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Estimation of slowly varying parameters in nonlinear systems via symbolic dynamic filtering ☆

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Abstract

This paper introduces a novel method for real-time estimation of slowly varying parameters in nonlinear dynamical systems. The core concept is built upon the principles of symbolic dynamic filtering (SDF) that has been reported in literature for anomaly detection in complex systems. In this method, relevant system outputs are measured, at different values of a critical system parameter, to generate an ensemble of time series data. The space of wavelet-transform coefficients of time series data is partitioned to generate symbol sequences that, in turn, are used to construct a special class of probabilistic finite state automata (PFSA), called the *D*-Markov machine. The parameter is estimated based on the statistical information derived from the PFSA. The bounds and statistical confidence levels, associated with parameter estimation, are also computed. The proposed method has been validated in real time for two nonlinear electronic systems, governed by Duffing equation and van der Pol equation, on a laboratory apparatus.

Keywords: Parameter estimation; Time series analysis; Symbolic dynamics

1. Introduction

Various analytical tools have been reported in literature for parameter estimation in nonlinear dynamical systems. One of the early methods [1] makes use of statistical linearization and other methods include unscented Kalman filtering (UKF) [2,3], Markov chain Monte Carlo (MCMC) [4], particle filtering (PF) [5], genetic algorithms (GA) [6], maximum likelihood estimation (MLE) [7] and wavelet-based tools [8].

Early detection of small changes has motivated the usage of symbolic dynamic filtering (SDF) [9] for parameter estimation in nonlinear dynamical systems. The core concept of SDF is built upon the fundamental principles of finite state automata, pattern recognition, and information theory, and relies on the following two basic assumptions:

- The system behavior is quasi-stationary at the fast-time scale of system dynamics.
- Observable non-stationary behavior of the dynamical system can be associated with parametric or non-parametric changes evolving at a slow-time scale.

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SDF is in some ways similar to commonly used Bayesian filtering tools like PF. While a PF [5] is constructed as a Markov process on a finite-dimensional state space, SDF is constructed as a finite state automaton with finite memory [9]. Specifically, the state variables in PF are real-valued and the automaton states of SDF may be considered analogous to energy states in quantum statistical mechanics [9]. Only the state transition matrix and hence the probabilities of the automaton states are allowed to vary with time. However, SDF differs from PF and other Bayesian filtering methods in many aspects. Existence of a dynamic model in the state-space setting is an underlying assumption for construction of PF and UKF. In contrast, SDF does not require a model for the system because a hidden Markov model (HMM) is generated from time series data. In addition, SDF is computationally a lot less intensive as compared to other techniques such as GA and PF that requires estimation of conditional probability density of a continuous variable. In the case of SDF, no such computation is necessary as state probability vectors are determined from a counting process. Thus, SDF is very easy to implement and is suitable for online parameter estimation.

An important advantage of SDF is its robustness to measurement noise [10]. A possible disadvantage of SDF could be loss of information in conversion of a time series data set to a symbol sequence due to coarse graining. However, with suitable partitioning and appropriate choice of SDF parameters, it has been observed that information necessary for accurate estimation is retained in symbol sequences [10].

Building on the earlier work of Ray [9], Rajagopalan and Ray [10], and Chin et al. [11], this paper makes use of SDF to formulate and validate, by laboratory experimentation, a novel concept for real-time estimation of a slowly varying parameter in nonlinear dynamical systems. Often such dynamical systems are stimulated with *a priori* known exogenous inputs, or are self-excited, to recognize behavior patterns of evolving dynamics from the observed quasi-stationary response. In the sequel, two nonlinear systems are analyzed and experimentally validated on a laboratory apparatus; one is the Duffing system [12] that is exogenously excited with a periodic input to capture possible bifurcation and chaotic behavior and the other is the van der Pol system [13] representing self-excited nonlinear oscillations.

The paper is organized in five sections including the present one. Section 2 briefly reviews the salient features of SDF. Section 3 describes the two nonlinear dynamical systems investigated in the paper and Section 3.3 presents a brief description of the laboratory apparatus that has been used for conducting the experiments. The experimental results, including the details of statistical parameter estimation, are discussed in Section 4. Section 5 summarizes and concludes the paper with recommendations for future work.

2. Review of Symbolic dynamic filtering (SDF)

This section summarizes the salient features of Symbolic dynamic filtering (SDF) that has been reported in literature [9,11,10,14]. Behavior identification in dynamical systems with a slowly varying parameter can be formulated as a two-time-scale problem if the physical process is assumed to have quasi-stationary dynamics on the *fast-time scale* and any observable non-stationary behavior is associated with changes occurring on the *slow-time scale*. In other words, the variations in the internal dynamics of the system are assumed to be negligible on the fast-time scale, while pattern changes due to parameter variations may become significant on the slow-time scale. In general, a long time span in the fast-time scale is a tiny (i.e., order(s) of magnitude smaller) interval in the slow-time scale, over which the system dynamics are assumed to have stationary behavior. Nevertheless, the notion of fast-time and slow-time scales is dependent on the specific application and operating environment.

From the above perspectives, the problem of slowly varying parameter estimation is categorized into two subsets: (i) The forward problem and (ii) the inverse problem.

2.1. Forward problem

The primary objective of the forward problem is identification of the changes in behavioral patterns of the system dynamics due to evolving parameter changes on the slow-time scale. Specifically, the forward problem aims at detecting the deviations in the statistical patterns in the time series data, generated at different time epochs in the slow-time scale, from the nominal behavior pattern. The solution procedure of the forward problem requires the following steps:

F1. *Collection of time series data sets* (*at the fast-time scale*) *from the available sensor*(*s*) *at* *different slow-time epochs*: Relevant information-bearing outputs of the nonlinear system under consideration are sampled at a suitable rate to generate an ensemble of time series data. The period of data acquisition is required to be significantly small compared to the period of parameter changes.

F2. *Generation of wavelet coefficient data sets*: Wavelet transform [15] is adopted because the time series data may be noisy and undergo non-stationary variations under slow variations of the parameters. In this context, an appropriate wavelet basis is chosen and the scales for wavelet analysis are then selected based on frequencies of interest [10].

F3. *Partitioning of the wavelet coefficient space for symbol sequence generation*: The partition is determined based on the data set under the nominal condition. This partition remains invariant in further analysis. Symbol sequences are generated from all data sets based on this partition.

F4. *Construction of a special class of probabilistic finite state automata (PFSA), called the D-Markov machine* [9]: The structure of the D-Markov machine remains fixed for all slow-time epochs but the elements of the resulting state transition matrix do change with variations of the parameter in the slow-time scale.

F5. *Computation of pattern vectors, defined as the (quasi-stationary) state probability vectors of the PFSA of the D-Markov machine at slow-time epochs*: Measures of the deviations of the current pattern vector from the nominal pattern vector, called *deviation measures*, are obtained in the slow-time scale as the parameter varies. An appropriate distance function from different viable options is adopted for this purpose [9,11].

2.2. Inverse problem

The objective of the inverse problem is to infer anomalies and to provide estimates of parameters associated with anomalies in real time. The decisions are based on the information generated from the forward problem. In case of deterministic systems, the solution of the inverse problem is straightforward, i.e., the estimates can be obtained by mapping the results of the forward problem. However, in case of stochastic systems, the problem may be ill-posed. That is, it may not always be possible to identify a unique anomaly pattern based on the observed behavior of the dynamical system. In such cases, estimates can only be obtained with certain confidence intervals (CIs).

The solution to inverse problem requires the following steps:

I1. Based on anomaly measure profiles generated in the forward problem, a statistical relationship is determined between anomaly measure and the value of parameter associated with the anomaly. In particular, probability distributions of parameter for various values of anomaly measures are obtained. Hypotheses tests are performed to determine goodness-of-fit of the distributions. For example, mean and variance associated with a two-parameter distribution provide adequate statistical information on the bounds and confidence levels of the estimated parameter.

I2. Time series data are collected (in the fast-time scale) under the similar operating conditions as in Step F1 of the forward problem. Data are analyzed as discussed previously to generate pattern vectors. The anomaly measure at the current time epoch is then calculated by quantifying the deviation of the current pattern vector from the nominal vector.

I3. Detection, identification and estimation of an anomaly (if any) based on the computed anomaly measure and the probability distribution derived in Step I1 of the inverse problem.

2.3. The parameter estimation procedure

The details of the forward problem in SDF, pertaining to the Duffing equation, have been reported in [9,10] and its performance is demonstrated to be superior to that of other standard pattern identification tools in [11,14]. The SDF analysis of van der Pol system is presented in this paper. Efficacy of the inverse problem in SDF for statistical estimation of system parameter(s) has not yet been demonstrated.

This paper presents a solution to the inverse problem in SDF and illustrates its usage in real-time parameter estimation for both externally stimulated and self-excited systems. The Duffing equation represents an externally stimulated system and the van der Pol equation represents a self-excited system. In the present experimental setting of both Duffing and van der Pol systems, a single critical parameter is changed in small steps to show that the SDF method is capable of distinguishing, in real

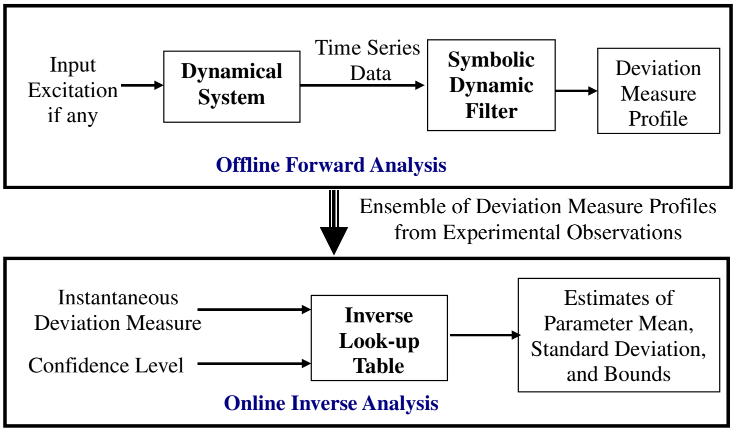

Fig. 1. Schematic diagram for symbolic dynamic filtering (SDF).

time, variations due to gradually evolving anomalies. For statistical analysis, the experiments in the forward problem are repeated a sufficiently large number of times by replacing selected component(s) (e.g., a resistor) in the laboratory apparatus, each time with a similar component. The objective here is to establish robustness of the estimate with respect to component-induced parametric uncertainties in identically manufactured systems.

Fig. 1 exhibits a schematic representation of the complete SDF procedure for parameter estimation, where the forward problem is solved offline to generate the information for online execution of the inverse problem that relies on the deviation measure at the current (slow-time) epoch in addition to the information generated in the forward problem.

3. Nonlinear system models and the laboratory apparatus

This section describes the governing equations of two nonlinear dynamical systems that are analyzed and validated on a laboratory apparatus.

3.1. The Duffing system

The externally excited Duffing system [12] is governed by a second-order non-autonomous nonlinear differential equation:

$$\frac{d^2y(t)}{dt^2} + \beta\frac{dy}{dt} + y(t) + y^3(t) = A\cos(\omega t). \qquad (1)$$

The amplitude A was equal to 22.0 and $\omega = 5.0\,\text{rad/s}$. Behavior of the Duffing system is sensitive to changes in the parameter β. For β in the range of 0.10–0.27, the behavior of the Duffing system in Eq. (1) is largely similar though there are small variations. However, as β increases to ≈ 0.28, the Duffing system undergoes a period doubling bifurcation and the behavior remains essentially unchanged for further increases in β. The phase plots for selected values of β can be found in [9]. In each experiment, 12,000 data points of the two phase variables y and $dy(t)/dt$ sampled at a uniform rate of 100 Hz were collected for different values of β.

3.2. The van der Pol system

The unforced van der Pol system [13] is governed by a second order autonomous nonlinear differential equation

$$\frac{d^2y(t)}{dt^2} - \mu(1-y^2(t))\frac{dy(t)}{dt} + y(t) = 0. \qquad (2)$$

Behavior of the van der Pol system is sensitive to changes in the parameter μ. For small values of μ, the stationary phase trajectory is a smooth orbit, largely similar to a circle of radius 2.0. As μ increases, this shape gets distorted and the distortion is very high around μ equal to 3.0. In each experiment, 4500 data points of the two phase variables y and $dy(t)/dt$ sampled at a uniform rate of 100 Hz were collected for different values of μ.

3.3. Description of the laboratory apparatus

The laboratory apparatus is a combination of hardware electronic circuit designed with resistors (R), capacitors (C) and operational amplifiers and a computer interfaced with the circuit. The circuit consists of R–C networks which model the linear dynamics of the process. The nonlinearity is generated in the computer. The adder sums up the input signal and the terms generated by the computer and feeds it back to the circuit, thereby making the overall system nonlinear.

Linear IC LM-741 chips were chosen to build the voltage amplifier and adder circuits since the operating frequencies are low ($<10\,\text{rad/s}$). The R–C values of both the networks were chosen to be equal. The resistance R was chosen to be $500\,\Omega$ and the capacitance C to be 1.00 mF. The software controlling the setup is coded in MATLAB. The software utilizes functions from the Data Acquisition toolbox of MATLAB. A single module of the apparatus can simulate differential equations of order four. Multiple modules in cascade can be used to realize higher-order equations.

Keithley KPCI 1801-HC plug-in board was used to interface the computer with the circuit. This board has the maximum sampling rate of 333 kilo-samples/s, 64 A/D channels (12 bit), 2 D/A channels and operates in the range of -5 to $+5$ V. The 12 bit A/D converters provide quantization levels of approximately 2.5 mV. The process dynamics in the experiments were of the order of hertz ($<5\,\text{Hz}$). The process is sampled at 2000 Hz. The derivative and other terms were computed numerically using such samples. This plug-in board can be used in conjunction with the Data Acquisition toolbox of MATLAB.

4. Experimental results and discussion

This section presents the experimental results of parameter estimation in Duffing system and the van der Pol system. The analysis problem for the Duffing system is explained in detail in [9,10]. A similar procedure is followed for the van der Pol system as well.

This section focuses on the inverse problem of parameter estimation based on computed values of the deviation measure (see item F5 in Section 2). The parameter is a slowly varying random process and is therefore assumed to be a random variable at each slow-time epoch, for which the deviation measures are the only observables. To account for the inherent uncertainties in the system components and to ensure robust estimation, a large number of experiments are performed and the deviation measures are calculated from observed time series data during every experiment, with the objective of estimating the unknown parameter.

The range of the computed deviation measure is discretized into finitely many levels. A pattern matrix is created where each column represents the spread of the parameter for a particular value of the deviation measure. A statistical distribution is hypothesized for the spread of the parameter and the goodness-of-fit of the hypothesized distribution is assessed with χ^2 and Kolmogorov–Smirnov tests. CIs are then assessed for confidence levels at 99% and 95%.

4.1. Results on the Duffing system

Section 4.1.1 briefly presents the results of forward problem. The inverse problem results are provided in Section 4.1.2.

4.1.1. Solution to forward problem

The first step in the forward problem is selection of the wavelet basis. Wavelet 'gaus1' [15] is chosen as the wavelet basis. The next step is selection of scales. Following the procedure described in [10], the scales are selected to be 64.76 and 71.23. The wavelet coefficients are obtained at these scales and stacked to form the scale series. The alphabet size is chosen to be $|\Sigma| = 6$. The maximum entropy (ME) partition is obtained with scale series data corresponding to the nominal condition at $\beta = 0.11$. The scale series is then converted to symbols based on the partition. With the selection of depth $D = 1$, the number of states in the D-Markov machine becomes $|\Sigma|^D = 6$.

Using the above partition, symbol sequences are generated from all other scale series data sets. As the dynamics of the system change due to variations in β, the statistics of the symbol sequences are also altered and so are the probability distributions. Because of the ME criterion, the probability distribution is uniform at the nominal condition of $\beta = 0.11$. As β gradually increases, the distribution deviates from being uniform. The angle measure, defined in [9], is used for quantifying the changes in the statistical patterns of the PFSA of the D-Markov machine. The profile of the angle measure, for one experiment, is shown in Fig. 2, where the dynamical system approaches a bifurcation point at $\beta \approx 0.29$.

4.1.2. Solution to inverse problem

Time series data are generated for different values of β, and the experiment is repeated about 40

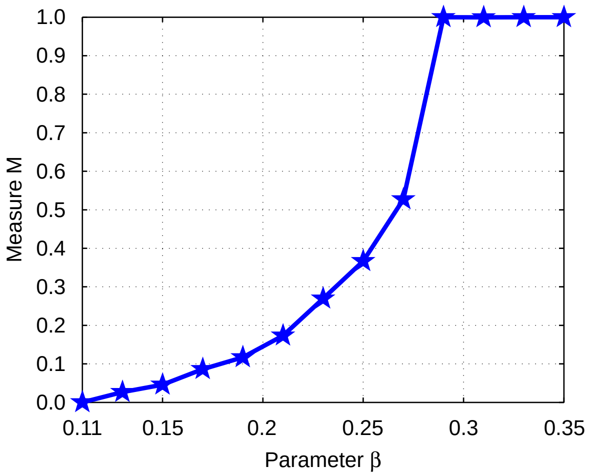

Fig. 2. A typical plot of deviation measure for the Duffing system.

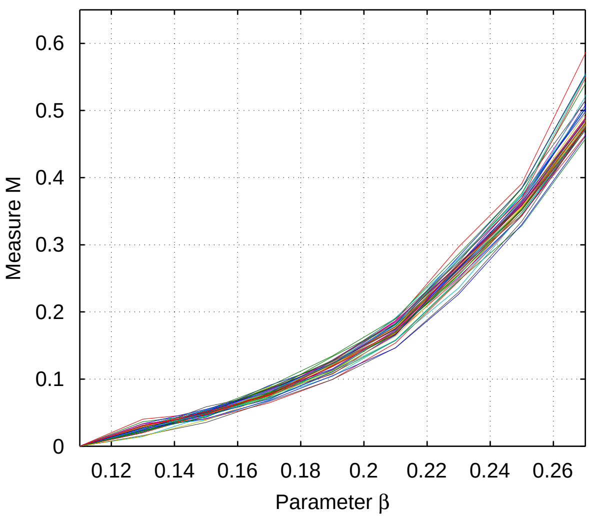

Fig. 3. Ensemble of deviation measure plots for the Duffing system.

number of times to acquire stochastic information on the system dynamics. For each run of the experiment, deviation measures M are calculated for different values of the parameter β. The ensemble of deviation measure plots is presented in Fig. 3. The profiles of individual samples are plotted up to $\beta = 0.27$ before the bifurcation point is reached, because the characteristics of the dynamical system drastically changes as the dynamical system approaches bifurcation condition at $\beta \approx 0.29$.

The range of deviation measure is discretized into $n = 20$ levels. A pattern matrix is constructed where each column represents the spread of the parameter β for a particular segment of the deviation measure. For the elements of each column, a two-parameter *log-normal* distribution is hypothesized, and its goodness-of-fit is examined by both χ^2 and Kolmogorov–Smirnov tests [16]. For each of the 20 data sets, the hypothesis of two-parameter *log-normal* distribution passed the χ^2-test at 10% significance level [16]. This satisfies the conventional standard of 5% significance level. Also, for each of the 20 measure levels, the hypothesis passed the Kolmogorov–Smirnov test at 30% significance level, which again exceeds the conventional standard of 5% significance level. The fitted two-parameter log-normal density plots are presented in Fig. 4 presents for selected values of deviation measures M.

Once the two-parameter log-normal distributions are obtained, the bounds of parameter estimation for different confidence levels are computed. Table 1 provides the mean and variance of the hypothesized distributions and the CIs. It is seen that the parameter estimate $\hat{\beta}$ is significantly greater than the standard deviation $\hat{\sigma}$. This suggests that the estimate is close to the true value of the parameter.

For testing validity of the proposed distribution, experiments were conducted with three arbitrarily chosen resistances belonging to the allowable tolerance levels. Time series data were generated at different β values and the measures were computed. The plot in Fig. 5 depicts the deviation measure M and the associated bounds for 90% CI. It is seen that all three measure plots lie within the 90% CI bounds.

4.2. *Results on the van der Pol system*

Section 4.2.1 presents the results of forward problem. The inverse problem is described in detail in Section 4.2.2.

4.2.1. *Solution to forward problem*

Wavelet ‘gaus1’ [15] is chosen as the wavelet basis. The suitable scales are found to be 28.57 and 50.00. The wavelet coefficients are obtained at these scales and stacked to form the scale series [10]. The alphabet size is also chosen to be $|\Sigma| = 6$. The ME partition is obtained with scale series data corresponding to the parameter $\mu = 0.2$. The scale series is then converted to symbols based on this partition. With depth $D = 1$, the number of states in the D-Markov machine becomes 6. Symbol sequences are then generated from all other scale series data sets, based on the partition mentioned above. As the

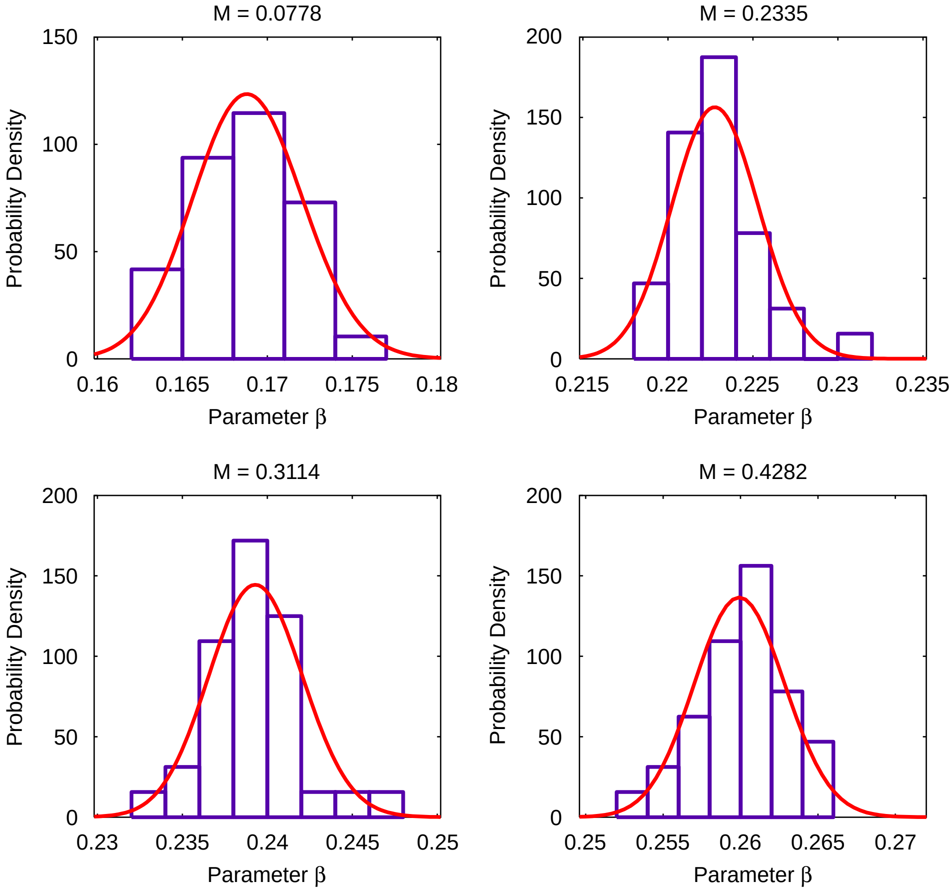

Fig. 4. Probability density plots for the Duffing system.

Table 1
Predicted value of β and confidence intervals for the Duffing system

Observed measure M	Estimates							
	$\hat{\beta}$	$\hat{\sigma}$	99%		95%		90%	
			β_{min}	β_{max}	β_{min}	β_{max}	β_{min}	β_{max}
0.0778	0.1689	3.233×10^{-3}	0.1607	0.1774	0.1626	0.1753	0.1636	0.1743
0.2335	0.2228	2.550×10^{-3}	0.2163	0.2295	0.2178	0.2278	0.2186	0.2270
0.3114	0.2393	2.762×10^{-3}	0.2323	0.2466	0.2340	0.2448	0.2348	0.2439
0.4282	0.2600	2.920×10^{-3}	0.2525	0.2676	0.2543	0.2657	0.2552	0.2648

dynamics of the van der Pol system change due to variations in the parameter μ, the statistics of the symbol sequences are altered and so are the probability distributions. As μ changes, the distribution deviates from being uniform. The angle measure [9] is used for quantifying the changes in the distributions. The profile of the angle measure, for one experiment, is shown in Fig. 6.

4.2.2. Solution to inverse problem

Time series data sets are generated for different values of the parameter μ, and the experiment is repeated about 40 times to acquire ensemble of statistical information about the system dynamics. For each run of the experiment, measures are calculated for different μ values. The ensemble of measure plots is presented in Fig. 7.

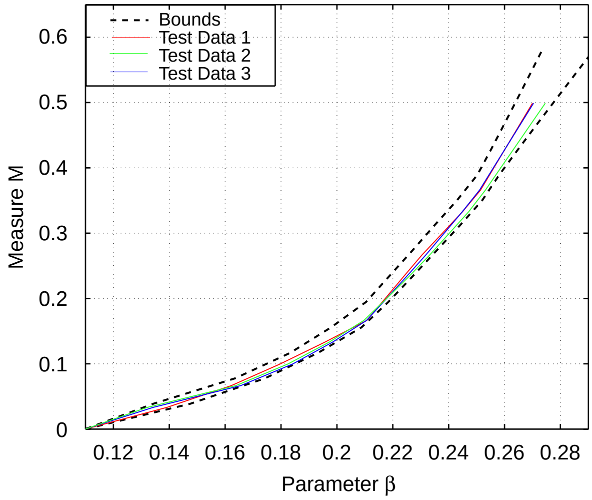

Fig. 5. Confidence intervals and test results for the Duffing system.

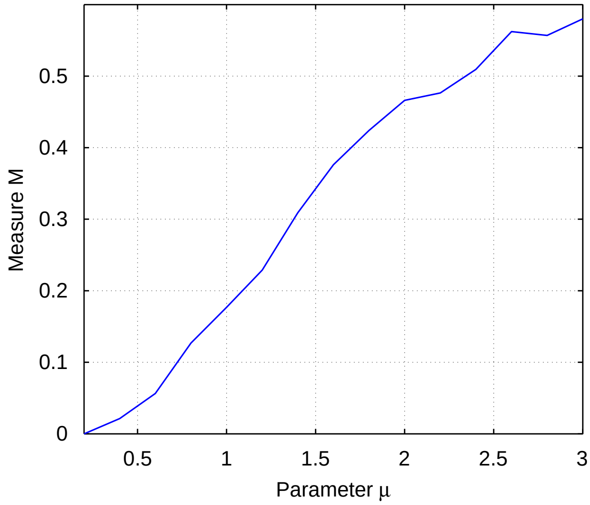

Fig. 6. A typical plot of deviation measure for the van der Pol system.

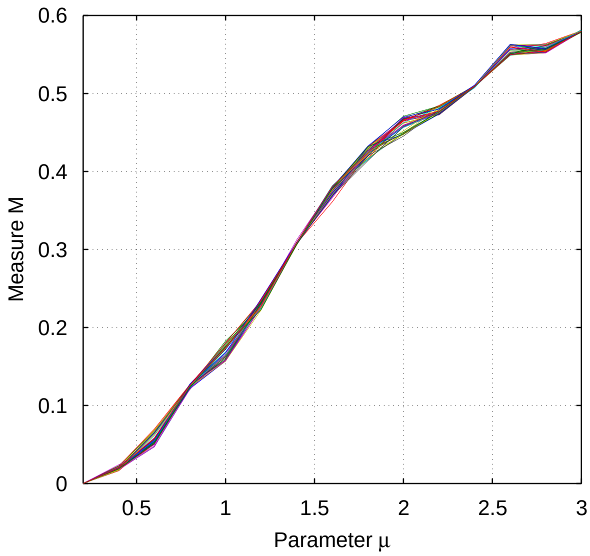

Fig. 7. Ensemble of deviation measure plots for the van der Pol system.

The range of measure is next discretized into $n = 20$ levels. A pattern matrix is constructed where each column represents the spread of the parameter μ for a particular value of measure. For the elements of each column, a two-parameter *log-normal* distribution is hypothesized, and its goodness-of-fit is examined by both χ^2 and Kolmogorov–Smirnov tests [16]. For all 20 measure levels, the hypothesis of two-parameter log-normal distribution passed the χ^2 test at 10% significance level the Kolmogorov–Smirnov test [16] at 30% significance level, which satisfies the conventional standard of 5% significance level. The probability density plots are presented in Fig. 8 for selected values of deviation measures.

Once the log-normal distributions are obtained, the bounds for different CIs are computed. Table 2 provides the mean and variance of the hypothesized distributions and the CIs associated with different confidence levels. It is seen that the parameter estimate $\hat{\mu}$ is significantly greater than the standard deviation $\hat{\sigma}$. This suggests that the estimate is very close to the true value of the parameter.

For testing validity of the proposed distribution, experiments were conducted with three arbitrarily chosen resistances belonging to the allowable tolerance levels. Time series data were generated at different μ values and the measures were computed. The plot in Fig. 9 depicts the deviation measure M and the associated bounds for 90% CI. It is seen that all three measure plots lie within the 90% CI bounds.

5. Summary, conclusions, and future work

This paper reports formulation and experimental validation of a novel statistical method for real-time parameter estimation in nonlinear systems. The core concept is built upon the principles of symbolic dynamic filtering (SDF), which has been reported in literature for anomaly detection in complex dynamical systems. Parameter estimation consists of the *forward problem* and the *inverse problem*.

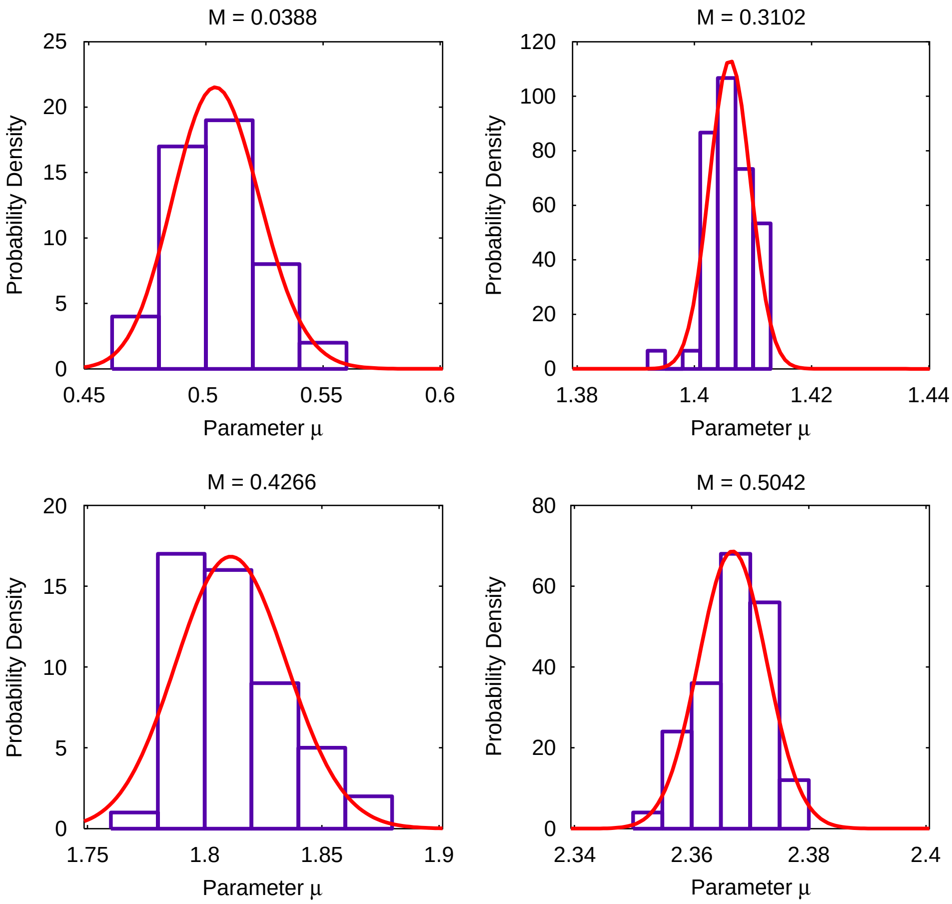

Fig. 8. Probability density plots for the van der Pol system.

Table 2
Predicted value of μ and confidence intervals for the van der Pol system

Observed measure M	Estimates							
	$\hat{\mu}$	$\hat{\sigma}$	99%		95%		90%	
			μ_{min}	μ_{max}	μ_{min}	μ_{max}	μ_{min}	μ_{max}
0.0388	0.5050	0.3×10^{-3}	0.4590	0.5549	0.4696	0.5424	0.4750	0.5362
0.3102	1.40618	0.012×10^{-3}	1.3970	1.4152	1.3992	1.4130	1.4003	1.4119
0.4266	1.8116	0.6×10^{-3}	1.7513	1.8737	1.7656	1.8585	1.7728	1.8510
0.5042	2.3670	0.0337×10^{-3}	2.352	2.382	2.3556	2.3784	2.3574	2.3766

In the forward problem, wavelet transforms of time series data sets of relevant system outputs are generated at different parametric values. Then, the space of wavelet coefficients is partitioned to obtain symbol sequences for construction of hidden Markov models (HMM) as probabilistic finite state automata (PFSA), called the *D*-Markov machine [9]. Distances between respective stationary statistics of the PFSA states, called deviation measures, are computed for different values of the parameter.

In the inverse problem, experiments are conducted to generate an ensemble of profiles of the computed deviation measure versus the parameter to be estimated. Standard statistical tools are applied to obtain the bounds and confidence levels associated with the estimate. A two-parameter

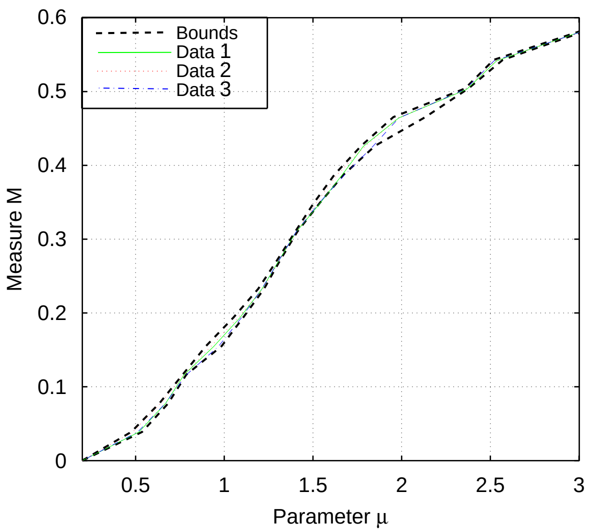

Fig. 9. Confidence intervals and test results for the van der Pol system.

log-normal distribution has been hypothesized to represent the spread of the estimated parameter based on an experimentally determined value of the deviation measure. The goodness of statistical fit has been determined by both χ^2 and Kolmogorov–Smirnov tests. The bounds and confidence levels associated with the parameter estimate are reported for two nonlinear systems, governed by Duffing system [12] and van der Pol system [13].

The reported work is a step toward building a real-time data-driven tool for estimation of slowly varying parameters in nonlinear dynamical systems. There are many potential applications of this tool, such as anomaly detection and early prognosis of failures in human-engineered systems. For example, the information on parameter deviations could be useful for execution of critical decision policies in real time. However, further theoretical and experimental research is necessary before its application in industry. Another important issue that has not been addressed in this paper is the impact of slow-scale (t_s) non-stationarity upon the assumption of fast-scale (t_f) stationarity when the ratio (t_f/t_s) is not sufficiently small.

While there are many other research issues that need to be addressed, the following research topics are being currently pursued.

- Extension of the single-parameter estimation to multi-parameter estimation, which renders the univariate statistical problem, presented in this paper, into a multivariate one.
- Construction of a real-time recursive algorithm that takes advantage of the previous data with trade-off between delay and robustness.
- Extension of the parameter estimation problem under different types (e.g., non-polynomial) of nonlinearities.
- Quantification of the effects of interactions between the slow-scale and fast-scale dynamics from the perspectives of singular perturbation [17].

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