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Brief paper

Real-time adaptation of decision thresholds in sensor networks for detection of moving targets[☆]

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ABSTRACT

This paper addresses real-time decision-making associated with acoustic measurements for online surveillance of undersea targets moving over a deployed sensor network. The underlying algorithm is built upon the principles of symbolic dynamic filtering for feature extraction and formal language theory for decision-making, where the decision threshold for target detection is estimated based on time series data collected from an ensemble of passive sonar sensors that cover the anticipated tracks of moving targets. Adaptation of the decision thresholds to the real-time sensor data is optimal in the sense of weighted linear least squares. The algorithm has been validated on a simulated sensor-network test-bed with time series data from an ensemble of target tracks.

1. Introduction

Detection of moving targets (e.g., undersea autonomous vehicles and weapon systems) in spatially-variable and uncertain environments is of prime importance in intelligence, surveillance and reconnaissance (ISR) systems. However, the situational context may prohibit placement of a single fixed long-term ISR system that can be fine-tuned to maximize performance in the area of interest. In such situations, distributed fields of passive sensor systems are often called for, as they allow the coverage of relatively large areas at a moderate cost (Culler, Estrin, & Srivastava, 2004; Li, Wong, Hu, & Sayeed, 2002; Zhao, Shin, & Reisch, 2008). A distributed system of small sensing nodes also provides a capacity for rapid deployment (e.g., many small assets are usually easier to position than a few large ones). In this context, the underwater target tracking must meet the demands of rapid deployment and wide area coverage for surveillance of moving targets in an uncertain environment.

When a large field of passive sensors is deployed for target tracking, the decision parameters of each sensor can be tuned according to the situation awareness. In particular, the sensor decision threshold D provides a cutoff level on the received energy of the acoustic signal above which a target detection is declared. Naturally, lower values of D provide a higher sensitivity to noise-induced false alarms, where random spikes in the background noise would cause a detection to be erroneously reported. In contrast, larger values of D reduce the effective detection range of the sensors, thus causing some targets to potentially move through the surveillance region undetected. By carefully examining the spatio-temporal sequence of reported detections across the entire field of sensors, a much lower level of threshold D may be applied, since random false alarms will rarely occur in patterns that are coincident with expected target motion behavior. The usage of moving target kinematics for multiple sensor detections is referred to as the track-before-detect strategy, and is commonly adopted in multi-sensor surveillance of moving targets.

Wettergren (2008) presented an application of track-before-detect strategies to undersea distributed sensor networks. In designing the deployment of a distributed passive sensor network that employs this track-before-detect procedure, it is imperative that the placement of sensors be commensurate with the expected detection range. With limited knowledge of expected target direction and environmental conditions (e.g., sensor performance variations over space), it is a common practice to assume uniform likelihoods of target motion direction and uniform environmental conditions; this assumption leads to a naturally optimal configuration of sensors in a circular ring with a small overlap between

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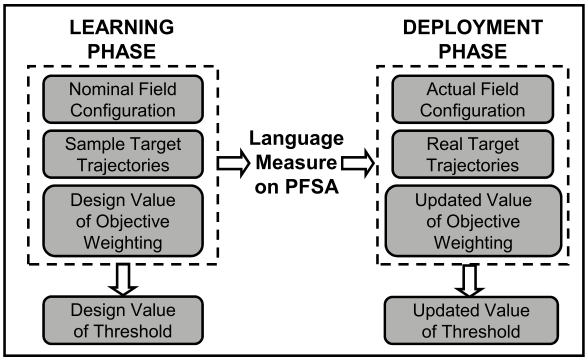


Fig. 1. Flow chart of the learning and adaptation phases.

coverage of individual sensors. Such a configuration would include a nominal setting of the decision threshold that is identical for all sensors. As situational information (e.g., from observation of a target as sensor time series) is gained from the system, it is desirable to adaptively improve the detection performance and reduce the probability of false alarms through adjustment of the decision threshold for the individual sensors in real time. From these perspectives, the decision thresholds at individual sensor nodes are adaptable parameters for maintaining a specified level of track-before-detect performance.

Given the *a priori* information: (i) the fixed sensor positions and (ii) the statistical distributions of expected target trajectories, the objective is to estimate track-dependent decision thresholds in real time by making robust trade-offs between minimization of the probability of false search (P_{FS}) and maximization of the probability of successful search (P_{SS}). The track-dependent decision thresholds are adapted using the concept of formal language-theoretic measure (Chattopadhyay & Ray, 2006) in the setting of probabilistic finite state automata (*PFSA*). The *PFSA* are constructed from symbol sequences generated from the observed time series data at each sensor location (Ray, 2004; Subbu & Ray, 2008). The flow chart in Fig. 1 depicts the process of learning and adaptation to illustrate how the formal language of the *PFSA* is derived to select the decision threshold for a real track.

To establish the feasibility of *PFSA*-based tools for estimation of sensor detection thresholds, simulation data sets have been constructed corresponding to a notional undersea sensor field surveillance scenario. These sensor data sets are generated on a simulation test-bed of noisy time series outputs. The test-bed is built on a typical sensor network that has been deployed to optimize its ability to track moving targets. The track-before-detect strategy has been used in a nominal sensing environment with an acceptable level of false search. As the target motion track is perturbed, the system performance degrades relative to the (*a priori* determined) optimal condition, which can be adaptively improved.

This paper addresses real-time adaptation of decision thresholds based on the time series information from sensor network nodes. Major contributions of this paper are listed below.

- Robust trade-off between probabilities of false search (P_{FS}) and successful search (P_{SS}) with variations in the target motion and uncertainties in the ambient/background noise distribution.
- Online correction of the offline estimate of decision threshold based on time series of the current track.
- Algorithm validation on a simulated sensor-network test-bed with time series from an ensemble of target tracks.

2. The track-before-detect strategy

This section formulates a track-before-detect strategy by developing a formal language theory-based optimization procedure for estimation of decision thresholds for off-nominal undersea target tracks as an alternative to conventional optimization methods. To this end, the following assumptions are made based on the standard characteristics of ocean environment and undersea sonar sensors (Urick, 1983):

- Deployment of passive omnidirectional sonar sensors in the sensor network with *a priori* known locations;
- Inverse relationship (e.g., inverse square law for deep water) of the transmission loss of the acoustic signal energy with respect to the sensor's distance from the target due to spherical spreading;
- Signal contamination with multiplicative Gaussian noise;
- Uniform ambient/background noise level for all sensors.

As a target travels across the region, each sensor picks up a noise-contaminated signal. A sensor that is closer to the target receives a stronger (i.e., larger magnitude) signal as compared to a sensor that is located farther away from the target. Each sensor in the network is modeled with a simple sonar equation (Urick, 1983), where the temporal positioning of signal energy is kinematically matched to the location of a moving target with constant source strength. The sonar equation represents the signal power excess (SE) received on the sensor, which is modeled in decibel (power) units as:

$$SE(t) = SL - NL - TL(x(t)) + DI \tag{1}$$

where t is the time, SL is the level of the source (i.e., target) energy emission, NL is the ambient noise level, TL is the (stochastically varying) transmission loss as a function of the current target's position $x(t)$, and DI is the directivity-induced noise compensation that accounts for the physical performance of the signal reception process. Such a performance model is typically used in naval systems and has been experimentally validated (Porter, 1993). The following model parameters have been used in the simulated scenario of the sensor network, which are consistent with the existing practice (Urick, 1983).

- Target acoustic energy emission level $SL = 100$ db;
- Ambient/background noise level $NL = 70$ db;
- Standard deviation of uncertainty in $TL = 2$ db;
- Parameter $DI = 0$ db due to sensor omnidirectionality.

The decision regarding the presence of a target is made collaboratively by multiple sensors in accordance with the track-before-detect paradigm. A target is said to be detected if, within a specified interval of time, a number k of sensors detect a signal that exceeds the specified decision threshold D; the length of the time interval is chosen based on the size of the surveillance region and the speed a typical target. In this study, k is chosen to be 3 following the standard practice (Wettergren, 2008). Fig. 2 illustrates the conceptual operation of the sensor field. The sensors are placed in an approximately circular shape, as shown by 'o' markers. In addition, the figure also shows an arbitrary track that a target may follow.

Identification of an optimal decision threshold D involves solution of a bi-objective optimization problem, where the two (conflicting) objectives are:

(1) Maximization of probability of successful search (P_{SS}).
(2) Minimization of probability of false search (P_{FS}).

For a given target track, P_{SS} as a function of D is evaluated by Monte Carlo simulation. For a given threshold D, the simulation of a target moving along a particular track is repeated 10,000 times. Then, $P_{SS}(D)$ is evaluated as the fraction of times the target is detected for the given D. On the other hand, P_{FS} is analytically evaluated (Wettergren, 2008) and depends on the ambient noise characteristics. Both $P_{SS}(D)$ and $P_{FS}(D)$ are monotonically decreasing functions of D.

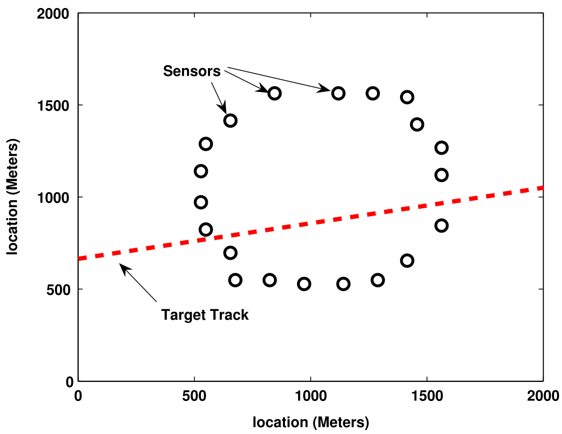


Fig. 2. Sensor placement and a target track.

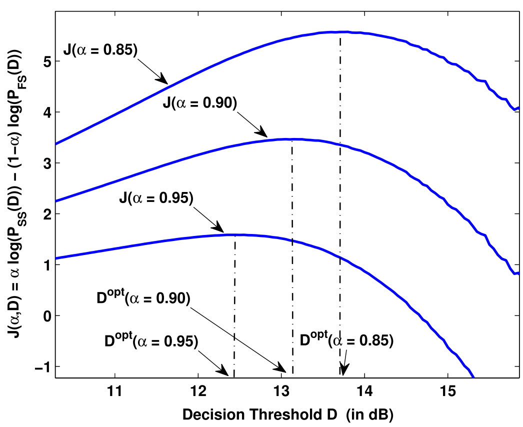


Fig. 3. Effect of threshold on the objective $J(\alpha, D)$ for the track in Fig. 2.

It has been observed that the non-dominated points in the plot of $\log(P_{SS})$ versus $\log(P_{FS})$ form a convex curve. Thus, a single objective function J is constructed by a weighted linear combination of the objective functions P_{SS} and P_{FS}.

$$J(\alpha, D) = \alpha \log(P_{SS}(D)) - (1-\alpha)\log(P_{FS}(D)) \tag{2}$$

where α is a scalar weight ($0 < \alpha < 1$). For a given weight α, the optimal detection threshold D^{opt} is obtained as:

$$D^{opt}(\alpha) = \arg\max_{D} J(\alpha, D). \tag{3}$$

The family of curves in Fig. 3 show the effects of the chosen decision threshold D on the objective function $J(\alpha, D)$ at different values of the parameter α for the track in Fig. 2. In this context, Fig. 4 is constructed to show the receiver operating characteristics (ROC) curve (i.e., the Pareto front) (Poor, 1988) for the track in Fig. 2. A point on the ROC curve provides the information on optimal value of the pair (P_{FS}, P_{SS}) for the respective α. This information is in agreement with the optimal decision threshold at the peak point of the ROC curve in Fig. 3 corresponding to the given value of α.

The above discussion evinces that, for a given α, the optimal decision threshold D^{opt} in Eq. (3) is dependent on the current track of the target. Although the information on the current track is not known *a priori*, an ensemble of time series data generated from the deployed sensor network is available. While the detection thresholds for feasible target tracks are determined offline to obtain an *a priori* expected value $\bar{D}$, the problem at hand is to identify the decision threshold D^{opt} online from the time series of the current target track.

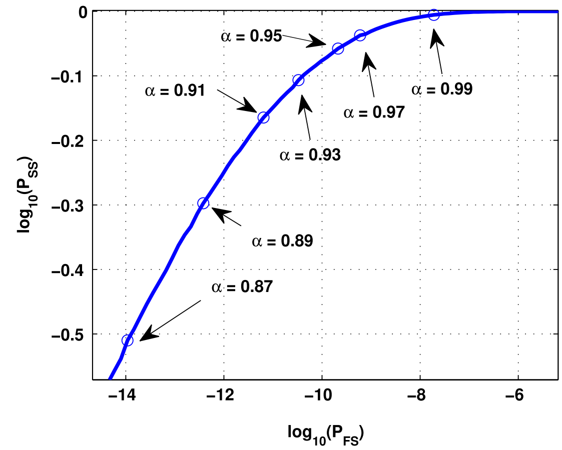


Fig. 4. Receiver operating characteristics for the track in Fig. 2.

3. Review of underlying mathematical concepts

This section reviews the concepts of symbolic dynamic filtering (*SDF*) (Ray, 2004; Subbu & Ray, 2008) and formal language measure (Chattopadhyay & Ray, 2006) that are used to compute decision thresholds for online surveillance of undersea targets moving over a deployed sensor network.

3.1. Symbolic dynamic filtering (SDF)

The *SDF* generates state probability vectors from probabilistic finite state automata (*PFSA*) to represent the evolving statistical patterns of the dynamical system. The performance of *SDF* relative to other classes of pattern recognition tools, such as Bayesian Filters and Artificial Neural Networks, has been reported in Rao, Ray, Sarkar, and Yasar (2009) from the perspectives of performance (e.g., capability for early detection of anomalies) and computational efficiency (e.g., execution time and memory requirements).

The *PFSA* are constructed via analytic signal space partitioning (*ASSP*) (Subbu & Ray, 2008) of the observed time series data for symbol sequence generation, which is an essential ingredient of *SDF*. A brief review of Hilbert-transform-based *ASSP* follows.

(1) *Analytic Signal Space Partitioning*: Hilbert transform of a real-valued signal $x(t)$ is obtained by the convolution:

$$\tilde{x}(t) \triangleq \mathcal{H}[x](t) = x(t) * \left(\frac{1}{\pi t}\right). \tag{4}$$

The (complex-valued) analytic signal (Cohen, 1995) of the real-valued signal $x(t)$ is defined as:

$$\mathcal{X}(t) \triangleq x(t) + i\,\tilde{x}(t). \tag{5}$$

Given a set of real-valued time series data, its Hilbert transform yields a pseudo-phase plot that is constructed from the analytic signal by a bijective mapping of the complex field onto $\mathbb{R}^2$, i.e., by plotting the real and the imaginary parts of the complex-valued signal on the abscissa and ordinate, respectively. The time-dependent analytic signal in Eq. (5) is now represented as a (one-dimensional) trajectory in the two-dimensional pseudo-phase space (Subbu & Ray, 2008).

Let Ξ be a compact region in the pseudo-phase space, which encloses the trajectory. The objective here is to partition Ξ into finitely many mutually exclusive and exhaustive segments, where each segment is labeled with a symbol; and the resulting set of symbols is called an alphabet Σ. The segments are determined by magnitude and phase of the analytic signal and also from density of data points in these segments. That is, if the magnitude and phase of a data point lies within a segment or on its boundary, then that data point is labeled with the corresponding symbol. This symbol generation process is called analytic signal space partitioning (*ASSP*) (Subbu & Ray, 2008).

One possible way of partitioning the region Ξ is to divide the magnitude and phase of the time-dependent analytic signal in Eq. (5) into uniformly spaced segments between their minimum and maximum values. This is called uniform partitioning. An alternative method, known as maximum entropy partitioning (Subbu & Ray, 2008), maximizes the entropy of the partition, which imposes a uniform probability distribution on the symbols. In this partitioning, parts of the state space with rich information are partitioned into finer segments than those with sparse information. The *ASSP* algorithm makes use of either one or both of these partitioning methods.

The space of the complex-valued analytic signal is partitioned in the angular and radial directions by either the uniform or the maximum entropy partitioning technique (Subbu & Ray, 2008). If $|\Sigma_R|$ and $|\Sigma_A|$ are the number of segments in the radial and angular directions, respectively, then the total number of symbols in the alphabet Σ is given by the product: $|\Sigma| = |\Sigma_R||\Sigma_A|$.

(2) *Construction of PFSA*: Given a symbol sequence derived from *ASSP* of observed time series data, the concept of d-Markov Machine (Ray, 2004) is adopted for construction of the *PFSA* with the stochastic matrix $\boldsymbol{\Pi}$ of state transitions being irreducible and acyclic (Bapat & Raghavan, 1997). (Note: The algorithm used for construction of the *PFSA* could be non-unique because it relies on the symbol sequence that is obtained by partitioning of the time series data, where symbolization may not be achieved through a generating partition (Ray, 2004).) The d-Markov machine has a state-space structure where the states of the machine are represented by blocks $\sigma_i\sigma_{i+1}\sigma_{i+2}\ldots\sigma_{i+d-1}$ of d consecutive symbols from the alphabet Σ. Thus, with cardinality $|\Sigma|$ of the alphabet and depth d of a symbol string of a state, the total maximum number of states in the d-Markov machine is given by $|\Sigma|^d$. Thus, the state machine moves from one state to another upon occurrence of a symbol. All symbol sequences that have the same last d symbols represent the same state. The states of the d-Markov machine at different time epochs should have identical probability distribution in the time scale in which the dynamical system is assumed to be (quasi)stationary.

3.2. Language measure for decision threshold computation

The *PFSA* constructed from a symbol sequence acts as a language generator and is represented as $G \triangleq (Q, \Sigma, \delta, \widetilde{\boldsymbol{\Pi}})$, where Q is the finite set of states with cardinality $|Q| = n$; Σ is the symbol alphabet and the Kleene closure of Σ is denoted as $\Sigma^\star$ that is the set of all finite-length strings of symbols including the empty string ε; the (possibly partial) function $\delta : Q \times \Sigma \to Q$ represents state transitions and $\delta^\star : Q \times \Sigma^\star \to Q$ is an extension of δ; and $\widetilde{\boldsymbol{\Pi}}$ is the symbol generation probability matrix, also called the morph matrix, which specifies the probability of symbol generation conditioned on individual states. The state transition matrix $\boldsymbol{\Pi}$ is derived from $\widetilde{\boldsymbol{\Pi}}$ and δ. If $\boldsymbol{\Pi}$ is an irreducible matrix, then $\boldsymbol{p}$ is the $(1 \times n)$ state probability vector which is the (sum-normalized) left eigenvector of $\boldsymbol{\Pi}$ corresponding to its unique unity eigenvalue (Bapat & Raghavan, 1997) and each element of $\boldsymbol{p}$ is strictly positive.

A signed measure of the language (Chattopadhyay & Ray, 2006) is obtained by assigning a weight to each of the states of the *PFSA*.

Definition 3.1. The state weight vector $\chi : Q \to \mathbb{R}$ assigns a signed real weight to each state q_i, $i = 1, 2, \ldots, n$. The $(1 \times n)$ state weight vector is denoted as:

$$\boldsymbol{\chi} = [\chi_1\ \chi_2\ \ldots\ \chi_n] \quad \text{where } \chi_j \triangleq \chi(q_j). \tag{6}$$

Definition 3.2. The symbol generation probabilities are specified as $\widetilde{\pi} : \Sigma^\star \times Q \to [0,\ 1]$ such that $\forall q_j \in Q, \forall \sigma_k \in \Sigma, \forall s \in \Sigma^\star$,

(1) $\widetilde{\pi}[\sigma_k, q_j] \triangleq \widetilde{\pi}_{jk} \in [0, 1]$; $\sum_k \widetilde{\pi}_{jk} = 1$;
(2) $\widetilde{\pi}[\sigma_k, q_j] = 0$ if $\delta(q_j, \sigma_k)$ is undefined; and $\widetilde{\pi}[\epsilon, q_j] = 1$;
(3) $\widetilde{\pi}[\sigma_k s, q_j] = \widetilde{\pi}[\sigma_k, q_j]\,\widetilde{\pi}[s, \delta(q_j, \sigma_k)]$.

The $\widetilde{\boldsymbol{\Pi}}$-matrix is defined as: $\widetilde{\boldsymbol{\Pi}}_{jk} = \widetilde{\pi}(q_j, \sigma_k)$, $q_j \in Q$, $\sigma_k \in \Sigma$.

Definition 3.3. The probabilistic state transition map of the *PFSA* is defined as a function $\pi : Q \times Q \to [0,\ 1]$ such that

$$\pi(q_j,\ q_k) = \begin{cases} 0 & \text{if } \{\sigma \in \Sigma : \delta(q_j,\ \sigma) = q_k\} = \emptyset \\ \displaystyle\sum_{\sigma\in\Sigma:\delta(q_j,\ \sigma)=q_k} \widetilde{\pi}(\sigma,\ q_j) \triangleq \pi_{jk} & \text{otherwise.} \end{cases} \tag{7}$$

The $\boldsymbol{\Pi}$-matrix is defined as $\boldsymbol{\Pi} \triangleq [\pi_{jk}]$.

Definition 3.4. The language measure (Chattopadhyay & Ray, 2006) of a *PFSA* with respect to a given state weight vector $\boldsymbol{\chi}$ is defined as:

$$\bar{\boldsymbol{\nu}}(\theta) = \theta\,[I - (1-\theta)\boldsymbol{\Pi}]^{-1}\boldsymbol{\chi}^T \tag{8}$$

where the scalar parameter $\theta \in (0, 1)$ ensures invertibility of the matrix on the right-hand side of Eq. (8) and implies a terminating automaton. As $\theta \to 0^+$, the terminating automaton converges to a non-terminating automaton. The following propositions are reported in Chattopadhyay and Ray (2006) along with their proofs.

Proposition 3.1. *The limiting measure vector* $\bar{\boldsymbol{\nu}}(0) \triangleq \lim_{\theta\to 0^+} \bar{\boldsymbol{\nu}}(\theta)$ *exists and is bounded from above such that* $\|\bar{\boldsymbol{\nu}}(0)\|_\infty \le \|\boldsymbol{\chi}\|_\infty$.

Proposition 3.2. *Given an irreducible and acyclic state transition matrix* $\boldsymbol{\Pi}$, *the measure vector in Eq. (8) reduces to:* $\bar{\boldsymbol{\nu}}(0) = \nu\mathbf{1}$, *where* $\mathbf{1} \triangleq [1\,1\ \ldots\ 1]^T$. *Then, the scalar measure* ν *is denoted as:* $\nu = \boldsymbol{p}\,\boldsymbol{\chi}^T$, *where* $\boldsymbol{p}$ *is the* $(1 \times n)$ *state probability vector which is the (sum-normalized) left eigenvector of* $\boldsymbol{\Pi}$ *corresponding to its unique unity eigenvalue (Bapat & Raghavan, 1997).*

4. Adaptation of the decision threshold

While the optimum threshold D^{opt} can be computed for a known track, the goal here is online adaptation of the threshold D for *a priori* unknown tracks, based on the in-situ sensor time series data. The decision threshold is computed as a positive real measure of the language of the *PFSA* generated from a symbol sequence obtained by partitioning of the time series. The language measure is derived in terms of the track-invariant state weight vector $\boldsymbol{\chi}$ of the *PFSA*, as described below.

The time series data collected from sensors are partitioned (ASSP) for conversion into a symbol sequence (Subbu & Ray, 2008). Then, by following the procedure outlined in Section 3, a *PFSA* is constructed from the concatenated symbol sequences (Ray, 2004) obtained from the relevant sensors. The *PFSA* has n states, where n is a positive integer, and is characterized by an $(n \times n)$ state transition matrix $\boldsymbol{\Pi}$ that is an irreducible acyclic stochastic matrix (Bapat & Raghavan, 1997) by construction and $\boldsymbol{p}$ is the associated state probability vector. Then, there exists a $(1\times n)$ state weight vector χ such that the (scalar) language measure of the *PFSA* is obtained by Proposition 3.2 as $\nu = \boldsymbol{p}\,\chi^T$.

Let $\bar{D}$ be the expected value of the track-dependent decision thresholds (as obtained using Eq. (3)) over the distribution of target tracks. Let the state weight vector χ be assigned such that the residue $(D_i - \bar{D})$ of the detection threshold for the ith track is identically equal to the language measure ν_i of the *PFSA* of the corresponding track. Then, the detection threshold D_i for the ith track is obtained as:

$$D_i - \bar{D} = \nu_i \Rightarrow D_i = \boldsymbol{p}_i \chi^T + \bar{D} \tag{9}$$

where $\boldsymbol{p}_i$ is the state probability vector for the ith track.

The sets of track data generated from the simulation test bed are divided into two mutually disjoint subsets—a training set and a test set. The estimate $\widehat{\chi}$ of the (track-invariant) state weight vector χ is computed from the ensemble of training data in terms of the respective decision thresholds for individual tracks in the *a priori* known training set.

Let ℓ_{train} be the number of tracks in training set. An $(\ell_{train} \times n)$ matrix $\mathbb{P}$ is constructed from the training set as

$$\mathbb{P} \triangleq [\boldsymbol{p}_1^T\, \boldsymbol{p}_2^T\, \ldots\, \boldsymbol{p}_{\ell_{train}}^T] \tag{10}$$

where the $(1 \times n)$ state probability vectors $\boldsymbol{p}_i, i \in \{1, 2, 3, \ldots, \ell_{train}\}$ are respectively obtained from the ℓ_{train} tracks in the training set. If the number of tracks in the training set is larger than the number of states in the *PSFA* (i.e. $\ell_{train} > n$) and if there is sufficient variety in the set of training tracks such that the matrix $\mathbb{P}$ has the full column rank n, then the $(n \times n)$ matrix $(\mathbb{P}^T\mathbb{P})$ is invertible.

A threshold residue vector $\boldsymbol{\Delta}$ consisting of the detection threshold for each track in the training set is defined as:

$$\boldsymbol{\Delta} \triangleq [D_1 - \bar{D}\; D_2 - \bar{D}\; \ldots\; D_{\ell_{train}} - \bar{D}]^T \tag{11}$$

where the positive scalars D_i, $i \in \{1, 2, 3, \ldots, \ell_{train}\}$, are the thresholds for the respective tracks in the training set and $\bar{D}$ is the average detection threshold, which are computed *a priori*. Accordingly, a measurement model of the $(\ell_{train} \times 1)$ threshold residue vector $\boldsymbol{\Delta}$ is formulated as:

$$\boldsymbol{\Delta} = \mathbb{P}\,\chi^T + \boldsymbol{\varepsilon} \tag{12}$$

where the measurement error vector $\boldsymbol{\varepsilon}$ is additive zero-mean and the (positive definite) error covariance matrix is $\mathbf{R}$.

An estimate $\widehat{\chi}$ of the track-invariant $(1 \times n)$ vector χ is obtained by the (weighted) linear least square method based on the information obtained from the training set. This task requires orthogonal projection of the $(\ell_{train} \times 1)$ threshold residue vector $\boldsymbol{\Delta}$ onto the column space of $\mathbb{P}$ such that

$$\widehat{\chi} = \left([\mathbb{P}^T\mathbf{R}^{-1}\mathbb{P}]^{-1}\mathbb{P}^T\mathbf{R}^{-1}\boldsymbol{\Delta}\right)^T. \tag{13}$$

Note that $\widehat{\chi}$ is an unbiased estimate of χ and if the measurement noise $\boldsymbol{\varepsilon}$ in Eq. (12) is jointly Gaussian, then $\widehat{\chi}$ is also the minimum-variance estimate of χ.

If the information on the measurement noise covariance matrix is not available, it is logical to assume (e.g., for identical sensors) that the measurement error covariance matrix $\mathbf{R} \sim \mathbf{I}_{\ell_{train}\times\ell_{train}}$. In that case, Eq. (13) reduces to

$$\widehat{\chi} = \left([\mathbb{P}^T\mathbb{P}]^{-1}\mathbb{P}^T\boldsymbol{\Delta}\right)^T. \tag{14}$$

In the deployment phase, following Eq. (14), a probabilistic-state-machine-based estimate of the threshold residue vector $\boldsymbol{\Delta}$ is obtained in terms of the estimate $\widehat{\chi}$ of the track-invariant state weight vector as:

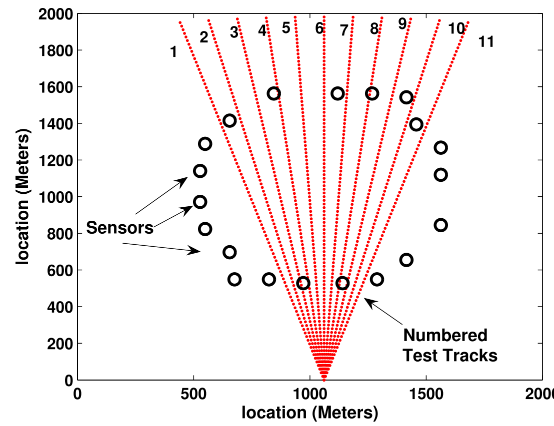


Fig. 5. Set of test tracks across a sensor field.

$$\widehat{\boldsymbol{\Delta}} = \mathbb{P}\widehat{\chi}^T \Leftrightarrow \widehat{D}_i = \boldsymbol{p}_i\,\widehat{\chi}^T + \bar{D}, \quad i \in \{1, 2, \ldots, \ell_{test}\}. \tag{15}$$

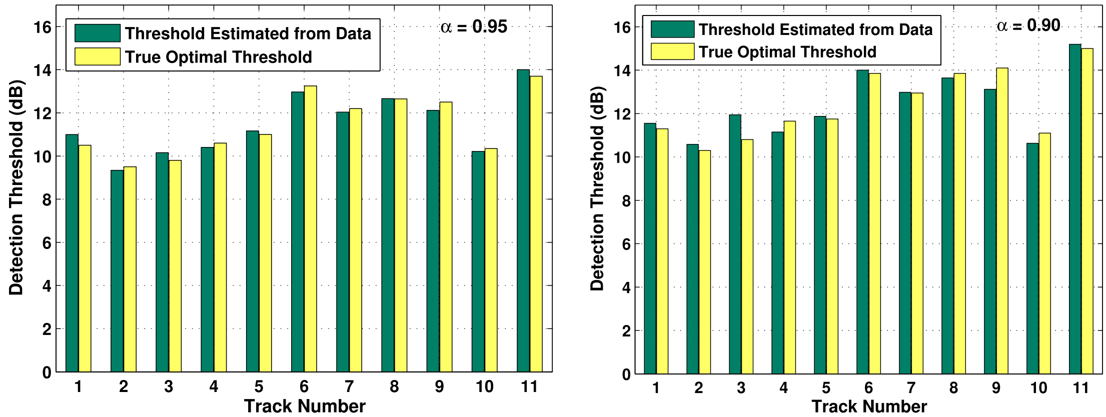
Remark 4.1. The algorithm in Eq. (15) for estimation of decision threshold is sufficiently fast for real-time execution on (limited memory) sensor nodes. In the present form, the algorithm is formulated based on the principle of linear least squares and is data driven in the absence of additional pertinent information such as a model of the underlying physical process and statistics of the environmental noise. Should this information be available, it is envisioned that combined model-based and data-driven algorithms for (possibly nonlinear) estimation of the state weight vector χ could be constructed for real-time execution on individual nodes in a sensor network.

Remark 4.2. The estimated state weight vector $\widehat{\chi}$ is a linear functional in the space $\mathbb{R}^n$ because $\widehat{\chi} : \mathbb{R}^n \to \mathbb{R}$ is a linear map, as seen in Eqs. (9) and (15). If the scalar parameter of decision threshold D is replaced by a parameter vector of dimension $m \le n$, then $\widehat{\chi}$ will become a linear transformation from $\mathbb{R}^n$ onto $\mathbb{R}^m$. In a more general case, a nonlinear transformation should be sought to address this identification problem. That is, it might be necessary to find a homeomorphism between the range space of $\widehat{\chi}$ and the space of the decision threshold vector that replaces the scalar D.

5. Results and discussion

This section presents the results of the proposed method of detection threshold estimation as applied to simulated data generated for a notional undersea sensor network. The nominal sensor configuration follows an approximately circular ring pattern, where the spacing between two neighboring sensors is set to be approximately one half of the expected detection range. Such a spacing provides multiple detection opportunities for targets that transit along paths near the center of the circular ring, even under somewhat noisy circumstances. The sensor characteristics, environment and target models are the same as described in Section 2.

The ensemble of time series data is obtained from a set of 20 sensors in the given sensor network of the simulation test bed for each of 21 different tracks. The optimal decision threshold for each target track corresponds to the cost weight α and is track-specific (see Eq. (3)). The objective here is to demonstrate the efficacy of *PFSA*-based estimation of the detection threshold for individual tracks. Data from 10 out of the 21 tracks have been used for training and the parameter vector χ is estimated according to Eq. (14). The



(a) Scalar weight $\alpha = 0.95$ in objective function J in Eq. (2).

(b) Scalar weight $\alpha = 0.90$ in objective function J in Eq. (2).

Fig. 6. Comparison of optimal decision threshold and estimation.

performance is then tested on the remaining 11 tracks. These 11 tracks are numbered as depicted in Fig. 5.

Fig. 6(a) and (b) present comparisons of the optimal decision threshold D^{opt} and the respective estimated values $\widehat{D}$ at the 11 test conditions for $\alpha = 0.95$ and $\alpha = 0.9$ respectively, where track numbers in both figures correspond to those in Fig. 5. These results are based on the information (i.e., $\widehat{\chi}$) generated in the training phase and are obtained by partitioning the space of complex-valued analytic signals (see Eq. (5)) with a symbol alphabet Σ having cardinality $|\Sigma| = 6$, where $|\Sigma_R| = 6$ and $|\Sigma_A| = 1$ (see Section 3.1). A probabilistic finite state automaton (*PFSA*) is then constructed from the generated symbol series. In this case, the *PFSA* has 6 states (Ray, 2004); consequently, the vector $\boldsymbol{\chi}$ lies in $\mathbb{R}^6$.

In both Fig. 6(a) and (b), the mean of the estimation error is negligibly small relative to the mean value of the optimal decision threshold, which implies that the estimate is practically unbiased. The *rms* value of the error $(D^{opt} - \widehat{D})$ is 0.3 db. If the threshold is assumed to be $\bar{D}$ instead of $\widehat{D}$ (i.e., no adaptation), the resulting *rms* error increases to 1.6 db.

6. Summary, conclusions, and future research

This paper addresses online surveillance of undersea targets as a real-time track-before-detect problem. As the target moves across the sensor field, each sensor collects time series data; the sensors that are closer to the path of the target capture stronger signals. A track-before-detect algorithm has been formulated to estimate the track-dependent decision threshold based on the ensemble of time series data from the sensor field. The objective here is to obtain an optimal trade-off between the probabilities of false search and successful search as well as adaptation to online time series data of the current track, which is robust relative to variations in the target's motion and uncertainties in the environmental noise distribution. The proposed probabilistic finite state automata (*PFSA*)-based algorithm is optimal in the sense of weighted linear least squares. The algorithm has been tested with sensor data from several tracks on a simulated sensor field. The results suggest that the *PFSA*-based approach is feasible for online estimation of decision thresholds, as needed for tracking of undersea targets. However, the proposed track-before-detect algorithm must be validated with rich experimental data to establish its efficacy for online surveillance of undersea targets.

As an extension of *PFSA*-based decision-making, future research is recommended in the following areas:

- *Track-dependent estimation of decision threshold for dynamic adaptation in a large sensor field*: The optimal decision threshold should be determined based on the local placement of sensors, where the size of the local region depends on the target speed and the time interval within which multiple sensors must detect the target.
- *Sensor placement for online tracking of target movements*. There is a need for development of a mathematically rigorous and computationally inexpensive formal-language-theoretic algorithm that will support the optimization objectives related to sensor placement. Once calibrated with the existing optimization algorithms, the proposed *PFSA*-based algorithm is expected to provide solutions to the online sensor placement problem for modest perturbations in the nominal target statistics and be locally executable on individual nodes of a sensor network in the undersea environment.

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