Spring 2008
ME (MATH) 577. Stochastic Systems for Science and Engineering

Take Home Examination #3; Due Date: March 17, 2008

Problem #1. Show, by Monte Carlo simulation of the integral of white Gaussian noise of constant intensity α, that the variance of the Wiener process, β_t, is αt where α is the constant diffusion parameter. Repeat the simulation exercise if the white noise is not of constant intensity, say, $\alpha(t) = \exp(-t)$. In that case, verify whether the variance of the non-constant diffusion Wiener process, β_t, is $1 - \exp(-t)$ or not.

Problem #2. Let a dynamical system be governed by the following stochastic differential equation:

$$\frac{d^2 y_t}{dt^2} + 3\frac{dy_t}{dt} + 2 y_t = w_t\,; \text{ given } E\begin{bmatrix} y_0 \\ \dot{y}_0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}; \text{ Cov}\left(\begin{bmatrix} y_0 \\ \dot{y}_0 \end{bmatrix}\right) = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix};$$

and w_t is the zero-mean stationary white Gaussian noise of intensity 0.01.

(a) Write a state-space model of the system and then find an analytical solution.

(b) Simulate the system on a computer and compare the results of Monte Carlo simulation with the analytical results. Explain how you have chosen the step size of integration.

Problem #3.

Problems 7A, 7B, 7C, and 7D of Bartle (p. 77)
Problems 7L and 7K of Bartle (p. 78)

Problem #4.

Problem 7.8 of 2^{nd} ed. Stark and Woods (p. 364).

Problem 7.14 of 2^{nd} ed. Stark and Woods (p. 366).

Problem 7.18 of 2^{nd} ed. Stark and Woods (p. 368).

Problem #5.

Problem 8.9 of 2^{nd} ed. Stark and Woods (p. 415).

Problem 8.17 of 2^{nd} ed. Stark and Woods (pp. 416-417).

Problem #6.

Problem 9.4 of 2^{nd} ed. Stark and Woods (p. 462) (Assume the process X_t to be zero-mean.)

Problem 9.7 of 2^{nd} ed. Stark and Woods (p. 463)

Problem 9.14 of 2^{nd} ed. Stark and Woods (p. 465).