# Fall 2002

# EE/ME 559 NONLINEAR CONTROL AND STABILITY

## Take-Home Examination #2; Due Date: October 29, 2002

**Problem #1**. Let $V:[0,\infty)\times\Re^n \to \Re$ be defined as: $V(t,x) = x(t)^T M(t)x(t)$ where $M$ is a continuous function of $t$ and the matrix M(t) is real and symmetric for each $t$. Determine necessary and sufficient conditions for $V$ to be: (a) positive definite; and (b) decrescent.

**Problem #2**. Show that the equilibrium point **0** is globally exponentially stable if there exists constants $a,b,c \in (0,\infty)$ and $p \in [1,\infty)$, and a continuously differentiable function $V:[0,\infty)\times\Re^n \to \Re$ such that

$$a\|x(t)\|^p \le V(t,x(t)) \le b\|x(t)\|^p \quad \text{and} \quad \dot{V}(t,x(t)) \le -c\|x(t)\|^p \, \forall t \in [0,\infty) \forall x \in \Re^n$$

**Problem #3**. (Floquet Theory) Consider the linear system $\dot{x}(t) = A(t)\,x(t)$ where $A(t)$ is $T$-periodic for some $T \in (0,\infty)$, i.e., $A(t) = A(t+T) \quad \forall t$. Let $\Phi(\bullet,\bullet)$ be the state transition matrix. Define a constant (square matrix) $B$ via the equation: $\exp(BT) = \Phi(T,0)$ and let $P(t) = \exp(Bt)\,\Phi(0,t)$. Show that:

(a) $P(t) = P(t+T) \quad \forall t$

(b) $\Phi(t,\tau) = P^{-1}(t)\exp[(t-\tau)B]P(\tau) \quad \forall t$

(c) The equilibrium point **0** is exponentially stable if and only if $B$ is Hurwitz, i.e. each eigenvalue of $B$ has a negative real part.

**Problem #4**. Exercise 4.17 (p. 184) of Khalil, 3rd ed.

**Problem #5**. Exercise 4.19 (p. 184) of Khalil, 3rd ed.