SPRING 2008
ME (Math) 577 Stochastic Systems
Take Home Examination #4; Due Date: April 16, 2008

Problem #1

Problem 9.15 of Stark and Woods (p. 466) 2^{nd} ed..

Problem 10.13 of Stark and Woods (pp.541-542), 2^{nd} ed..

Problem #2: This problem is intended for practice of Kalman filter design rather than the development of its theory. Consider the following the RC-network in Fig. 1 which is excited by a zero-mean WSS Gaussian white noise w_t of intensity Q. The measured output is the voltage z_t across the capacitor C_2 in Fig. 1 where the measured voltage is contaminated with a zero-mean WSS Gaussian white noise v_t of intensity R. Also assume that the parameters of the circuit in Fig. 1 as: $R_1 = R_2 = 1\Omega$ and $C_1 = C_2 = 1$ farad; and the statistics of the voltages across C_1 and C_2 at time t=0 are independent and identically distributed as N(0, 0.0001).

(a) State your assumptions to construct a Kalman filter for state estimation in both discrete-time and continuous-time settings.

(b) Write down the governing equations for a discrete-time Kalman filter and a continuous-time Kalman filter.

(c) Derive the plant model and the measurement model in the continuous-time state-variable setting.

(d) Obtain a discrete-time state-space model for a constant sampling period T.

(e) Select appropriate values of the parameters T, Q and R (you may try T in the range of 0.03 sec to 0.1 sec; R in the range of 0.001 $volt^2$ to 5.0 $volt^2$; Q in the range of 10 $volt^2$ to 50 $volt^2$) to execute the filter equations in the discrete-time setting. Based on the values of the parameters Q, R and T, obtain the continuous-time parameters. Find the optimal gain matrices for both discrete-time and continuous-time filters.

Write your own software in the Matlab environment and verify the results by comparison with those obtained from the standard Matlab m-files (e.g., dlqe and lqe).

Examine the effects of changing T on accuracy of the result (e.g., norm of the state error covariance matrix).

Examine the effects of changing the relative magnitudes of Q and R on the filter performance (e.g., norm of the filter gain matrix).

Verify that, for small T's, the results generated by the discrete-time and continuous-time filters are close to each other.

(f) Show, by simulation, that the innovation sequence of the discrete-time filter is indeed white if the filter is optimal. Also demonstrate that the innovation sequence may no longer be white if the plant model or the noise statistic is altered. In particular, demonstrate the fault detection capability of the Kalman filter by injecting an abrupt change in plant model parameter(s).

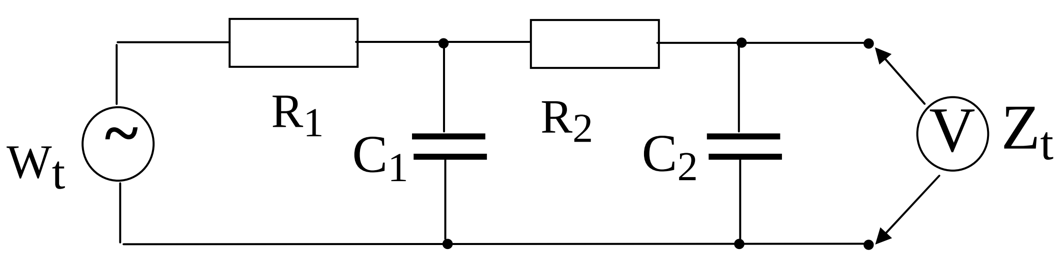

Figure 1 Circuit Diagram of the R-C network