Spring 2008

ME 577 Stochastic Systems for Science and Engineering # 961225
MATH 577 Stochastic Systems for Science and Engineering # 106750

Instructor: Professor Asok Ray

Office Location: 329 Reber Building; Tel: (814) 865-6377; email: axr2@psu.edu

Web Page Address: http://www.mne.psu.edu/ray/graduate_courses.html

Lecture Class Time and Location: MW 2:30 PM to 3:45 PM at 206 Hammond

Recitation Class Time and Location: T 5:45 PM to 7:00 PM at 214 Reber

Instructional Objectives:

The objectives of this second-level graduate course are: (1) development of the theory of stochastic processes and stochastic differential equations for applications to science and engineering; and (2) preparation for advanced course work and research in stochastic systems modeling and analysis, estimation, and control. The course is developed for advanced graduate students and researchers in engineering, applied mathematics, statistics, physics, meteorology, geo-sciences, and life sciences.

Course Outline:

Concepts of probability space, random variables, and functions of random variables from the measure-theoretic point of view. Case studies with finite and σ-finite measures. Notion of Radon-Nikodym derivative as applied to construction of density functions and likelihood ratios. Generalized Chebyshev inequality. Characteristic and moment generating functions. Concept of sufficient statistic. Examples to illustrate physical significance of the mathematical concepts from application perspectives. **3 weeks**

Introduction to stochastic processes. Concepts of separability and measurability. Stochastic convergence and continuity. Completeness of the Borel-measurable space relative to different topologies. Introduction to Markov processes including the birth and death process. Properties of stochastic matrices and the Perron-Frobenius theorem. Introduction to Martingales and the Strong Law of Large Numbers. **3 weeks**

Mean square calculus in the $L_2(P)$ space. Wiener process and white Gaussian noise. Fractional Brownian motion (fBm) and fractional Gaussian noise (fGn). Solutions of stochastic integrals and linear stochastic differential and difference equations. Introduction to the discrete-time and continuous-time minimum-variance filter in the setting of orthogonal projection in a separable Hilbert space. Stochastic controllability and observability. Stability analysis of the continuous-time and discrete-time Kalman filter. Shaping filters. **6 weeks**

Introduction to Nonlinear stochastic differential equations of *Itô* and Stratonovich. Kolmogorov forward and backward diffusion equations and their relationship to *Itô* equations. **3 weeks**

Individual Research Projects: Single selection from the following topics -- Hidden Markov modelling (HMM); Extended Kalman filtering (EKF); Rao-Blackwellised Particle Filtering (RBPF); Fractional Brownian motion (fBm) modelling.

Computing Laboratory: Experiments on computation of multi-variate characteristic and moment generating functions; simulation of randomly varying dynamical processes for solving problems in science and engineering; experiments on the Kalman-Bucy filter realized as an innovation-whitening filter. Design of shaping filters and approximate nonlinear filters.

Prerequisites: ME 550 or MATH 501; and EE 560 or MATH (STAT) 418

Reference Books (Reserved in the Library):

R. G. Bartle, *The Elements of Integration and Lebesgue Measure*, Wiley Interscience, 1966 (Reprinted 1995).

J.L. Doob, *Stochastic Processes*, John Wiley, 1953 (Reprinted 1990).

A.H. Jazwinski, *Stochastic Processes and Filtering Theory*, Academic Press, 1970.

P.S. Maybeck, *Stochastic Models, Estimation, and Control*, Volumes 1, 2 and 3, Academic Press, 1979, 1982, 1982.

A.W. Naylor and G.R. Sell, *Linear Operator Theory in Engineering & Science*, Springer-Verlag, 1982.

B. Oksendal, *Stochastic Differential Equations*, 6th ed., Springer, Berlin, 2003.

A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd ed., McGraw Hill, 1991.

W. Rudin, *Real and Complex Analysis*, 3rd ed., McGraw Hill, 1987.

H. Stark and J.W. Wood, *Probability, Rrandom Processes, and Estimation Theory for Engineers*, Prentice Hall, 1994.

E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems*, Springer-Verlag, New York, 1985.