Inverted Pendulum Experiment

Figure 1 shows the Inverted Pendulum Experiment. It consists of a pendulum rod which supports the sliding balance rod. The balance rod is driven by a belt and pulley which in turn is driven by a drive shaft connected to a DC servo motor below the pendulum rod. There are two series of weights included that affect the physical plant: brass counter weights connected underneath the pivot plate with adjustable height and weight and brass “donut” weights attached to both ends of the balance rod.

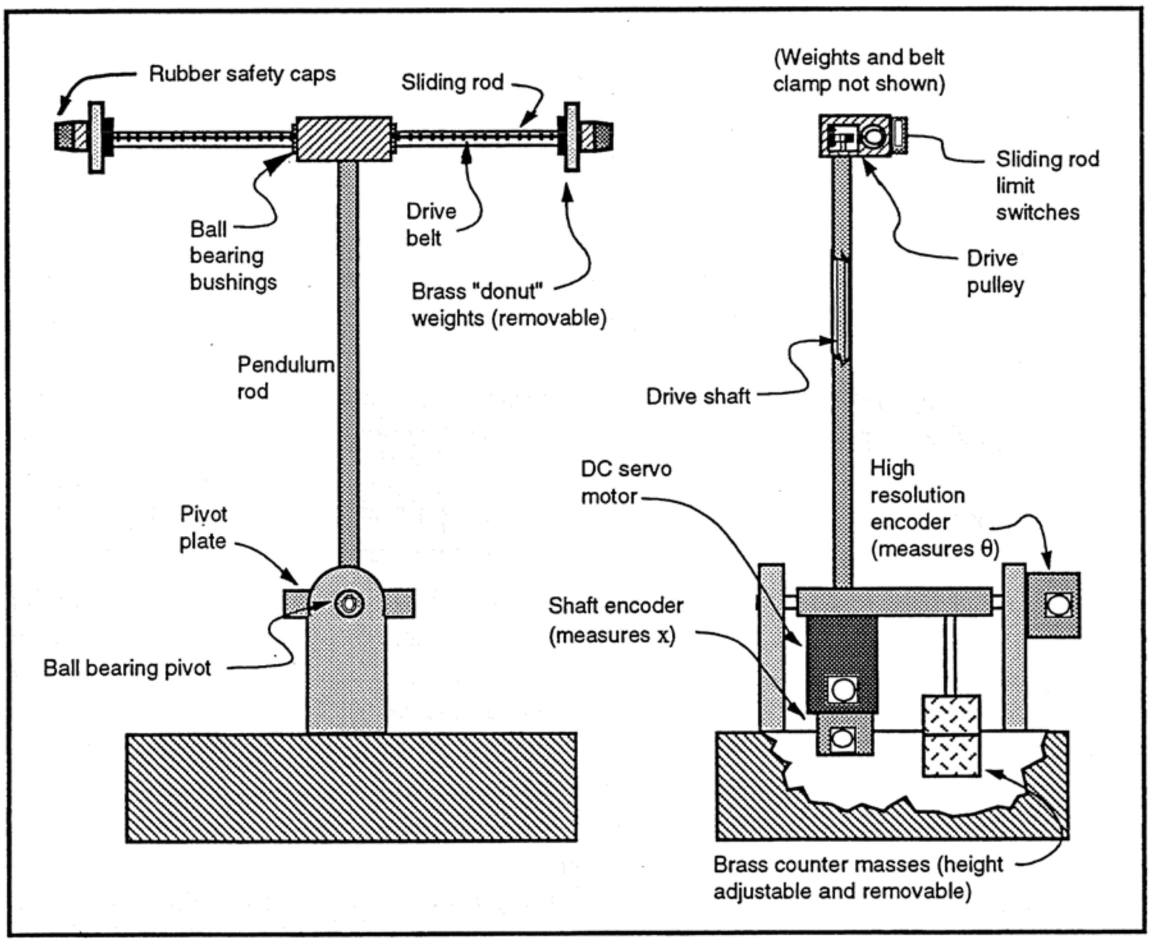

Figure 1: Model 505 ECP Inverted Pendulum Apparatus

Checkout Procedure

Step 1: Enter the ECP program by double clicking on its icon. You should see the Background Screen. Gently move the sliding and pendulum rods back and forth. You should observe some following errors and changes in encoder counts. The Control Loop Status should indicate "OPEN" and the Controller Status should indicate “OK”.

Step 2: Make sure that you can rotate the rods freely. Now press the black "ON" button to turn on the power to the Control Box. You should notice the green power indicator LED lit, but the motor should remain in a disabled state. Do not touch the mechanism whenever power is applied to the Control Box since there is a potential for uncontrolled motion of the rods unless the controller has been safety checked.

Introduction Exercises

A. Read the safety information for the Inverted Pendulum Experiment in Chapter 2 of the equipment manual (See also Appendix B on the course website).

B. Identify the control elements and signals in the Inverted Pendulum Experiment*.

Sensor:
Controller:
Actuator Output:

Actuator:
Reference Input:
System Output:

*Refer to the Introduction Literature.

Procedures

Be sure to read pages 60-63 before beginning the experiments. Be sure to note that the plant configurations for these experiments are described on page 62 of the ECP manual. Include all plots generated for the experiment.

Experiment 6.1: System Identification

Follow the instructions in Experiment 6.1 (System Identification) in the ECP Manual.

The following quantities should be found experimentally from the System Identification procedure. Be sure to include any applicable units.

Experiment 6.1.1: k_a = _____________
k_x = _____________
Experiment 6.1.2: J in terms of T:
J = ______________
J_o^* = _____________

The plot for 6.1.2 is on page: ________

What is the effect of k_a and k_x on the system? The block diagram on Figure 6.1-2 may be helpful.

Experiment 6.2: Successive Loop Closure/ Pole Placement Design

Follow the instructions in Experiment 6.2 and next two pages to setup the pole placement to gain control of the angle of the pendulum.

The following quantities should be found. Be sure to include applicable units:

Experiment 6.2.1.
Eq. (6.2-5) k_{hw} = ____________
Eq. (6.2-3) m^* = ____________

Step 2: k_p (10 Hz) = ____________
k_d (10 Hz) = ____________
Tested 90% rise time: ___________
Plot pg:______________

Experiment 6.2.2.
Comparing Eqs. (6.2-10 to 12), determine:
k^* = ___________
d_2 = ___________
d_1 = ___________
d_o = ___________
n_2 = ___________
n_1 = ___________
n_o = ___________
From the next two pages, determine:
r_1 = ___________
r_o = ___________
s_1 = ___________
s_o = ___________
k_{pfc} = _____________

Step 10: Plot pg:____________

Are the runs and results consistent with what described in the manual Steps 8 to10. Briefly comment on differences if any.

Determination of the controller coefficients, s_1, s_o, r_1, r_o, in S(s) and R(s)

Equation 6.2.12 in the manual leads to

$$D_{cl}(s) = (d_2 r_1 + n_2 s_1)s^3 + [(d_1 r_1 + d_2 r_o) + (n_1 s_1 + n_2 s_o)]s^2 + [(d_o r_1 + d_1 r_o) + (n_o s_1 + n_1 s_o)]s + (d_o r_o + n_o s_o)$$

Equation 6.2.13 leads to

$$D_{cl}(s) = s^3 + 5\pi s^2 + 8\pi^2 s + 6\pi^3 = s^3 + a_2 s^2 + a_1 s + a_o$$

Equating the coefficients of like terms in the above two polynomial equations yields

$$d_2 r_1 + n_2 s_1 = 1 \qquad (1)$$

$$d_1 r_1 + n_1 s_1 + d_2 r_o + n_2 s_o = a_2 \qquad (2)$$

$$d_o r_1 + n_o s_1 + d_1 r_o + n_1 s_o = a_1 \qquad (3)$$

$$d_o r_o + n_o s_o = a_o \qquad (4)$$

Also, from your earlier work or from Eq. 6.2-10

$$d_1 = n_1 = 0$$

Then Eqs. (1) to (4) reduce to

$$d_2 r_1 + n_2 s_1 = 1 \qquad (1a)$$

$$d_2 r_o + n_2 s_o = a_2 \qquad (2a)$$

$$d_o r_1 + n_o s_1 = a_1 \qquad (3a)$$

$$d_o r_o + n_o s_o = a_o \qquad (4a)$$

Solving 1a & 3a, we get

$$r_1 = \frac{n_0 - a_1 n_2}{d_2 n_0 - d_0 n_2} \tag{5}$$

$$s_1 = \frac{d_0 - a_1 d_2}{n_2 d_0 - n_0 d_2} \tag{6}$$

Similarly, solving 2a and 4a gives

$$r_0 = \frac{a_2 n_0 - a_0 n_2}{d_2 n_0 - d_0 n_2} \tag{7}$$

$$s_0 = \frac{a_2 d_0 - a_0 d_2}{n_2 d_0 - n_0 d_2} \tag{8}$$

Where

$$a_0 = 6\pi^3$$
$$a_1 = 8\pi^2$$
$$a_2 = 5\pi$$

Determination of k_{pfc} in the block diagram 6.2-3

Assuming c(s) indicated in Fig. 6.2-3 is close to unity and using the notation in Eq. (6.2-10), the close-loop transfer function of the system based on the block-diagram in Fig. 6.2-3 is:

$$T_{cl}(s) = \frac{k_{pfc}}{R(s)} \frac{k * \frac{N_{ax}(s)}{D_{ax}(s)}}{1 + k * \frac{N_{ax}(s)}{D_{ax}(s)} \frac{S(s)}{R(s)}}$$

The system gain k_{pfc} may be determined by setting $T_{cl}(0) = 1.0$