Rectilinear System

Introduction

For this lab, there are three separate experiments that will be performed. The first experiment will calculate all the system parameters that will be used in later parts of the lab. Experiment 2 has three sub parts. Part A consists of creating P and PD controllers that are manually displaced. Part B then inputs a step function into three controller setups; under-damped, critically damped, and over-damped. Finally in Part C, a complete PID controller is implemented and studied. In Experiment 3, the modes of a two degree of freedom system are studied.

Hardware

The Rectilinear Control Plant is shown in Figure 1. The apparatus consists of three mass carriages interconnected by bi-directional springs. The mass carriage suspension is an anti-friction ball bearing type with approximately +/- 3 cm of available travel. The linear drive is comprised of a gear rack suspended on an anti-friction carriage and pinion (pitch dia. 7.62 cm (3.00 in)) coupled to the brushless servo motor shaft. Optical encoders measure the mass carriage positions – also via a rack and pinion with pinion pitch diameter 3.18 cm (1.25 in).

![Figure 1: Rectilinear Apparatus](image)
Safety
As with every lab, read Appendix B on the course website prior to starting any experiment.

In this and all work of this lab, be sure to stay clear of the mechanism before implementing a controller. Selecting Implement Algorithm immediately implements the specified controller; if there is an instability or large control signal, the plant may react violently. If the system appears stable after implementing the controller, first displace it with a light, non sharp object (e.g. a plastic ruler) to verify stability prior to touching plant.

Hardware/Software Equipment Check
Prior to starting the lab, make sure the equipment is working by conducting the following steps:

Step 1: Enter the ECP program by double clicking on its icon. You should see the Background Screen. Gently move the first mass carriage by hand. You should observe some following errors and changes in encoder counts. The Control Loop Status should indicate "OPEN" and the Controller Status should indicate “OK”.

Step 2: Make sure that you can oscillate the mass carriages freely. Now press the black "ON" button to turn on the power to the Control Box. You should notice the green power indicator LED lit, but the motor should remain in a disabled state. Do not touch the apparatus whenever power is applied to the Control Box since there is a potential for uncontrolled motion of the masses unless the controller has been safety checked.

Experiment 1: System Identification
Following the procedure below, the relevant parameters such as the mass, spring stiffness and damping coefficients will be determined. There are many system parameters to calculate, so be patient.
Procedure:

Before starting, identify which mass carriage is labeled, one, two and three. It is very important not to confuse the order!

1. Set up the experiment. Clamp the second mass to put the mechanism in the configuration shown in Figure 2a above using a shim (e.g. 1/4 inch nut) between the stop tab and stop bumper so as not to engage the limit switch (see Section 2.2). Note that in this lab you have been provided with three different types of springs. There are 3 low stiffness springs and they have white paint on their ends. There are 3 medium stiffness springs and they have silver paint on their ends. There are 4 high stiffness springs and they have been left unpainted. Verify that the
medium stiffness spring (nominally 400 N/m (2.25 lb/in.)) is connecting the first and second mass carriages.

2. Secure four 500g masses on the first and second mass carriages.

3. Set up the data acquisition. With the controller powered up, enter the Control Algorithm box via the Set-up menu and set \( T_s = 0.00442 \). Enter the Command menu, go to Trajectory and select Step, Set-up. Select *Open Loop Step* and input a step size of 0 (zero), a duration of 3000 ms and 1 repetition. Exit to the background screen by consecutively selecting OK. This puts the controller in a mode for acquiring 6 sec of data on command but without driving the actuator. This procedure may be repeated and the duration adjusted to vary the data acquisition period.

4. Select the relevant encoders to gather data. Go to Set up Data Acquisition in the Data menu and select Encoder #1 and Encoder #2 as data to acquire and specify data sampling every 2 (two) servo cycles (i.e. every 2 \( T_s \)'s). Select OK to exit. Select Zero Position from the Utility menu to zero the encoder positions.

5. Read this step fully before running the experiment. Select Execute from the Command menu. Prepare to manually displace the first mass carriage approximately 2.5 cm. Exercise caution in displacing the carriage so as not to engage the travel limit switch. With the first mass displaced approximately 2.5 cm in either direction, select Run from the Execute box and release the mass approximately 1 second later. The mass will oscillate and attenuate while encoder data is collected to record this response. Select OK after data is uploaded.

6. Export the test data from ECP to MATLAB. Plot the encoder 1 data in MATLAB, clearly labeling the plot. Be sure to use the Data Cursor for data analysis.

7. Determine the frequency from the tests. Choose several consecutive cycles (say ~5) in the amplitude range between 5500 and 1000 counts. Divide the number of cycles by the time taken to complete them being sure to take beginning and end times from the same phase of the respective cycles. Convert the resulting frequency in Hz to radians/sec. This *damped frequency*, \( \omega_d \), approximates the *natural frequency*, \( \omega_n \), according to:

\[
\omega_{n_{m11}} = \frac{\omega_{d_{m11}}}{\sqrt{1 - \zeta_{m11}^2}} = \omega_{d_{m11}} \quad (for \ small \ \zeta_{m11})
\]

(Equation 1-1)
where the "m11" subscript denotes mass carriage #1, trial #1. (Hint, use the Data Cursor feature in MATLAB figure window to get the beginning and end times and amplitudes)

8. Prepare to perform a similar test. Remove the four masses from the first mass carriage and repeat Steps 5 through 7 to obtain \( \omega_{nm12} \) (natural frequency for mass carriage #1, trial #2) for the unloaded carriage. If necessary, repeat Step 3 to reduce the execution (data sampling only in this case) duration.

9. Calculate the damping ratio. Measure the initial cycle amplitude \( X_0 \) and the last cycle amplitude \( X_n \) for the \( n \) cycles measured in Step 8. Using relationships associated with the logarithmic decrement:

\[
\frac{\zeta_{m12}}{\sqrt{1 - \zeta_{m12}^2}} = \frac{1}{2\pi n} \ln \left( \frac{X_0}{X_n} \right) \quad \zeta_{m12} = \frac{1}{2\pi n} \ln \left( \frac{X_0}{X_n} \right) \quad (for \ small \ \zeta_{m12})
\]

(Equation 1-2)

find the damping ratio \( \zeta_{m12} \) and show that for this small value the approximations of Equation 1-1 and 1-2 are valid.

10. Repeat Steps 5 through 9 for the second mass carriage using the set up in figure 2b. Here in Step 6 you will need to remove Encoder #1 position and add Encoder #2 position to the plot set-up. Hence obtain \( \omega_{nm21} \), \( \omega_{nm22} \) and \( \zeta_{m22} \). How does this damping ratio compare with that for the first mass?

11. Connect the mass carriage extension bracket and dashpot to the second mass as shown in Figure 2c. Open the damping (air flow) adjustment knob 2.0 turns from the fully closed position. Repeat Steps 5, 6, and 9 with four 500 g masses on the second carriage and using only amplitudes \( \geq 500 \) counts in your damping ratio calculation. Hence obtain \( \zeta_d \) where the "d" subscript denotes "dashpot".

12. Each brass weight has a mass of 500 ± 10 g. Calling the mass of the four weights combined \( m_w \), use the following relationships to solve for the unloaded carriage mass \( m_{c2} \), and spring constant \( k \).

\[
k/(m_w + m_{c2}) = (\omega_{nm21})^2 \quad \text{(Equation 1-3)}
\]

\[
k/m_{c2} = (\omega_{nm22})^2 \quad \text{(Equation 1-4)}
\]

Find the damping coefficient \( c_{m2} \) by equating the first order terms in the equation form:

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1Note that the calculated masses \( m_{c1} \) and \( m_{c2} \) will include the reflected inertias of all connected elements – e.g. motor pinion and armature.
Repeat the above for the first mass carriage, spring and damping $m_{c1}$, $c_{m1}$ and $k$ respectively.\(^\text{2}\)\(^\text{3}\)

Calculate the damping coefficient of the dashpot, $c_d$.

*Note that the total damping coefficient for your experiment with the dashpot connected is $c = c_{m2} + c_d$. You first figure out $c$ from the experimental result and then determine $c_d$.

13. Remove the carriage extension bracket and dashpot from the second mass carriage, replace the medium stiffness spring with a high stiffness spring (800 N/m nominally), and repeat Steps 5 through 9 to obtain the resulting natural frequency $\omega_{m23}$ and $\omega_{m24}$. Repeat this frequency measurement using the least stiff spring (nominally 200 N/m) to obtain $\omega_{m25}$ and $\omega_{m26}$. Calling the value of stiffness obtained in Step 12 above $k_{\text{med stiffness}}$, calculate $k_{\text{high stiffness}}$ and $k_{\text{low stiffness}}$ from the frequency measurements of this step.

14. Find the hardware gain ($k_{hw}$\(^4\)) by using equation 1-6 and values

$$k_{hw} = k_c k_a k_{mp} k_e k_{ep} k_s$$  \hspace{1cm} \text{(Equation 1-6)}

where:

$k_c$, the DAC gain, = 10V / 32,768 DAC counts
$k_a$, the Servo Amp gain, = approx 2 (amp/V)
$k_t$, the Servo Motor Torque constant = approx 0.1 (N-m/amp)
$k_{mp}$, the Motor Pinion pitch radius inverse = 26.25 m\(^{-1}\)
$k_e$, the Encoder gain, = 16,000 pulses / $2\pi$ radians
$k_{ep}$, the Encoder Pinion pitch radius inverse = 89 m\(^{-1}\)
$k_s$, the Controller Software gain, = 32 (controller counts / encoder or ref input counts)\(^5\)

The final report is expected to include:
A diagram identifying the control elements and signals in the experiment, including:

Sensor:  \hspace{2cm} Actuator:

\(^2\)Step 12 may be done later, away from the laboratory, if necessary.

\(^3\)The resulting value for $k$ should be very close to that measured when considering the second mass case. You may use the average of the two for your identified $k$ value.

\(^4\)It contains software gain also. This software gain, $k_s$ is used to give higher controller-internal numerical resolution and improves encoder pulse period measurement for very low rate estimates.

\(^5\)These are the counts that are actually operated on in the control algorithm. i.e. The system input (trajectory) counts and encoder counts are multiplied by 32 prior to control law execution.
Controller:     Reference Input: 
Actuator Output:    System Output: 

Seven (7) MATLAB Plots, with two (2) Data Cursor Points on the plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation. Plots include:
- Plot of Mass 1 Trial 1 (with four weights)
- Plot of Mass 1 Trial 2 (without weights)
- Plot of Mass 2 Trial 1 (with four weights)
- Plot of Mass 2 Trial 2 (without weights)
- Plot of Mass 2 with dashpot connected (with four weights)
- Plot of Mass 2 Trial 3 (with the high-stiffness spring and with weights)
- Plot of Mass 2 Trial 4 (with the low-stiffness spring and with weights)

Calculations showing how you found the following values, along with units for EVERY quantity found. Use equations 1-1 – 1-5.

- Mass 1 Trial 1 natural frequency, $\omega_{nm11}$
- Mass 1 Trial 2 natural frequency, $\omega_{nm12}$
- Mass 1 Trial 2 damping ratio, $\zeta_{m12}$

- Mass 2 Trial 1 natural frequency, $\omega_{nm21}$
- Mass 2 Trial 2 natural frequency, $\omega_{nm22}$
- Mass 2 Trial 2 damping ratio, $\zeta_{m22}$
- Mass 2 damping ratio with dashpot connected, $\zeta_d$
- Mass 2 Trial 3 natural frequency, $\omega_{nm23}$
- Mass 2 Trial 4 natural frequency, $\omega_{nm24}$

- Mass of brass weights, $m_w$
- Mass of Carriage #1 plus driving unit, $m_{c1}$
- Mass of Carriage #2, $m_{c2}$
- Mass 1 & driving unit damping coefficient, $c_{m1}$
- Mass 2 damping coefficient, $c_{m2}$
- Dashpot damping coefficient, $c_d$ *
- Low stiffness spring constant, $k_{low}$
- Medium stiffness spring constant, $k_{med}$
- High stiffness spring constant, $k_{high}$
- Hardware gain $k_{hw}$

For all the questions highlighted, the questions should be copied and pasted into your lab report and explicitly answered immediately thereafter.
**Experiment 2: Rigid Body PD and PID Control**

In this experiment, there are three separate parts. The first is manual displacement of a P and PD controller. Part B then includes a step input to various PD controllers. Finally Part C concludes with adding integral action to the PD controller. This experiment demonstrates some key concepts associated with proportional plus derivative (PD) control and subsequently the effects of adding integral action (PID). This control scheme, acting on plants modeled as rigid bodies finds broader application in industry than any other.

The block diagram for forward path PID control of a rigid body is shown in Figure 3a where friction is neglected. Figure 3b shows the case where the derivative term is in the return path. Both implementations are found commonly in application and, as the student should verify, both have the identical characteristic roots. They therefore have identical stability properties and vary only in their response to dynamic inputs.

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The student may want to later verify that for the relatively high amount of control damping in the scheme that follows – induced via the parameter $k_d$ – that the plant damping is very small.
The closed loop transfer functions for the respective cases are:

\[
\frac{c(s)}{r(s)} = \frac{(k_{hw}/m)(k_ds^2+k_ps+k_i)}{s^3+(k_{hw}/m)(k_ds^2+k_ps+k_i)} \quad \text{(Equation 2-1a)}
\]

\[
\frac{c(s)}{r(s)} = \frac{(k_{hw}/m)(k_ps+k_i)}{s^3+(k_{hw}/m)(k_ds^2+k_ps+k_i)} \quad \text{(Equation 2-1b)}
\]

For the first portion of this exercise we shall consider PD control only (ki=0). For the case of kd in the return path the transfer function reduces to the equation below, students should show how this equation is obtained by using the block-diagram algebra on the control loop (the answer given below does have an error that we are INTENTIONALLY leaving in this laboratory manual to avoid copying/pasting the answer).

\[
\frac{c(s)}{r(s)} = \frac{(k_{hw}/m)(k_p)}{s^2+(k_{hw}/m)(k_ds+k_p)} \quad \text{(Equation 2-2)} \quad \text{HAS AN ERROR! 😁}
\]

\[
\omega_n = \sqrt{\frac{k_p k_{hw}}{m}} \quad \text{(Equation 2-3)}
\]

\[
\zeta = \frac{k_d k_{hw}}{2m \omega_n} = \frac{k_d k_{hw}}{2\sqrt{mk_p k_{hw}}} \quad \text{(Equation 2-4)}
\]

\[
\frac{c(s)}{r(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad \text{(Equation 2-5)}
\]

The effect of kp and kd on the roots of the denominator (damped second order oscillator) of Equation 2-2 is studied in the work that follows.

(Note that frequency needs to be converted to rad/s in the calculation of kp etc.)

**Experiment 2a: Proportional & Derivative Control Actions**

In this portion of the lab, the values found from Experiment 1 will be used to create proportional and PD controllers

1. Using the results of Experiment 1 construct a model of the plant with four 500g mass pieces on the first mass carriage with no springs or damper attached. You may neglect friction.

2. Set-up the plant in the configuration described in Step 1. There should be no springs or damper connected to the first carriage and the other carriages should be secured away from the range of motion of the first carriage.
3. Calculate the gain. From Equation 2-3 determine the value of $k_p$ ($k_d=0$) so that the system behaves like a $\sqrt{2}$ Hz spring-mass oscillator.

4. Set up the ECP data acquisition hardware. Collect Encoder #1 and Commanded Position information via the Set-up Data Acquisition box in the Data menu. Set up a closed-loop step of 0 (zero) counts, dwell time $= 3000\text{ ms}$, and 1 (one) rep (via Trajectory in the Command menu).

5a. Set up the controller. Enter the Control Algorithm box under Set-up and set $T_s=0.0042\text{ s}$ and select Continuous Time Control. Select PID and Set-up Algorithm. Enter the $k_p$ value determined above for $\sqrt{2}$ Hz oscillation ($k_d$ & $k_i = 0$, do not input values greater than $k_p = 0.087$) and select OK.

5b. Place the first mass carriage at approximately the -0.5 cm (negative is toward the motor) mark. Select Implement Algorithm, then OK.

6. Enter Execute under Command. Prepare to manually displace the mass carriage roughly 2 away from the motor. Select Run, displace the mass and release it. Do not hold the mass position for longer than about 1 second as this may cause the motor drive thermal protection to open the control loop.

7a. Export the data from ECP to MATLAB and plot the encoder 1 data in MATLAB.

7b. Use the Data cursor to determine the frequency of oscillation. If the experimental frequency is way off from $\sqrt{2}$, repeat the experiment, and if still off recheck your values. If this does not work, experimentally determine the right values to get $\sqrt{2}$ Hz.

7c. Double the value for $k_p$. Repeat Steps 5 & 6. Plot the data in MATLAB.

8. Add the derivate controller aspect. Determine the value of the derivative gain, $k_d$, to achieve $k_d k_w = 50 \text{ N/(m/s)}$. Repeat Step 5, except input the above value for $k_d$ and set $k_p$ & $k_i = 0$. (Do not input values greater than $k_d = 0.04$).

9. After checking the system for stability by displacing it with a ruler, manually move the mass back and forth to feel the effect of viscous damping provided by $k_d$. Do not excessively coerce the mass as this will again cause the motor drive thermal protection to open the control loop.

10. Repeat Steps 8 & 9 for a value of $k_d$ five times as large (again, $k_d \leq 0.04$). Can you feel the increased damping?

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7Here due to friction the system, which is ideally quasi-stable (characteristic roots on the jω axis), remains stable for small $k_p$. For larger values, the time delay associated with sampling may cause instability.

8For the discrete implementation you must divide the resulting value by $T_s$ for the controller input value. Here, since the PD controller is improper, the backwards difference transformation: $s = (1-z^{-1})/T_s$ is used.
Based on the work above, how does increasing \( k_p \) affect the dynamics of the system?

The final report is expected to include:

Two (2) MATLAB Plots, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of \( k_p \)
- Plot of \( 2k_p \)

Calculations showing how you found the following values, along with units for EVERY quantity found.
- Proportional gain \( k_p \)
- Proportional gain \( 2k_p \)
- Frequency of the system for \( k_p \)
- Frequency of the system for \( 2k_p \)
- Derivative gain \( k_d \)

For all the questions highlighted, the questions should be copied and pasted into your lab report and explicitly answered immediately thereafter.

**Experiment 2b: PD Control Design**

For this part of Experiment 2, a step input will be used instead of manually displacing the masses.

11. From Eq's (2-2,2-3,2-4) design controllers (i.e. find \( k_p \) & \( k_d \)) for a system natural frequencies \( \omega_n = 4 \) Hz, and three damping cases: 1) \( \zeta = 0.2 \) (under-damped), 2) \( \zeta = 1.0 \) (critically damped), 3) \( \zeta = 2.0 \) (over-damped).

12. Implement the underdamped controller (via PI + Velocity Feedback) and set up a trajectory for a 2500 count closed-loop Step with 1000 ms dwell time and 1 rep.

13. Execute this trajectory and plot the commanded position and encoder position in MATLAB (Plot them both on the same vertical axis so that there is no graphical bias.)

14. Repeat Steps 12 & 13 for the critically damped and over-damped cases. If the plots do not look critical, under, or over damped repeat the experiment, and if still off recheck your values. Save your plots for later comparison.

The final report is expected to include:

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Recall that for discrete implementation, you must divide the \( k_d \) values by \( T_s \) for controller input.
Four (4) MATLAB Plots, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.

- Plot of the under-damped response
- Plot of the critically damped response
- Plot of the over-damped response
- Plot of all three cases along with the commanded positions

Calculations showing how you found the following values, along with units for EVERY quantity found.

- Proportional gain \( k_p \)
- Under-damped Derivative gain \( k_d \)
- Critically damped gain \( k_d \)
- Over-damped Derivative gain \( k_d \)

For all the questions highlighted, the questions should be copied and pasted into your lab report and explicitly answered immediately thereafter.

**Experiment 2c: Adding Integral Action**

In part c, a full PID controller will be designed, and added effects of the integral gain will be studied.

18a. Now compute \( k_i \) such that \( k_i k_{hw} = 7500 \) N/(m-sec). \(^{10}\)

18b. Implement a controller with this value of \( k_i \) and the critically damped \( k_p \) & \( k_d \) parameters from Step 11. (Do not input \( k_i >3.0 \)) \(^{11}\). Be certain that the following error seen in the background window is within 20 counts prior to implementing.

18c. Execute a 2500 count closed-loop step of 1000 ms duration (1 rep).

18d. Plot the encoder #1 response and commanded position in MATLAB

19. Increase \( k_i \) by a factor of two, implement your controller (do not input \( k_i >3.0 \)) and plot its step response. Manually displace the mass by roughly 5 mm. Can you feel the integral action increasing the restoring control force with time? (Do not hold for more than about 2 seconds to avoid excessive force build-up and hence triggering the motor thermal protection.) In Step 19, what happens when you let go of the mass?

How do you feel/observe the integral action affect the system behavior/response? Does this effect become stronger or weaker when the value of \( k_i \) is increased?

The final report is expected to include:

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\(^{10}\) For discrete implementation you must multiply the resulting value of \( k_i \) by \( T_s \) before inputting into the controller.

\(^{11}\) For discrete implementation, do not input \( k_i >0.3* T_s \).
Three (3) MATLAB Plots, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.

- Plot of the response with $k_i$
- Plot of the response with $2k_i$
- Plot of the commanded position, $0^*k_i$ (from Experiment 2b), $1^*k_i$, and $2^*k_i$

Calculations showing how you found the following values, along with units for EVERY quantity found.

- Integral gain $k_i$
- Integral gain $2k_i$

For all the questions highlighted, the questions should be copied and pasted into your lab report and explicitly answered immediately thereafter.

**Experiment 3:**

**Free vibration of a two-degree-of-freedom system**

In this final experiment, there are 3 springs and 2 masses being implemented and no control algorithm is used. This experiment demonstrates the various modes a 2 degree of freedom system can take.

Create the system shown below using **carriages 2 and 3**. Put five weights in each carriage and use three medium-stiffness springs. Make sure the parts are tightly connected with springs in good pre-compression (for that, you may need to use one mass block as a stopper to help fix the third carriage at a closer position toward the second carriage).

![System Diagram]

The system has two modes and two natural frequencies.

$$\omega_{n_1} = \sqrt{\frac{k}{m}}$$

Equation 3-1 the natural frequency of the 1st mode

$$\omega_{n_2} = \sqrt{\frac{3k}{m}}$$

Equation 3-2 the natural frequency of the 2nd mode
Experiment 3a: First-mode experiment
With an initial condition of displacing both masses in the same direction by the same amount and then releasing will excite the first mode only. Perform such an experiment by displacing the masses by 2.0 cm to the right and then release. Observe that the two masses will free-vibrate in fairly synchronized manner; the frequency of the vibration should be roughly equal to \( \omega_{n1} \). Set up your system for data acquisition, free from any controls. Conduct the experiment and record the free vibrations of the two masses to a complete stop. Plot the displacements of the two masses along one common axis. Inspect your plot to see that the two masses move roughly together. Repeat the experiment with initial displacement of the masses to the left by 2.0 cm and record the resulting free vibration.

From the plots, estimate the natural frequency of the free vibration. Take the average of the two experiments (they should be very close).

The final report is expected to include:

Two (2) MATLAB Plots, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of the free vibration with initial condition of 2.0 cm to the right
- Plot of the free vibration with initial condition of 2.0 cm to the left

*Note it is very important to add a legend to these plots!

Calculations showing how you found the following values, along with units for EVERY quantity found.
- Mass of the carriage and five weights
- Spring Stiffness
- Frequency of the first mode
- Experimental natural frequency of the first mode
- Percent error of the calculated and experimentally determined first modes

Experiment 3b: Second-mode experiment
With an initial condition of displacing the two masses in opposite directions by the same amount and then releasing will excite the second mode only. Perform such an experiment by pushing the masses toward each other by 2.0 cm each and then release. Observe that the two masses will free-vibrate in fairly opposite direction and significant faster than that in the first-mode experiment; the frequency of the vibration should be roughly equal to \( \omega_{n2} \). Conduct the experiment and record the free vibrations of the two masses to a complete stop. Inspect your plot to see that the two masses move opposite to each other. Repeat the experiment by pulling the two masses away from each other by 2.0 cm each and record the resulting free vibration.
From the plots, estimate the natural frequency of the free vibration. Take the average of the two experiments.

Cite one main reason below that may be responsible for the discrepancies in $\omega_{n_1}$ and $\omega_{n_2}$.

The final report is expected to include:

Two (2) MATLAB Plots, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of the free vibration with initial condition toward each other
- Plot of the free vibration with initial condition away from each other

*Note it is very important to add a legend to these plots!

Calculations showing how you found the following values, along with units for EVERY quantity found.
- Mass of the carriage and five weights
- Spring Stiffness
- Frequency of the second mode
- Experimental natural frequency of the second mode
- Percent error of the calculated and experimentally determined second modes

Experiment 3c: Free vibration that excites both modes

Arbitrary initial conditions will in general excite both modes of the system so that the free vibration will contain a mixture of both frequencies. To see this, conduct an experiment by holding one mass while displacing the other by 2.0 cm in any direction and release. Observe the somewhat irregular vibratory motions of the two masses. Record the vibration of either mass 1 or mass 2.

The final report is expected to include:

One (1) MATLAB Plot, with two (2) Data Cursor Points on plots required to find the natural frequency of the system, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of the free vibration with initial condition listed above