ME 578 Theory and Applications of Wavelets

Take-Home Examination #1; Due Date: January 28, 2009

Problem#1. Solve all problems of Chapter 1 of Kaiser.

Problem#2. Given $f(t) = exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right)$ where $t \in \Re$ and $\sigma \in \Re^+$,

(a) Verify whether $f \in L_1(\Re) \cap L_2(\Re)$. If so, show that $\|f\|_{L_2(\Re)} = \|\hat{f}\|_{L_2(\Re)}$.

(b) For different numerical values of μ and σ, use the Matlab function FFT to analyze $\hat{f}$ and then plot the results to compare with the analytically determined $\hat{f}$.

(c) For different numerical values of μ and σ, use Matlab function IFFT to reconstruct f from $\hat{f}$ and then plot the results to compare with results with the original f.

(d) Investigate the effects of μ and σ on $\hat{f}$.

(e) Now consider the following two-dimensional Gaussian function

$$f(t_1,t_2) = exp\left(-\frac{1}{2\left(1-r^2\right)}\left[\left(\frac{t_1-\mu_1}{\sigma_1}\right)^2 - 2r\left(\frac{t_1-\mu_1}{\sigma_1}\right)\left(\frac{t_2-\mu_2}{\sigma_2}\right) + \left(\frac{t_2-\mu_2}{\sigma_2}\right)^2\right]\right)$$

to investigate the effects of $\mu_1, \mu_2, \sigma_1, \sigma_2$, and r on $\hat{f}(\xi_1,\xi_2)$ by using the Matlab function FFT2.