Spring 2009

ME 577 Stochastic Systems for Science and Engineering # 390193
MATH 577 Stochastic Systems for Science and Engineering # 427414

Instructor: Professor Asok Ray

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Lecture Class Time and Location: MWF 2:30 PM to 3:20 PM at 371 Willard

Recitation Class Time and Location: T 5:45 PM to 7:00 PM at 214 Reber

Instructional Objectives:

The objectives of this second-level graduate course are: (1) development of the theory of stochastic processes and stochastic differential equations for applications to science and engineering; and (2) preparation for advanced course work and research in stochastic systems modeling and analysis, estimation, and control. The course is developed for advanced graduate students and researchers in engineering, applied mathematics, statistics, physics, meteorology, geo-sciences, and life sciences.

Course Outline:

Concepts of probability space, random variables, and functions of random variables from the measure-theoretic point of view. Case studies with finite and σ-finite measures. Notion of Radon-Nikodym derivative as applied to construction of density functions and likelihood ratios. Borel-Cantelli lemmas and their applications. Generalized Chebyshev inequality, Chernoff bound, and Cramer-Rao lower bound. Characteristic and moment generating functions. Examples to illustrate physical significance of the mathematical concepts from application perspectives. **2 weeks**

Stochastic processes and random fields: Concepts of separability and measurability. Stochastic convergence and continuity. Completeness of the Borel-measurable space relative to different topologies. Markov processes including the birth and death process. Properties of stochastic matrices and the Perron-Frobenius theorem. Introduction to Martingales and the Strong Law of Large Numbers. Introduction to Markov random fields, Gibbs-Markov equivalence, and Hammersley-Clifford Theorem. Recurrence and ergodicity: Birkhoff's ergodic theorem, and concepts of mixing and entropy. **5 weeks**

Mean square calculus in the $L_2(P)$ space. Wiener process and white Gaussian noise. Fractional Brownian motion (fBm) and fractional Gaussian noise (fGn). Solutions of stochastic integrals and linear stochastic differential and difference equations. Introduction to the discrete-time and continuous-time minimum-variance filter in the setting of orthogonal projection in a separable Hilbert space. Stochastic controllability and observability. Uniform asymptotic stability of the continuous-time and discrete-time Kalman filters. Shaping filters. **5 weeks**

Introduction to Nonlinear stochastic differential equations of *Itô* and Stratonovich. Kolmogorov forward and backward diffusion equations and their relationship to *Itô* equations. Applications to state estimation and dynamic programming. **3 weeks**

Individual Research Projects: Single selection from the following topics -- Hidden Markov modelling (HMM); Rao-Blackwellised Particle Filtering (RBPF); Fractional Brownian motion (fBm) modelling; Random field modelling of images.

Computing Laboratory: Experiments on computation of multi-variate characteristic and moment generating functions; simulation of randomly varying dynamical processes for solving problems in science and engineering; experiments on the Kalman-Bucy filter realized as an innovation-whitening filter. Design of shaping filters and approximate nonlinear filters.

Prerequisites: ME 550 or MATH 501; and EE 560 or MATH (STAT) 418

Reference Books (Reserved in the Library):

R. G. Bartle, *The Elements of Integration and Lebesgue Measure*, Wiley Interscience, 1966 (Reprinted 1995).

J.L. Doob, *Stochastic Processes*, John Wiley, 1953 (Reprinted 1990).

R. Durrett, *Probability: Theory and Examples*, 2nd ed., Wadsworth Publishing Company, Belmont, CA, 1996.

A.H. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press, 1970.

A.W. Naylor and G.R. Sell, *Linear Operator Theory in Engineering & Science*, Springer-Verlag, 1982.

B. Oksendal, *Stochastic Differential Equations*, 6th ed., Springer, Berlin, 2003.

W. Rudin, *Real and Complex Analysis*, 3rd ed., McGraw Hill, 1987.

H. Stark and J.W. Wood, *Probability, Rrandom Processes, and Estimation Theory for Engineers*, Prentice Hall, 1994.

E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems*, Springer-Verlag, New York, 1985.