{\partial Y^{-1}}{\partial p}+Y^{-1}A+A^TY^{-1}+Y^{-1}B_1B_1^TY^{-1}-C_2^TC_2+C_1^TC_1, \tag{27}$$

Thus, Eq. (24) becomes equivalent to:

$$Q_X<0,\ \ Q_Y-Q_X<0,\ \text{and}\ \ X-Y^{-1}<0;\ \ \forall p\ \ \forall\dot{p}. \tag{28}$$

The formulation of Eq. (28) can be expressed as parameterization in terms of a free pair of function-valued matrices, $Q(p,\dot{p})>0$ and $S(p,\dot{p})>0$, as follows:

$$\dot{p}\frac{\partial X}{\partial p}+A^TX+XA+C_1^TC_1-XB_2B_2^TX+XB_1B_1^TX+Q(p,\dot{p})=0, \tag{29}$$

$$\dot{p}\frac{\partial Y^{-1}}{\partial p}+Y^{-1}A+A^TY^{-1}+Y^{-1}B_1B_1^TY^{-1}-C_2^TC_2+C_1^TC_1$$
$$+S(p,\dot{p})=0\ \ \ MAp\ \ \ \forall\dot{p}, \tag{30}$$

$$X(p)-Y^{-1}(p)<0, \tag{31}$$

$$Q(p,\dot{p})-S(p,\dot{p})<0. \tag{32}$$

Any pair of positive-definite matrices, $Q>0$, $S>0$, which could be dependent on both p and $\dot{p}$, determines a feasible observer-embedded LPV L_2-gain controller in terms of $X>0$, $Y>0$. The features of the proposed LPV L_2-gain control law are summarized below:

- ***Feature 1***: The internal structure of the feasible observer-embedded LPV L_2-gain controller can be realized in the sense that increasing Q emphasizes control and increasing S emphasizes estimation.
- ***Feature 2***: The allowable parameter variation rate should be bounded if Eqs. (29)–(32) yield feasible solutions. Specifically, Eqs. (29) and (30) represent two partial differential equations in terms of two independent variables p and $\dot{p}$ without specified boundary conditions. The pair of algebraic inequalities in Eqs. (31) and (32) serve as constraints in the searching domain (X, Y, R, S). The solution of Eqs. (29)–(32) is strongly dependent on boundary conditions that can be chosen as the freely regulated positive-definite matrices R and S.
- ***Feature 3***: The control and estimation laws are parametrically dependent on the scheduling parameter p but not on $\dot{p}$. So, there is no need for on-line measurements of the parameter variation rate $\dot{p}$. Furthermore, the allowable parameter variation rate is bounded, or leads to $\dot{p}(\partial X/\partial p)+Q(p,\dot{p})=0$ that may no longer represent a parameter in an LPV system.

Let the parameter variation rate $\dot{p}$ be bounded within a rectangle

$$\breve{\beta}\leqslant\dot{p}\leqslant\hat{\beta}, \tag{33}$$

where $\{\breve{\beta},\hat{\beta}\}$ represents vertex of the rectangle, and the parameter p is bounded by $0\leqslant p\leqslant\pi$ without losing the generality. In such case, the following differential inequalities can be formulated to help solve a feasible solution of Eqs. (29)–(32):

$$\begin{bmatrix} -\beta\frac{\partial X^{-1}}{\partial p}+AX^{-1}+X^{-1}A^T-B_2B_2^T & X^{-1}C_1^T & B_1 \\ C_1X^{-1} & -I & 0 \\ B_1^T & 0 & -I \end{bmatrix}<0;$$
$$\beta\in\{\breve{\beta},\hat{\beta}\}, \tag{34}$$

$$\begin{bmatrix} +\beta\frac{\partial Y^{-1}}{\partial p}+Y^{-1}A+A^TY^{-1}-C_2^TC_2 & Y^{-1}B_1 & C_1^T \\ B_1^TY^{-1} & -I & 0 \\ C_1 & 0 & -I \end{bmatrix}<0;$$
$$\beta\in\{\breve{\beta},\hat{\beta}\}, \tag{35}$$

$$\begin{bmatrix} Y^{-1} & I \\ I & X^{-1} \end{bmatrix}>0;\ \ X^{-1}>0;\ \ Y^{-1}>0, \tag{36}$$

$$\hat{S}-\hat{Q}>0;\ \ \breve{S}-\breve{Q}>0, \tag{37}$$

where the nonconvex formulation consists of a set of differential linear matrix inequalities (LMIs) in Eqs. (34)–(36) and a feasibility index in Eq. (37) with

$$\hat{Q} = -\left(\hat{\beta}\frac{\partial X}{\partial p} + A^T X + XA + C_1^T C_1 - XB_2 B_2^T X + XB_1 B_1^T X\right), \tag{38}$$

$$\hat{S} = -\left(\hat{\beta}\frac{\partial Y^{-1}}{\partial p} + Y^{-1}A + A^T Y^{-1} + Y^{-1}B_1 B_1^T Y^{-1} - C_2^T C_2 + C_1^T C_1\right), \tag{39}$$

and similarly for $\check{Q}$ and $\check{S}$. The inequalities in Eq. (37) make the embedding of a feasible observer conservative relative to an LMI-based synthesis like (Wu, 2000 [10]). A feasible solution of Eqs. (34)–(37) is to be searched from the three convex differential LMIs in Eqs. (33)–(36) until Eq. (37) is satisfied.

An LPV feasible solution (X^{-1}, Y^{-1}) for Eqs. (34)–(37) is considered to be a perturbation of the inverse of gain-scheduled solution (X_0^{-1}, Y_0^{-1}) via Fourier-sin series expansion as:

$$X^{-1}(p) = X_0^{-1}(p) + \sum_{k=1}^{n} X_k \sin(kp); \quad Y^{-1}(p) = Y_0^{-1}(p) + \sum_{k=1}^{n} Y_k \sin(kp), \tag{40}$$

where the solution (X_0, Y_0) of Eq. (40) is the stable solution of the following two gain-scheduled Riccati equations for $0 \leqslant p \leqslant \pi$:

$$A^T X_0 + X_0 A + C_1^T C_1 - X_0 B_2 B_2^T X_0 + X_0 B_1 B_1^T X_0 = 0, \tag{41}$$

$$A Y_0 + Y_0 A^T + B_1 B_1^T - Y_0 C_2^T C_2 Y_0 + Y_0 C_1^T C_1 Y_0 = 0, \tag{42}$$

which can be solved based on the Riccati operator on proper Hamiltonian matrices (Doyle et al., 1988 [12]) for $\forall p \in [0, \pi]$. Using the Fourier expansion in Eq. (40), it follows that the LPV solution (X, Y) and the gain-scheduled solution (X_0, Y_0) have the same boundary conditions at both ends of the parameter domain $p = 0$ and $p = \pi$. The numerical procedure to find a feasible solution of Eqs. (34)–(37) is presented as follows:

[***Step 1***]: Start at $n = 0$. If the pair $(X^{-1}, Y^{-1}) = (X_0^{-1}, Y_0^{-1})$ satisfies Eqs. (34)–(37), then stop and the gain-scheduled solution is the choice among the feasible LPV solutions; else go to Step 2.

[***Step 2***]: Increase n by 1, [i.e., $n \leftarrow (n+1)$]. Use a numerical tool (for example, MATLAB LMI Toolbox) to obtain a feasible solution of Eqs. (34)–(36) in terms of the decision matrices $X_0^{-1}(p)$, X_1, $X_2 \cdots X_n$, $Y_0^{-1}(p)$, Y_1, $Y_2 \cdots Y_n$.

[***Step 3***]: If the feasibility index of matrix pairs $(\hat{S} - \hat{Q}, \check{S} - \check{Q})$ is positive definite as in Eq. (37), then stop; else go back to Step 2.

[***Step 4***]: Having positivity of the feasibility index in Step 3, if a feasible solution is found to satisfy Eqs. (34)–(36), then stop and this solution is the choice among the feasible LPV solutions; else go back to Step 1 to obtain a feasible solution for the revised robust performance (i.e., nominal performance plus stability robustness) criteria of the generalized plant in Eq. (1).

Simulation Experiments

This section presents the results of simulation experiments to elucidate LPV L_2-gain control design. The set of simulation experiments is based on an exact model (i.e., with no modeling uncertainties) of a two-degree-of-freedom mass-spring-damper vibration system with varying damping and stiffness under exogenous inputs of plant disturbances and sensor noise. The mean values of the first mass, second mass, first damping coefficient, second damping coefficient, first spring constant, and second spring constant are set at: $m_1 = 1$; $m_2 = 1$; $\zeta_1 = 0$; $\zeta_2 = 0$; $k_1 = 1$; and $k_2 = 3$, respectively. The control law processes the (measured)

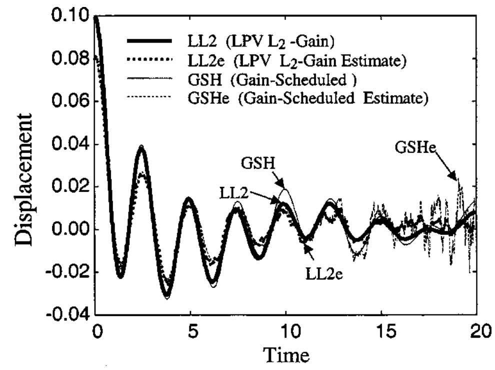

Fig. 1 First mass displacement with one-dimensional (1-D) scheduling

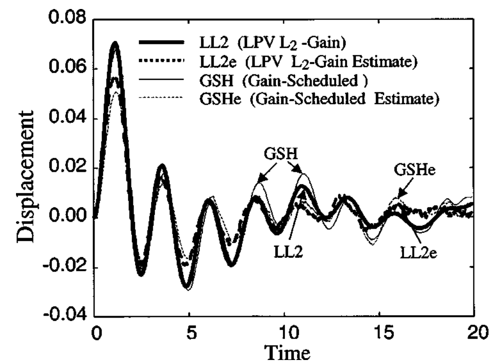

Fig. 2 Second mass displacement with 1-D scheduling

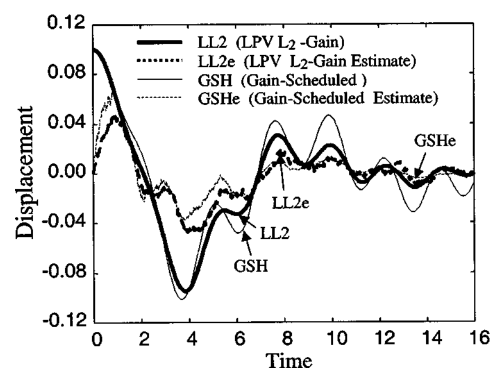

Fig. 3 First mass displacement with 2-D scheduling

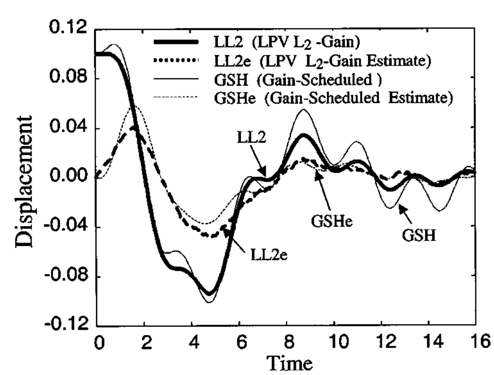

Fig. 4 Second mass displacement with 2-D scheduling

displacement signal of the second mass to determine the control force applied on the first mass. The temporal evolution of the disturbance is generated by zero-mean band-limited Gaussian noise. These signals are used as the (force) input to the first mass and as sensor noise in the displacement measurement of the second mass. The two exogenous signals of noise are independent and identically distributed.

Four curves in each of Figs. 1–4 present transient responses of mass displacements and their estimates under LPV-L_2-gain control and gain-scheduled H_∞ control. While the gain-scheduled H_∞ control is obtained by H_∞/μ-synthesis at each operating point without considering the variation rate of the scheduling parameters, the LPV L_2-gain control takes into account the impact of the parameter variation rate within an allowable bound. Each of these two control laws is tested for full state as well as output feedback where the state vector x is replaced by its estimate $\hat{x}$. For each controller, the solid line represents a mass displacement (i.e., state x_1 or x_2) derived from the plant model; and the dotted line represents the estimated value (i.e., state $\hat{x}_1$ and $\hat{x}_2$) of the corresponding mass displacement derived from the observer. The initial conditions of the plant states and the observer states are intentionally set at different values of $[0.1\ 0.1\ 0.0\ 0.0]^T$ and $[0.0\ 0.0\ 0.0\ 0.0]^T$, respectively, to examine the ability of each controller to maintain the steady-state state estimation errors to small values in the presence of exogenous disturbances.

In Figs. 1 and 2, the scheduling parameter is the damping ratio ζ_1 that varies in a single-frequency motion within the range of $[-0.3,\ 0.3]$ with the maximum absolute variation rate of 0.05 while the remaining five parameters are held constant at their respective mean values. In Figs. 3 and 4, in addition to ζ_1, we introduce another scheduling parameter k_2 that varies in a single-frequency motion within the range of [2,4] and with the maximum absolute variation rate of 0.25. The remaining four parameters are held constant at their respective mean values. From the temporal trajectories of the mass displacements and their estimates, it appears that LPV L_2-gain control exhibits superior system performance and estimation accuracy compared to gain-scheduled H_∞ control. This feature of LPV L_2-gain control becomes more significant as the dimension of the scheduling parameter increases.

Summary and Conclusions

This paper formulates a procedure for synthesis of observer-embedded linear parameter varying (LPV) L_2-gain control laws using linear matrix inequalities (LMIs). The LMIs are formulated to solve a feasible observer-embedded LPV L_2-gain control law that does not require on-line measurements of the scheduling parameter variation rate. Results of simulation experiments on a two-degree-of-freedom mass-spring-damper system are presented to evaluate the LPV L_2-gain control relative to gain-scheduled H_∞ control.

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Receding Horizon Stabilization of a Rigid Spacecraft With Two Actuators

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In this paper, a receding-horizon control, using systematic projection on a Chebyshev's polynomial basis, is proposed for the stabilization of a rigid spacecraft operating with only two actuators. The proposed scheme privileges the speed of the algorithm. Simulations on SPOT4 spacecraft with a robustness test are provided. [DOI: 10.1115/1.1591806]

1 Introduction

If the stabilization of the angular velocities of a rigid spacecraft operating with two torques instead of three usually available can be achieved with a smooth static state feedback (Aeyels and Szafranski, 1988 [1]), it is no longer true for the attitude since the necessary condition for static continuous feedback stabilization then fails to be satisfied (Byrnes and Isidori, 1991 [2]). The stabilization of the rigid spacecraft in failure mode hence requires discontinuous (see, e.g., Crouch, 1984 [3]; Krishnan et al., 1992 [4]) or time varying (see Morin and Samson, 1997 [5], and the references therein) tools; this last approach often resulting in costly in term of energy or wear rapid oscillations.

The proposed approach is based on receding horizon. In order to avoid the usual heavy computer cost of this strategy, the minimization is avoided; on the other hand, sole the attractivity can formally be established. The approach gives reasonable variations

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