Fall 2009

ME 550. Foundations of Engineering Systems Analysis

Take_Home Examination #3 **Due Date: November 13, 2009**

Problem #1. (a) Let $T=[0,\infty)$. Cite examples to show that $L_s(T)\not\subset L_r(T)$ for $r,s\in[1,\infty]$ and $r\neq s$. Hence, conclude that if μ is a σ-finite (e.g., Lebesgue) measure on $\Re$, then $L_s(\mu)\not\subset L_r(\mu)$ for $r,s\in[1,\infty]$ and $r\neq s$.

(b) Let $T=[-\pi,\pi)$. Show that $L_s(T)\subset L_r(T)$ for $1\le r\le s\le\infty$. Hence, conclude that if ν is a finite (e.g., probability) measure on $\Re$, then $L_s(\nu)\subset L_r(\nu)$ for $1\le r\le s\le\infty$. **10 pts**

Problem #2. Let the sequence $\xi=\left\{\frac{1}{k}:k\in\mathbf{N}\right\}$.

(a) Show that $\xi\in\ell_p\ \forall p\in(1,\infty]$.

(b) Make use of Fourier Series Theorem to show that $\|\xi\|_{\ell_2}=\frac{\pi}{\sqrt{6}}$ and $\|\xi\|_{\ell_4}=\frac{\pi}{\sqrt[4]{90}}$ **20 pts**

Problem #3. Let $V=F^n$ and $W=F^m$. Let $\Im:V\to W$ be a linear transformation such that $\Im\in F^{m\times n}$. Let $\|\Im\|_p$ denote the norm induced by the p^{th} Holder norm for $p\in[1,\infty]$. Then, show that

(a) $\|\Im\|_1=\max_{j\in\{1,\cdots,n\}}\left(\sum_{k=1}^{m}|\Im_{kj}|\right)$ that is the maximum absolute column sum.

(b) $\|\Im\|_\infty=\max_{k\in\{1,\cdots,m\}}\left(\sum_{j=1}^{n}|\Im_{kj}|\right)$ that is the maximum absolute row sum. **20 pts**

Problem #4. Let $A\in L(V,W)$.

(a) Show that $A\in B(V,W)$ if V is finite-dimensional. Is it necessary that W is also finite-dimensional?

(b) Show that $A\in B(V,W)$, A^{-1} exists, and $A^{-1}\in B(\mathit{range}(A),V)$ if and only if $\exists\beta\ge\alpha>0$ such that $\alpha\|x\|_V\le\|Ax\|_W\le\beta\|x\|_V\ \forall x\in V$. **10 pts**

Problem #5. Let $p\in[1,\infty)$ and $\frac{1}{p}+\frac{1}{q}=1$. For $y\in L_q([0,1])$, show that $f\in L_p^*([0,1])$ if $f(x)=\int_0^1 dt\,x(t)\,y(t)$ **10 pts**

Problem #7. Let $V=C_\infty[0,1]$ be the space of continuous real-valued functions with the domain $[0,1]$ and the norm $\|\bullet\|_\infty$.

Let $y\in V$. Let the functional f be defined as $f(x)=\int_0^1 dt\,x(t)\,y(t)\quad\forall x\in V$.

(a) Show that $f\in V^*$ where V* is the dual space of V.

(b) Find the norm $\|f\|$.

(c) Let y(t)=t. Find an $x\in V$ such that $\|x\|_\infty=1$ and $|f(x)|=\|f\|$. **30 pts**