Torsion Experiment

Introduction
For the Torsion lab, there are two required experiments to perform and one extra credit assignment at the end. In experiment 1, the system parameters need to be identified so that they can be used in later experiments to build a controller. Experiment 2 is broken into 4 sub-parts. First, the disks are manually displaced with a P or a PD controller. After this, a step input is implemented to 3 different systems; under-damped, critically damped and over-damped. Next, a full PID controller is implemented and tested, and comparisons are drawn between the PD and PID controllers. The final test in Experiment 2 then evaluates the frequency response of the under-damped, critically damped and over-damped systems tested earlier.

Hardware
The system we use in this experiment is Model 205. Figure 1 shows the Model 205 Torsion Experiment consisting of three disks supported by a torsionally flexible shaft that is suspended vertically on anti-friction ball bearings. The shaft is driven by a brushless servo motor connected via a rigid belt (negligible tensile flexibility) and pulley system.
with a 3:1 speed reduction ratio. An encoder located on the base of the shaft measures the angular displacement, \( \theta_1 \) of the first disk, \( J_1 \). Two additional encoders measure the displacements of the other two disks as shown. The torsional mechanism represents many physical plants including rigid bodies; flexibility in drive shafts, gearing and belts; and coupled discrete vibration with actuator at the drive input and sensor collocated or at flexibly coupled output (non-collocated).

**Safety**

As with every lab, first read Appendix B on the course website. For this lab experiment

- Ensure the masses are firmly secured by screws.
- Be sure to stay clear of the mechanism when turning on the controller. Selecting Implement Algorithm immediately implements the specified controller; if there is an instability or large control signal, the plant may react violently. If the system appears stable after implementing the controller, first displace the disk with a light, non sharp object (e.g. a plastic ruler) to verify stability prior to touching plant.

**Hardware/Software Equipment Check**

Before starting the lab, make sure the equipment is working by following the steps below:

**Step 1:** Enter the ECP program by double clicking on its icon. You should see the Background Screen. Gently rotate the drive or load disk by hand. You should observe some following errors and changes in encoder counts. The Control Loop Status should indicate "OPEN" and the Controller Status should indicate “OK”.

**Step 2:** Make sure that you can rotate the disks freely. Now press the black "ON" button to turn on the power to the Control Box. You should notice the green power indicator LED lit, but the motor should remain in a disabled state. Do not touch the disks whenever power is applied to the Control Box since there is a potential for uncontrolled motion of the disks unless the controller has been safety checked.
**Experiment 1: System Identification**

In practice, the system parameters of a piece of equipment, such as the inertia, spring constant, and damping ratios are often unknown. In this section of the lab, these unknown parameters will be determined using a process called system identification. These same parameters will be used later to implement various controllers.

**Procedure:**

1. Secure four 500g masses on the upper and lower disks as shown in the figure. Verify that the masses are secured and that each is at a center distance of 9.0 cm from the shaft center-line.

2. Clamp the center disk to put the mechanism in the configuration shown in Figure 1-1a using the 1/4" bolt, square nut, and clamp spacer. Only light torquing on the bolt is necessary.

3. Set the time sample interval and time duration for sampling using the following procedure: With the controller powered up, enter the Control Algorithm box via the Set-up menu and set $T_s = 0.00442$ and Continuous Time. Enter the Command menu, go to Trajectory and select Step, Set-up. Select Open Loop Step and input a step size of 0 (zero), a duration of 4000 ms and 1 repetition. Exit to the Background Screen by consecutively selecting OK. This puts the controller board in a mode for acquiring 8 sec of data on command but without driving the actuator. This procedure may be repeated and the duration adjusted to vary the data acquisition period.

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![Diagram](image-url)  
**a) Set-up To Begin Plant Identification Procedure**

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![Diagram](image-url)  
**b) Transfer Function Configuration**
Figure 1  Configurations For Plant Identification

4. Select the encoder(s) needed for this lab and how often to measure them using the following steps: Go to Set up Data Acquisition in the Data menu and select Encoder #1 and Encoder #3 as data to acquire and specify data sampling every 2 (two) servo cycles, i.e. every 2 T_s's. Select OK to exit. Select Zero Position from the Utility menu to zero the encoder positions.

5. Collect data for the experiment using the following steps: Prepare to manually displace the upper disk approximately 20 deg. Exercise caution in displacing the inertia disk; displacements beyond 40 deg may damage and possibly break the flexible drive shaft. (Displacements beyond 25 deg will trip a software limit which disables the controller indicated by "Limit Exceeded" in the Controller Status box in the Background Screen. To reset, simply reselect Execute from the Command menu.) Select Execute from the Command menu. With the upper disk displaced approximately 20 deg (≤1000 encoder counts as read on the Background Screen display) in either direction, select Run from the Execute box and release the disk approximately 1 second later. The disk will oscillate and slowly attenuate while encoder data is collected to record this response. Select OK after data is uploaded.

6. Export the data from ECP to MATLAB, and plot the Encoder 3 data using MATLAB. Be sure to clearly label the plots.

7. Find the natural frequency of the system using the following steps: Choose several consecutive cycles (say 5 to 10) in the amplitude range between 100 and 1000 counts. Divide the number of cycles by the time taken to complete them being sure to take beginning and end times from the same phase of the respective harmonic cycles. Convert the resulting frequency in Hz to radians/sec. This damped frequency, ω_d, approximates the natural frequency, ω_n, according to:

\[ \omega_n = \sqrt{\omega_d^2 \left(1 - \frac{\omega_d^2}{\omega_n^2}\right)} \]  \hspace{1cm} (Equation 1-1)

where the "d31" subscript denotes disk #3, trial #1.

To find the exact time, use the Data Cursor tool in the MATLAB figure window from the plot you just created in step 6. Use these values to find the frequency. When pasting in the plot to your lab report, make sure to include the beginning and ending points you used for your calculations.
8. Now prepare for the next test, and repeat the above process. To do this, remove the four masses from the third (upper) disk and repeat Steps 5 through 7 to obtain $\omega_{nd32}$ for the unloaded disk. If necessary, repeat Step 3 to reduce the execution (data sampling) duration.

Make sure to export the data from ECP to MATLAB and create a new figure for this data, and clearly label the plot and the beginning and end points as you did in the step before.

9. Find the damping ratio of the system using the following steps:
   Measure the reduction from the initial cycle amplitude $X_o$ to the last cycle amplitude $X_n$ for the $n$ cycles measured in Step #8. Using relationships associated with the logarithmic decrement:

   \[
   \frac{\zeta_{d32}}{1 - \zeta_{d32}^2} = \frac{1}{2\omega n} \ln \left( \frac{X_o}{X_n} \right) \rightarrow \zeta_{d32} \approx \frac{1}{2\omega n} \ln \left( \frac{X_o}{X_n} \right) \text{ for small } \zeta_{d32}
   \]

   (Equation 1-2)

   find the damping ratio $\zeta_{d32}$ and show that for this small value the approximations of Equations 1 and 2 are valid.

   By using the Data Cursor from the last step, you also have the beginning and ending amplitudes.

10. Find the natural frequency and damping ratio for Disk 1. To do this, repeat Steps 5 through 9 for the lower disk, disk #1. Make sure you acquire data for Encoder 1 instead of Encoder 3. Obtain $\omega_{nd11}$, $\omega_{nd12}$ and $\zeta_{d12}$. How does this damping ratio compare with that for the upper disk?

11. Calculate by hand the inertia of the disks 1 and 3 together. Use the following information pertaining to each mass piece to calculate the portion of each disk's inertia attributable to the four masses for the "d31" and "d11" cases.

   Brass Mass (incl. bolt & nut) = 500g (± 5g)
   Diameter of Brass Mass = 5.00 cm (± 0.02 cm)

12. Calling this lumped inertia $J_m$ (i.e. that associated with the four masses combined), use the following relationships to solve for the unloaded disk inertia $J_{d3}$, and upper torsional shaft spring $k_{d3}$.

   \[
   k_{d3}/(J_m + J_{d3}) = (\omega_{nd3})^2 \quad \text{(Equation 1-3)}
   \]

   \[
   k_{d3}/J_{d3} = (\omega_{nd32})^2 \quad \text{(Equation 1-4)}
   \]
13. Find the damping coefficient $c_{d3}$ by equating the first order terms in the equation form:

$$s^2 + 2\zeta_n\omega_n s + \omega_n^2 = s^2 + c/s + k/J$$  \hspace{1cm} \text{(Equation 1-5)}

Repeat this for the lower unloaded disk inertia (this includes the reflected inertias of the motor, belt, and pulleys), spring and damping $J_{dl}$, $c_{dl}$ and $k_{dl}$ respectively.\(^1\)

Now all dynamic parameters have been identified! Values for $J_1$ and $J_2$ for any configuration of masses may be found by adding the calculated inertia contribution of the masses to that of the unloaded disk\(^2\).

**The final report is expected to include:**

A diagram identifying the control elements and signals in the Torsion Experiment.

Sensor: \hspace{1cm} \text{Actuator:}
Controller: \hspace{1cm} \text{Reference Input:}
Actuator Output: \hspace{1cm} \text{System Output:}

Four (4) MATLAB Plots, with two (2) Data Cursor Points on each plot, along with titles, labels and legends if necessary that **clearly** show which plot corresponds to which situation.

- Disk 3 Trial 1
- Disk 3 Trial 2
- Disk 1 Trial 1
- Disk 1 Trial 2

Calculations showing how you found the following values, along with units for EVERY quantity found. Use equations 1-1 – 1-5.

- 4 Natural frequencies ($\omega_{n31}$ $\omega_{n32}$ $\omega_{n11}$ $\omega_{n12}$)
- 2 Damping ratios ($\zeta_{d32}$ $\zeta_{d12}$)
- *Inertia of the 4 masses ($J_m$)
- Inertia of Disk 1 ($J_{d1}$)
- Inertia of Disk 3 ($J_{d3}$)
- Damping constant on Disk 3 ($c_{d3}$)
- Damping constant on Disk 1 ($c_{d1}$)
- Spring constant on Disk 3 ($k_{d3}$)
- Spring constant on Disk 1 ($k_{d1}$)

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\(^1\)Steps 11, 12 and 13 may be done later, away from the laboratory, if necessary.

\(^2\)In plant configurations where a disk is used in the center location, the inertia and damping parameters may be assumed to be the same as for the upper disk.
* First, calculate the inertia of each weight about its center of gravity \( J_{cg} = 0.5m*r^2 \). Then, use the parallel axis theorem to get the inertia about the center of rotation \( J = J_{cg} + mR^2 \). Then multiply four to get \( J_m \).

For all the questions highlighted, the questions should be copied and pasted into your lab report and explicitly answered immediately thereafter.

**Experiment 2.a: Rigid Body PD and PID Control**

In this part of the lab, a proportional controller will be implemented, so that the system will act like a specific frequency spring. After this the proportional gain will be increased. After these tests a damping term will also be included.

Note: You will need this value: \( k_{hw} = 17.4 \text{ N-m/rad} \)

**Procedure:**

- **Proportional & Derivative Control Actions**
  1. Using the results from Experiment 1 to construct a model of the plant with two mass pieces at 9.0 cm radial center distance on the bottom disk – **both other disks removed**. You may neglect friction.
  2. Set-up the plant in the configuration described in Step 1.
  3. From Equation 2-3 determine the value of \( k_p \) \((k_d=0)\) so that the system behaves like a 1 Hz spring-inertia oscillator.

\[
\omega_n = \sqrt{\frac{k_p k_{hw}}{J}}
\]  

(Equation 2-3)

4. Set the time sample interval and time duration for sampling using the following procedure: Set-up to collect Encoder #1 and Commanded Position information via the Set-up Data Acquisition box in the Data menu. Set up a closed-loop step of 0 (zero) counts, dwell time = 5000 ms, and 1 (one) rep (Trajectory in the Command menu).

5. Now, set up the controller. Enter the Control Algorithm box under Set-up and set \( T_s = 0.00442 \text{ s} \) and select Continuous Time Control. Select PI + Velocity Feedback (this is the return path derivative form) and Set-up Algorithm. Enter the \( k_p \) value determined above for 1 Hz oscillation \((k_d \& k_i = 0, \text{ do not input values greater than } k_p = 0.16)\) and select OK. Select Implement Algorithm, then OK.

6. Run the experiment: Prepare to manually rotate the lower disk roughly 60 deg. Select Execute under Command. Then select Run, rotate about 60 deg. and release disk. Do not hold the rotated disk position for
longer than 1-2 seconds as this may cause the motor drive thermal protection to open the control loop.

7.a Export the data to MATLAB. Plot the encoder 1 data

Calculate the frequency by using the Data Cursor Tool in the MATLAB Figure. Be sure to show the calculations and units.

For system stability, do not input values greater than \( k_p = 0.16 \).

7.b Repeat the test with a new proportional gain. Double the value of \( k_p \).

Repeat steps 5, 6 and 7.a with the new value of \( k_p \). Again, Export the data to MATLAB. Plot the encoder 1 data

8. Determine the value of the derivative gain, \( k_d \), to achieve \( k_d k_h w = 0.1 \text{N-m/(rad/s)} \). \(^3\) Repeat Step 5, except input the above value for \( k_d \) and set \( k_p \) & \( k_i \) = 0. (Do not input values greater than \( k_d = 0.1 \)).

9. After checking the system for stability by displacing it with a ruler, manually move the disk back and forth to feel the effect of viscous damping provided by \( k_d \). Do not excessively coerce the disk as this will again cause the motor drive thermal protection to open the control loop.

10. Repeat Steps 8 & 9 for a value of \( k_d \) five times as large (Again, \( k_d \leq 0.1 \)). Can you feel the increased damping?

The final report is expected to include:

Two (2) MATLAB Plots, with two (2) Data Cursor Points on each plot, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of \( k_p \)
- Plot of \( 2 k_p \)

Calculations showing how you found the following values, along with units for EVERY quantity found.
- Inertia of the System \( J \) for this experimental setup
- Calculation for \( k_p \)
- Experimental frequency calculations using the \( k_p \) value calculated.
- Calculations for \( k_d \)

\(^3\)For the discrete implementation you must divide the resulting value by \( T_s \) for the controller input value

Here, since the PD controller is improper, the backwards difference transformation: \( s = (1-z^{-1})/T_s \) is used.
Experiment 2.b PD Control Design & Step Response

This part is a continuation of Experiment 2.a with the same experimental setup. In this section the $k_p$ and $k_d$ values will be used to calculate 3 separate scenarios; under damped, over damped and critically damped.

11. Perform the calculations used to find the gain values for the following tests. From Equations 2-2 through 2-4 design controllers (i.e. find $k_p$ & $k_d$) for a system natural frequency $\omega_n = 2$ Hz, and three damping cases: 1) $\zeta = 0.2$ (under-damped), 2) $\zeta = 1.0$ (critically damped), 3) $\zeta = 2.0$ (over-damped).

\[
c(s) = \frac{k_p k_{hw}/J}{s^2 + (k_{hw}/J)(k_ds + k_p)} \quad \text{(Equation 2-2)}
\]

\[
\omega_n \Delta = \sqrt{\frac{k_p k_{hw}}{J}} \quad \text{(Equation 2-3)}
\]

\[
\zeta = \frac{k_d k_{hw}}{2J \omega_n} = \frac{k_d k_{hw}}{2\sqrt{Jk_p k_{hw}}} \quad \text{(Equation 2-4)}
\]

12. Set up the controller and the input. Implement the underdamped controller (via PI + Velocity Feedback) and set up a trajectory for a 2500 count closed-loop Step with 2000 ms dwell time and 1 rep.

13. Run the test. Execute this trajectory and plot the commanded position and encoder position in MATLAB (Plot them both on the same vertical axis so that there is no graphical bias.)

14. Repeat Steps 12 & 13 for the critically damped and over-damped cases. Save your plots for later comparison.

The final report is expected to include:

Four (4) MATLAB Plots, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Under-damped step response
- Critically damped step response
- Over-damped step response
- All three cases plotted on one graph with the commanded position (Hint: use the Legend command and different colors and lines to plot each response clearly)

Calculations showing how you found the following values, along with units for EVERY quantity found.
- $K_p$
- Under-damped $k_d$
- Critically damped $k_d$
- Over-damped $k_d$

*If the calculated $k_p$ does not give you the right $\omega_n$, experimentally determine a $k_p$ which does.

**Experiment 2.c Adding Integral Action**

In this part of the lab, a full PID controller will be implemented. By adding integral action to the controller, the settling time and overshoot will be impacted.

18a. Perform the calculations and program the controller. Now compute $k_i$ such that $k_i k_{h_w} = 3 \text{ N-m/(rad-sec)}$. Implement a controller with this value of $k_i$ and the critically damped $k_p$ & $k_d$ parameters from Step 11. (Do not input $k_i > 0.4$).

18b. Execute a 2500 count closed-loop step of 5000 ms duration (1 rep).

18c. Plot the encoder #1 response and commanded position in MATLAB

19. Experimentally determine a value of $k_i$ that visibly gives you a better response judged by its transient response and steady-state error in comparison to the previous run. This $k_i$ may be smaller than the one used in Step 18.

The final report is expected to include:

Three (3) MATLAB Plots, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Calculated $k_i$
- Experimentally “better” $k_i$
- A plot containing values of $k_i = 0$ (from Experiment 2.b), the calculated $k_i$ and the “better” $k_i$, along with the commanded position (Hint: use the Legend command and different colors and lines to plot each response clearly)

Calculations showing how you found the following values, along with units for EVERY quantity found.
- $k_i$

Observe these results and briefly describe the effects of adding integral action to the controller. Is this what you expect?

**Experiment 2.d Frequency Response**

In this portion of the lab, the input will now be a series of increasing frequency sine waves, used to determine the frequency performance of the controller.

15a. Implement the under damped controller from Step 11.
15b. Set up a trajectory for a 400 count closed-loop Sine Sweep from 0.1 Hz to 20 Hz of 60 seconds duration with Logarithmic Sweep checked. (You may wish to specify Encoder #1 data only via Set-up Data Acquisition. This will reduce the acquired data size.)

16a. Execute the trajectory

16b. Plot the Encoder 1 frequency response using Linear Time and Linear amplitude for the horizontal and vertical axes in MATLAB. The data will reflect the system motion seen as the sine sweep was performed.

16c. Now plot the same data using Logarithmic Frequency and Db amplitude. By considering the amplitude (the upper most portion of the data curve) you will see the data in the format commonly found in the literature for Bode magnitude plots. Can you easily identify the resonance frequency and the high frequency (>5 Hz) and low frequency (< 0.8 Hz) gain slopes? (i.e. in Db/decade).

17. Repeat Step 16 for the critically damped and over damped cases

The final report is expected to include:

Six (6) MATLAB Plots, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation.
- Plot of under-damped linear time and amplitude
- Plot of critically damped linear time and amplitude
- Plot of over-damped linear time and amplitude
- Plot of under-damped frequency response (logarithmic axes) * mark the resonance peak with the Data Cursor
- Plot of critically damped frequency response (logarithmic axes)
- Plot of over-damped frequency response (logarithmic axes)

Calculations showing how you found the following values, along with units for EVERY quantity found.
- The percent error between the natural frequency from Experiment 2.b and the resonance frequency
- Note the gain slopes

Extra Credit!

The following is optional. It was in old versions of the lab, and students are welcome to try it to see if this improves understanding of the system behavior. If understanding is notably demonstrated in the lab writeup, then a minor level of extra credit, no more than 10%, will be given.
Part (2) Two degree-of-freedom system (follow instruction below)

Mount all three disks to the device. Add a pair of weights to both lower and middle disks at 9.0 cm radius. Unfasten the middle disk and fasten the top disk to the frame so that it can’t move. The system now has two degrees of freedom at the two moving disks, and the shaft segments connecting the three disks act as torsional springs.

Set up a sine sweep action similar to what you did in Step 15 except this time setting the trajectory to sweep frequencies from 0.5 Hz to 10Hz. You should acquire position data for both disk 1 and disk 2. Execute the sine sweep and observe the responses from both the disks. They are not in phase but you should see the responses increase, decrease, increase again and decrease again as the sweep frequency increases, suggesting two natural frequencies. Plot Encoder 1 position in Log frequency and Db amplitude, from which to estimate the captured two natural frequencies (the peaks). Repeat it for Encoder 2 position to see if the captured peaks are the same frequencies.

The final report is expected to include:

Two (2) MATLAB Plots, along with titles, labels and legends if necessary that clearly show which plot corresponds to which situation. Use the Data Cursor to indicate the two natural frequency peaks in each plot.

- Plot of Encoder 1
- Plot of Encoder 2

Calculations showing how you found the following values, along with units for EVERY quantity found.

- The two natural frequencies for Disk 1
- The two natural frequencies for Disk 2