ME 578 Theory and Applications of Wavelets

Take-Home Examination #6; Due Date: March 18, 2005

Solve Exercises 8.1 to 8.8 in p. 197 of Kaiser. (See attached copies of pages from Kaiser)

Conjecture 8.9. *ϕ^2 and ϕ^3 are exact.*

Equations (8.96) can be derived from the Poisson summation formula in combination with (8.7); they are related to the beautiful Strang–Fix approximation theory (Strang and Fix [1973]; Strang [1989]), which has been generalized recently by de Boor, DeVore, and Ron (1994).

8.5 Plots of Daubechies Wavelets

Figures 8.1 through 8.4 show the results of computing the minimal-support, minimal-phase Daubechies scaling functions and wavelets by the cumulant algorithm, using four iterations. The initial values of ϕ are marked with $\circ$, and the first three iterations are marked with $*$, $\times$, and $+$. The fourth iteration for ϕ is a solid curve, and the wavelet ψ as computed from the third iteration of ϕ is a dashed curve. The vertical dotted line is $t = \tau$.

Exercises

8.1. Verify that $P(z)$, defined by (8.6), satisfies the orthonormality condition (8.2) if $W(z)$ is a solution of (8.16), with $\mathcal{W}_N(S)$ given by (8.15).

8.2. Prove (8.23).

8.3. Let ϕ be the scaling function (8.41). For any $M \in \mathbf{N}$, let $u_{3n} = 1, u_{3n+1} = -1$, $n = 0, 1, \ldots, M-1$, with all other $u_k = 0$. Compute and compare $\sum_k |u_k|^2$ and $\| \sum_k u_k \phi_k \|^2$. What happens as $M \to \infty$? Explain why ϕ is "bad."

8.4. Find the wavelet corresponding to the scaling function (8.41).

8.5. Compute the next recursion for ϕ from (8.62), i.e., find $\phi(.25), \phi(.75), \ldots, \phi(2.75)$.

8.6. Use a computer to find and plot $\phi^2(t)$ by the method of Section 8.2: Compute $\hat{\phi}$ to some accuracy, then take its inverse Fourier transform. (Experiment with the accuracy.)

8.7. Use a computer to find and plot $\phi^2(t)$ by diadic interpolation, to the scale $\Delta t = 2^{-4}$.

8.8. Plot the minimum-phase, minimum-support scaling function $\phi^{10}(t)$ by the method of cumulants, using three iterations. (The coefficients $h_n = c_n/\sqrt{2}$ are listed in Daubechies (1992), p. 195.)