### Topology optimisation for energy management in underwater sensor networks

Devesh K. Jha, Thomas A. Wettergren, Asok Ray, and Kushal Mukherjee

*Department of Mechanical & Nuclear Engineering, Pennsylvania State University, University Park, PA 16802-1412, USA; Naval Undersea Warfare Center, Newport, RI 02841-1708, USA and Department of Mechanical & Nuclear Engineering, Pennsylvania State University, University Park, PA 16802-1412, USA; United Technology Research Center, Cork, Ireland*

(Received 28 October 2014; accepted 5 February 2015)

In general, battery-powered sensors in a sensor network are operable as long as they can communicate sensed data to a processing node. In this context, a sensor network has two competing objectives: (1) maximisation of the network performance with respect to the probability of successful search for a specified upper bound on the probability of false alarms, and (2) maximisation of the network’s operable life. As both sensing and communication of data consume battery energy at the sensing nodes of the sensor network, judicious use of sensing power and communication power is needed to improve the lifetime of the sensor network. This paper presents an adaptive energy management policy that will optimally allocate the available energy between sensing and communication at each sensing node to maximise the network performance subject to specified constraints. Under the assumptions of fixed total energy allocation for a sensor network operating for a specified time period, the problem is reduced to synthesis of an optimal network topology that maximises the probability of successful search (of a target) over a surveillance region. In a two-stage optimisation, a genetic algorithm-based meta-heuristic search is first used to efficiently explore the global design space, and then a local pattern search algorithm is used for convergence to an optimal solution. The results of performance optimisation are generated on a simulation test bed to validate the proposed concept. Adaptation to energy variations across the network is shown to be manifested as a change in the optimal topology: specified minimum life of the sensor network

Keywords: underwater sensor network; energy management; Pareto optimisation; adaptation

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(r_{\text{em}}, f)$</td>
<td>acoustic attenuation</td>
</tr>
<tr>
<td>$a(f)$</td>
<td>absorption coefficient related to acoustic emission</td>
</tr>
<tr>
<td>$E_i$</td>
<td>energy available to sensing node $i$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>out-degree of node $i$</td>
</tr>
<tr>
<td>$E$</td>
<td>$[E_1, E_2, \ldots, E_M]^T$</td>
</tr>
<tr>
<td>$E_k$</td>
<td>set of edges in a directed graph</td>
</tr>
<tr>
<td>$f$</td>
<td>signal frequency</td>
</tr>
<tr>
<td>$G$</td>
<td>directed graph</td>
</tr>
<tr>
<td>$M$</td>
<td>number of sensing nodes</td>
</tr>
<tr>
<td>$N(f)$</td>
<td>noise power spectrum density</td>
</tr>
<tr>
<td>$N^i$</td>
<td>expected number of transmission trials at node $i$</td>
</tr>
<tr>
<td>$n^i$</td>
<td>average number of transmitted packets from node $i$</td>
</tr>
<tr>
<td>$r_c$</td>
<td>radius of communication vector $[r_c^1, r_c^2, \ldots, r_c^M]^T$</td>
</tr>
<tr>
<td>$r_{\text{d}}$</td>
<td>radius of detection for sensor $i$</td>
</tr>
<tr>
<td>$r_{\text{d}}^i$</td>
<td>radius of detection vector $[r_{\text{d}}^1, r_{\text{d}}^2, \ldots, r_{\text{d}}^M]^T$</td>
</tr>
<tr>
<td>$S$</td>
<td>surveillance region</td>
</tr>
<tr>
<td>$T$</td>
<td>life of the sensor network</td>
</tr>
<tr>
<td>$T^{\text{goal}}$</td>
<td>specified minimum life of the sensor network</td>
</tr>
<tr>
<td>$V$</td>
<td>set of vertices in a directed graph</td>
</tr>
<tr>
<td>$W_i^j$</td>
<td>nominal power draw for sensor $i$</td>
</tr>
<tr>
<td>$W_{i,c}^j$</td>
<td>average communication power for sensor $i$</td>
</tr>
<tr>
<td>$W_{i,d}^j$</td>
<td>detection power for sensor $i$</td>
</tr>
<tr>
<td>$W_{\text{TR}}^j$</td>
<td>total power available at sensor $i$</td>
</tr>
<tr>
<td>$W_{\text{TR}}$</td>
<td>power for transmission of packets at node $i$</td>
</tr>
<tr>
<td>$W_{\text{TR}}^1, W_{\text{TR}}^2, \ldots, W_{\text{TR}}^M$</td>
<td></td>
</tr>
<tr>
<td>$W_N^1, W_N^2, \ldots, W_N^M$</td>
<td></td>
</tr>
<tr>
<td>$W_{\text{TR}}^1, W_{\text{TR}}^2, \ldots, W_{\text{TR}}^M$</td>
<td></td>
</tr>
<tr>
<td>$W_{d}^1, W_{d}^2, \ldots, W_{d}^M$</td>
<td></td>
</tr>
<tr>
<td>$W_c^1, W_c^2, \ldots, W_c^M$</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>position of the cell $k$ in the surveillance region</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>probability of successful packet transmission</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>receiver noise bandwidth</td>
</tr>
<tr>
<td>$\pi_{\text{det}}$</td>
<td>probability of detection</td>
</tr>
<tr>
<td>$\pi_{\text{fa}}$</td>
<td>probability of false alarm</td>
</tr>
</tbody>
</table>

*Corresponding author. Email: axr2@psu.edu

© 2015 Taylor & Francis
2. D.K. Jha et al.

\( \pi_{\text{fa}}^{\text{max}} \): maximum allowable probability of false alarm

\( \pi_{\text{ss}} \): probability of successful search of a target

\( \pi_{\text{goal}} \): specified probability of successful search

**Acronym**

CFAR: constant false alarm rate

FAR: false alarm rate

GA: genetic algorithm

PS: pattern search

SNR: signal-to-noise ratio

1. Introduction

Sensor networks for underwater persistent surveillance are equipped with a small number of affordable sensors that are expected to yield a high probability of correct detection as long as possible, while not exceeding a specified rate of false alarms; such sensor networks are designed to be cost effective, reliable, and long-lasting. For example, in littoral undersea networks, the operational period may extend up to few months without the need for any external intervention. These sensor networks have peculiar characteristics: they have large propagation delay, high error rate, low bandwidth, and limited energy (Ovalidis, Savage, & Kanakaris, 2010). Improving energy efficiency in such networks is important since replacement of batteries is expensive. For fixed (i.e., immobile) sensor networks deployed for persistent surveillance, there are primarily two time-dependent sources of power consumption:

- sensing/detection of targets,
- communicating the information of detection.

Efficient management of the available energy across a battery-powered sensor network is necessary to achieve the competing objectives of: (1) improvement of the network performance over a fixed time-horizon, and (2) enhancement of the network lifetime while satisfying hard constraints, such as false alarms and changes in the resource availability. Consequently, there could be an inconsistency in design and operation (Jaleel et al., 2013) as the dynamics of sensing and communication in the network are decoupled. For example, the network communication topology is not considered while deciding the sensing capabilities (or variables) for the network. In this paper, a network design procedure is presented with the goal of maintaining a minimum desired performance level when there are hard constraints of resource availability for the expected life of the network.

In a majority of the work reported in open literature, efforts have been expended to develop energy-efficient data-packet routing protocols and to build efficient detection models; a clear trade-off between performance and longevity of the network has been apparently ignored. The work, proposed in this paper, differs from those reported in the existing literature in the sense that tools of network topology optimisation have been employed to judiciously allocate the available battery power between the operations of sensing and node-to-node communications to maximise the performance of the sensing nodes as a network. While a majority of the existing work is concentrated on finding the best communication and detection rules (which are not necessarily energy-efficient), the goal here is to improve the network performance for given routing protocols with fixed criteria for operational activities (e.g., target detection). All such design procedures miss a critical component, i.e., making a trade-off between performance and network lifetime. The current paper addresses the problem of striking a trade-off between performance and sustainable lifetime of networks by making optimal energy allocation between sensing and communication at the node level. To this end, it is imperative to develop optimisation tools for maximising the network performance under a variety of operating conditions.

The objective of this paper is to construct a reliable and long lasting sensor network that will adaptively make trade-off decisions between the sensing power and the communication power; such decisions should be independent...
of each sensing node. From these perspectives, optimised node-level energy management algorithms have been developed to extend the average battery life at the sensing nodes by maintaining an acceptable level of the sensor network’s performance. For example, to increase the detection capabilities, a larger number of sensing channels on a node may need to be activated, which could draw more power; similarly, communications with distant nodes, while reducing false alarms via decision fusion, would also draw more power. As the network operations evolve, the intermittency of target events may cause some of the sensors to lose much of their energy reserves, and the remaining sensors may be required to conserve available energy to maintain the same level of network performance. It is noted that this paper is not proposing a new sensor scheduling algorithm, rather it is optimising the connectivity of the sensor nodes so that the network performance is maximised. A two-stage optimisation procedure is proposed to solve the resulting problem.

(1) The optimisation procedure is initiated with a genetic algorithm (GA) that efficiently explores the global design space to yield a nearly optimal solution.

(2) Solutions of the GA are fed into a pattern search (PS) algorithm that produces an optimal solution by gradient-free local search.

The above two-stage optimisation procedure yields the node-level power for sensing and communication, which will maintain the required performance over the desired life of the sensor network.

The paper is structured as follows. Starting with a description of the sensor power dynamics, the paper formulates the fixed-time-horizon power trade-off problem with the probability \( \pi_{ss} \) (Wettergren, 2008) of successful search as the cost function, where \( \pi_{ss} \) is shown to be a function of the network topology under specified constraints. This function is then optimized using a GA search and is followed by PS to yield the optimal network topology. All the sensing nodes communicate with the sink node that acts as the computation node of the network; the sink node also keeps an account of the energy levels across the network. Even though the sensors start with homogeneous batteries with the same amount of energy, variations in the energy level occur because different sensors may use different amounts of energy for detection and communication purposes. Based on the current energy levels, the sink node recalculates the optimal topology which is then broadcasted across the network to adapt to energy variations over the network’s lifetime. The Pareto-optimal surface (Miettinen, 1998) is obtained as a trade-off between network lifetime and the performance of the network (measured in terms of probability of successful search. Even though this paper uses models relevant to underwater communication and detection, the overall framework of energy management presented could be very well suited for other types of sensor network.

2. Network communication and target detection

This section presents simple models of network communication and target detection for underwater surveillance. Figure 1 shows the schematic of a typical sensor network deployed for surveillance, where the sensing nodes communicate with the sink node to provide the information on both successful and false detections. The numerical results presented in this paper are based on the models detailed in the current section. However, the applicability of the proposed framework is not limited to any particular model.

2.1 Network communication model

A network communication link is said to be functional if the signal-to-noise ratio (SNR) is above a nominal specified value. For underwater applications, acoustic modems are known to be the most efficient communications mechanism (Urick, 1994), where messages are transmitted with a communication radius \( r_c \) for sensor \( i \) at a frequency \( f \) and the resulting narrow-band SNR is approximated as

\[
\text{SNR}(r_c, f) = \frac{W_{TR}}{A[r_c, f] \cdot N[f] \cdot \Delta f},
\]

where \( W_{TR} \) is the transmitted power for communication (for sensor \( i \)), \( A[r_c, f] \) is the acoustic attenuation, \( N[f] \) is the noise power spectrum density, and \( \Delta f \) is the receiver noise bandwidth that is normalized to unity. The superscript \( i \) is dropped for simplicity in the following equations, because

![Figure 1. Detection, false alarm, and communication in a typical sensor network. Arrows denote the directed links for communication.](image-url)
acoustic attenuation of all sensor signals is given as

$$A[r_c, f] = c_0 \cdot (r_c)^t \cdot a[f]^{11},$$  \hfill (2)

where $c_0$ is a constant for unit normalisation, $\eta$ is the spreading factor, and $a[f]$ is the absorption coefficient. While detection problems typically make the distinction between spherical ($\eta = 2$) and cylindrical ($\eta = 1$) spreading, the acoustic communications community resorts to a value of $\eta = 1.5$, which is referred to as the practical spreading coefficient. The last term in Equation (2) is the absorption coefficient that is given by Thorp’s formula (Urick, 1994) that is valid in the range of hundreds of Hz to 50 kHz:

$$\eta = \text{hyp}.$$  \hfill (3)

The noise power spectrum density $N[f]$ in Equation (1) results from a number of sources, such as turbulence, shipping, waves, and thermal effects. A reasonable approximation is to take the net effects of noise (Urick, 1994) as follows:

$$10 \log_{10} N[f] \approx 50 - 18 \log_{10} f, \hfill (4)$$

where the frequency $f$ is in the units of Hz. This approximation is valid for the frequency range of tens of Hz up to tens of kHz in a wide variety of ocean regions (Urick, 1994).

The power used for packet transmission determines the radius of communication for individual nodes. It follows from Equations (1) to (4) that $W_{\text{TR}}$ is a monotonically increasing function of communication radius $r_c$, and the nominal SNR threshold required to establish a link. The vector representing the communication radii of all sensing nodes $r_c$ defines the topology of the sensor network. For fixed sensor locations, each sensing node has neighbours at fixed distances, i.e., the power required for establishing communication links with the neighbours is fixed. Hence, for a finite number of sensors, there is a finite set of power requirement parameters that are used by a sensor to establish communication links with other nodes in the network.

### 2.2 Target detection model

A constant false alarm rate (CFAR) (Poor, 1994) model has been used for target detection, where

1. each sensing node in the surveillance region has a CFAR, and
2. each sensor has a sensing radius $r_d$, within which the sensor detects a target with probability $\pi_{\text{det}}$.

The probability of successful search, $\pi_{\text{ss}}$, over the surveillance region has been listed in the flowchart shown in Figure 2 and are briefly explained next.

As the sensing radius is calculated under the assumption of Gaussian noise by using the following relationship:

$$r_d^i = k_d (W_d^i)^{0.25}, \hfill (5)$$

where $k_d$ is the proportionality constant that is assumed to be the same for all sensors. For a specified value of CFAR, the probability of successful search $\pi_{\text{ss}}$ can be enhanced by increasing the power for detection (Urick, 1994). However, without using additional power for detection, an increase in $\pi_{\text{ss}}$ may also increase the false alarm rate (FAR). Figure 3 elucidates a typical surveillance region with the corresponding sensor footprint calculated according to the detection model in the presence of additive Gaussian noise. The probability of successful search is calculated by taking the mean of the probability of detection over the surveillance region. For enhancement of computation, the surveillance region is discretised into a finite number of grid cells, where each cell is labelled by its centre at the location $z[x, y]$. The probability of target detection at the cell, labelled by $z$, is then calculated as

$$\pi_{\text{det}}[z] = 1 - (1 - \pi_{\text{det}})^{k_z}, \hfill (6)$$

where $k_z$ is the number of sensors that can detect a possible target in the cell $z$. A binary function, $\chi'(z)$ is defined as

$$\chi'(z) \triangleq \begin{cases} 1 & \text{if node } i \text{ can detect a target at } z \\ 0 & \text{otherwise,} \end{cases} \hfill (7)$$
where the sensing node \(i \in 1, 2, \ldots, M\). Then, \(k_i\) is calculated as

\[
k_i = \sum_{i=1}^{M} x'(z).
\] (8)

The probability of successful search \(\pi_{ss}\) is obtained by taking the average over the entire surveillance region \(S\) as

\[
\pi_{ss} = \frac{\sum_{z \in S} \pi_{ss}(z)}{\text{Area}[S]},
\] (9)

where \(\text{Area}[S]\) denotes the area of the surveillance region \(S\).

Remaining details of the passive sensing used for detection are available in VanTrees (2001) and Picinbono (1995). It is noted that, at any \(t \in [0, T_{\text{cal}}]\), the performance measure \(\pi_{ss}\) is a function of the vector \(W_{T\_R}\) as a result of the Equations (5)–(9).

### 2.3 Network connectivity model

The network is modeled as a directed graph \(G(V, E)\), where \(V = V_s \cup V_{\text{sink}}\) with \(V_s\) being the set of sensing nodes and \(V_{\text{sink}}\) the (singleton) set of the sink node. If there are \(M\) nodes in \(V_s\), the cardinality of the node set \(V\) is \(M_V = (M + 1)\), because there is only one sink node. The edge set \(E\) represents a collection of \(M_E\) edges that provide interconnections or links in the network. The sensor field under consideration is constrained with fixed sensor locations, so that every sensor has the information about all other sensors and the sink node.

The directed graph \(G\) is required to have the property that there exists a path from every sensing node to the sink so that the data packets, sent by each sensor, are able to reach the sink node. It is noted that this notion is different from that of connectivity of directed graphs, where there must exist a path between every pair of nodes. Hence, the concepts of graph Laplacian and its spectrum may not be used to ensure the connectivity. For a given topology of the network, the routes from every sensing to the sink are found by using Dijkstra’s algorithm (LaValle, 2006). If there exists a route from each sensing node to the sink, then the network is said to be connected. That is, every node is connected to the sink node via the shortest possible route for a given network topology; it is noted that the routes, thus found, will depend on the network topology. The metric used in measuring the distance of such routes is the energy requirement for establishment of communication links. Hence, the route requiring the minimum amount of energy for data packet transmission to the sink is the shortest route. Presence of a route, either single-hop or multi-hop, from each of the sensing nodes to the sink ensures the network connectivity.

An important characteristic of a connected network is its tendency to use single-hop links against multi-hop links. The preference of multi-hop or single-hop communication would depend on the relative position of the sensors and the processing node (e.g., the sink node). The number of single-hop links to the sink might increase the energy used in communication by individual nodes. However, it should make the network robust to link failures as there are more nodes which can act as bottleneck nodes, other sensors can transmit through nodes directly connected to the sink. This would, in general, also depend on the relative placement of the sensors and variations in energy availability across the network. It is noted that, at any time \(t \in [0, T_{\text{cal}}]\), the connectivity of \(G(t)\) depends on the vector \(W_{T\_R}\) and thus \(W_{c}\).

### 2.4 Network protocol model

For perfect communication, all packets are received at the sink in the first attempt and there are no retries or loss of information. Associated with the sensing nodes, the packet drop probability is \(\alpha = 0\). Data packets are sent to the sink by finding the shortest route through the network depending on the topology. Every time a sensor detects the possibility of a target being present, it transmits data over a short time interval (e.g., on the order of seconds); otherwise, the sensors are directed to communicate with the sink at a predefined frequency so that the sink can maintain a count of live sensors and their battery levels.

For imperfect communication, there is a packet drop probability associated with packets sent by the sensing nodes, i.e., the packet drop probability \(\alpha > 0\). The expected number of trials needed to send the data packets to the sink is calculated by using the packet drop probability. Hence, the situation of imperfect communication is modeled by increasing the packet transmission time by an appropriate factor depending on the packet drop probability. Although this paper makes the assumption of perfect communication with no limit on the amount of data at a node, the situation...
of imperfect communication can be analysed by increasing the time for packet transmission by an appropriate factor that is dependent on the packet drop probability. Such a model is presented below.

Given the packet drop probability \( \alpha \in (0, 1) \), the expected value of the number of trials at node \( i \) is evaluated as

\[
N^i(\alpha) = \left[ 1 \cdot (1 - \alpha) + 2 \cdot \alpha \cdot (1 - \alpha) + \cdots + N \cdot \alpha^{N-1} \cdot (1 - \alpha) + \cdots \right]
\]

\[
= \frac{1}{1 - \alpha}. \tag{10}
\]

It follows from Equation (10) that, for perfect communication (i.e., \( \alpha = 0 \)), the parameter \( N^i(0) = 1 \).

The expected number, \( n^i \), of data packets transmitted per unit time by node \( i \) is obtained as

\[
n^i \propto \text{FAR} \cdot d^i \cdot N^i(\alpha), \tag{11}
\]

where the network design parameters, FAR, and out-degree \( d^i \) of the node \( i \), are available for any topology of the sensor network.

The expected energy requirements for data transmission is obtained from \( N^i(\alpha) \). The network protocol model together with the communication model, described in Section 2.1, determines the transmission power vector \( \mathbf{W}_{TR} \) and thus the average communication power vector \( \mathbf{W}_c \). For imperfect communication (i.e., \( \alpha > 0 \)), the average power for communication for a node \( i \) would increase by a factor of \( N^i(\alpha) \), i.e.,

\[
W^i_c(\alpha) = W^i_c(0) \frac{1}{1 - \alpha}, \text{ where } W^i_c(\alpha) \propto n^i.
\]

3. Problem formulation

From the above perspectives, the problem of energy management in underwater sensor networks is treated as adaptive optimal sharing of the available battery power between sensing and communication. This multi-objective cost functional leads to non-dominant optimisation. Such a problem is formulated as a Pareto-optimal trade-off (Miettinen, 1998) between network performance and lifetime in the underwater sensor network. In this setting, each passive sensing node needs to adaptively select the appropriate sensing radius based on the current level of its battery life and the projected remaining life of the sensor network that requires communications connectivity to remain functional. From this perspective, sensor networks have two competing objectives.

- Maximisation of network performance with respect to the probability of successful search with a specified FAR for a given coverage area.
- Maximisation of the network’s operational life.

The goal here is to synthesise an adaptive energy management policy for a given sensor network that will optimally allocate available power between the operations of sensing and communication at each node to maximise the network performance under the following assumptions.

1. The \( 2 - D \) sensor network consists of \( M \) sensing nodes (without the sink node) that are located at fixed positions \( i = 1, 2, \ldots, M \) to perform cooperative surveillance. (It is noted that the locations are randomly generated but they are fixed.) Each sensing node is allocated a fixed amount of energy (i.e., the \( i \)th sensing node has energy \( E_i \)) and this allocated energy is to be shared for sensing, communication and associated fixed nominal overhead expenses.

2. Sensing nodes have power requirements for basic operational overhead (e.g., for central processing) as well as for receiver capabilities, with a constant nominal power draw \( W^i_{N} \) for each sensing node \( i \).

3. The average power for communication is obtained by time averaging of expected communication power for the remainder of the network life.

4. The communication of data packets from all sensors to sink is perfect (i.e., no lost packets). As such, no acknowledgement of the data packets received are sent by the sink back to the sensing nodes.

5. The packet size of target detection messages, transmitted by a sensing node, is small (e.g., in the order of a few seconds) compared to the network operational time.

6. Target detection algorithms are constructed for specified CFAR.

7. The sensing radius is calculated in the presence of Gaussian noise by using the relationship, \( r^i \propto (W^i_d)^{0.25} \).

8. The sensing radius at a node is calculated from the sensing power of a signal in the presence of Gaussian noise (Poor, 1994) by using the square-law detection model (Picinbono, 1995).

9. The network remains connected, i.e., data packets from all sensing nodes reach the processing node (i.e., the sink node) in a finite number of hops.

Remark 3.1: The network is designed based on the steady-state behaviour when it is allowed to have a specified number of false alarms. Hence, all calculations for energy allocation are made using the allowable number of false alarms. During operation, however, the network may come across multiple targets.

The life \( T \) of a sensor network is defined as the time over which the network maintains an effective probability of successful search \( \pi_{ss} \) of at least \( \pi_{ss}^{\text{goal}} \), i.e.,

\[
T \triangleq \inf \left\{ t : \pi_{ss}[t] > \pi_{ss}^{\text{goal}} \right\}. \tag{12}
\]
In the above setting, a dual problem is to design the sensor network to remain functional for a specified time \( T^{\text{goal}} \) and maximise the minimal value of the probability of successful search \( \pi_{ss} \) for the network over the specified life. Sufficient information has to reach a sink node for verification of target detection, which uses multi-sensor decision fusion rules (e.g., \( k \)-detections or track-before-detect) (Mukherjee, Ray, Wettergren, Gupta, & Phoha, 2011; Wettergren, 2008) to decide the presence of a target or a false alarm. However, the sink node has no sensing capabilities of its own.

As discussed earlier, the sensors start with batteries having fixed energy and the detection rules are based on CFAR. Given these design criteria and the expected network lifetime \( T^{\text{goal}} \), each sensing node calculates its total available power. To meet its expected performance, each sensing node \( i \) allocates its total available power \( W_i \) as: (1) sensing power \( W_s^i \) and (2) the average power for communicating detection information \( W_c^i \). Sensors communicate only when they detect the presence of a target (including false alarms), which is a sparse and an intermittent event. Hence, the power used for transmission \( W_{TR} \) of data packets is different from the average power used for communication \( W_c^i \) that is calculated by time averaging of the power \( W_{TR}^i \) for transmission of data packets.

For a sensing node \( i \), the total power balance is then expressed as

\[
W^i(t) = W_s^i(t) + W_d^i(t) + W_c^i(t) \quad \forall t \in \left(0, T^{\text{goal}}\right],
\]

where \( W^i_N(t) \) is considered to be a constant in this paper. As shown in Section 2, the variables \( W_s^i \) and \( W_c^i \) could be expressed as functions of \( r_s^i \) and \( r_c^i \), respectively, when other factors, such as SNR and \( a(t) \), are kept constant. The vectors \( r_s^i \) and \( r_c^i \) (and thus \( W_d^i \) and \( W_c^i \)) together define the network performance by determining \( \pi_{ss} \) and the connectivity of the sensors with the sink. Under the assumption of constant \( W^i_N \), the optimisation problem is to maximise the network performance \( \pi_{ss} \) under the constraints of fixed network life and energy availability by identifying the optimal vectors \( W_s^i \) and \( W_c^i \). More formally, the objective is to find \( W_s^i \) and \( W_c^i \) (and thus the corresponding \( r_s^i \) and \( r_c^i \)) such that

\[
\arg\max (W_s^i, W_c^i) = (W_d^i, W_c^i) \pi_{ss}(W_d^i, W_c^i)
\]

under the constraints imposed by the following conditions.

- Power balance in Equation (13).
- \( \pi_{ss}[t] > \pi_{ss}^{\text{goal}} \forall t \in \left(0, T^{\text{goal}}\right] \)
- \( \mathcal{G}(t) \) is connected \( \forall t \in \left(0, T^{\text{goal}}\right] \)
- \( \pi_{fs}(t) \leq \pi_{fs}^{\text{max}} \forall t \in \left(0, T^{\text{goal}}\right] \)

The above static optimisation process finds out the best network topology (i.e., \( r_c^i \)) that maintains the network connectivity and maximises \( \pi_{ss} \). It is noted that every feasible network topology, represented by the vector \( r_c^i \), uniquely identifies a detection radius vector \( r_s^i \) (see Equation (13)). However, the converse is not true (see Section 4 for details).

The network needs to be self-adaptive to the changes in the remaining battery life and target behaviour to maintain a satisfactory performance level. The associated dynamic (or adaptive) optimisation is executed every time a significant event is detected. The network is supposed to operate over a sufficiently large period of time; real-time adaptation for the network is not required. The network reconfigures over a slow time scale as compared to the dynamics of the likely events (targets) in the network. In this setting, behavioural changes in network topology could be measured in terms of the ratio of multiple hop and single-hop paths present in the network under the optimal and average operating conditions.

To obtain the optimal performance conditions over different operational conditions of the network, a Pareto surface (Miettinen, 1998) is generated as a trade-off between network lifetime and network performance for successful detection of targets, which helps the choice of an operating point. Depending on the situation, the network may select another search strategy by reducing \( T^{\text{goal}} \) or vice versa.

The problem of sensor network design, addressed in the paper, is stated as follows.

Design of an energy-constrained sensor network that will optimally allocate available power at each node for sensing and communication to maximize the network performance under constraints of fixed life. It is noted that the network performance is measured in terms of \( \pi_{ss} \) and its connectivity which are global variables. As a result, the decision variables (i.e., choice of sensing and communication power) of sensors are tightly coupled and need to be decided simultaneously.

### 4. Proposed approach

The major steps involved in solution of the optimisation problem are shown as a flowchart in Figure 4. As discussed in the last section, the optimisation process optimally allocates available power at each node for sensing and communication to maximise the network performance that is measured in terms of probability, \( \pi_{ss} \), of successful detection of a target, which is obtained from Equations (5) to (9) and represents the global behaviour of the network. Under the current detection model, presented in Section 2.2, \( \pi_{ss} \) is obtained as a function of \( W_{TR} \). On the other hand, the network connectivity (or topology) depends on the power vector \( W_c \) and hence on \( r_c \). Thus, every sensor must decide which neighbouring node it should communicate with so that the network remains connected and its performance is maximised. It is noted that these variables need to be decided simultaneously for all the sensors, because the objective function i.e., \( \pi_{ss} \) is a global property of the network.

Figure 5 shows the profiles of variations of communication radius with sensing radius at different nominal thresh-
olds of SNR to set-up inter-node communication links, under the constraints of a CFAR and fixed energy. These relationships are obtained by using Equations (1)–(3) in Section 2.1 at a frequency of approximately 1 kHz. At a fixed alarm rate, the sensing radius is evaluated from the energy left for detection (for a fixed time-horizon) after meeting the communication requirements (over the same time-horizon). It is seen in Figure 5 that the sensing radius is less sensitive to changes in the communication radius at relatively smaller SNR as the family of curves becomes more flat at larger SNR. In essence, the communication radius has less significant effects on the sensitivity of the sensing radius at larger SNR. The implication is that, with a small SNR, a good part of the available power would be used to ensure successful communication and it would consequently result in a narrow range of detection radius. Furthermore, since communication power $\tilde{W}_c$ varies superlinearly with communication radius $r_i^c$ at high SNR, the bulk of the available power would be used for communication even with the nearest neighbours resulting in low detection radius. Optimisation will be meaningful only in scenarios where changes in the communication radius have significant effects on the sensing radius. Otherwise, the network can only have a small range of feasible probability of successful search $\pi_{ss}$ while still being connected and the optimal behaviour would not significantly differ from average behaviour in those cases. Therefore, while designing a network, the battery energy levels at the sensing nodes should be chosen appropriately with due consideration to the communication SNR. In other words, an ill-designed sensor network may suffer from having a narrow region of performance, where a trade-off between sensing and communication may not able to significantly improve the network performance.

Since the sensor locations are assumed to be fixed, every sensor should have neighbours at fixed distances (e.g., as seen in Figure 6). Thus, there exists a discrete finite set of communication radii for each sensor (e.g., if $|V_s \cup V_{sink}| = M + 1$, then $r_i^c \in \{r_{i1}^c, \ldots, r_{iM}^c\}$). Consequently, new links in the network are established only at discrete values $\tilde{W}_c$ and there are discrete values for feasible $\tilde{W}_c$, which can be used for communication. The sensing nodes would encounter only a discrete set of power levels that they may use for communication. Using any
other power level in between these discrete levels for the same task may not have any advantage for communication as no new links could be established. It is reasonable to use discrete power levels of communication in the optimisation process, while the remaining power at each node is available for sensing of targets. Using the current model for communication, the vectors $W_T$ and $W_c$ could be expressed as implicit functions of the communication radius vector $r_c$. The vector $r_c$ is an $M$-dimensional discrete vector (e.g., $r_c = \{r_c^1, \ldots, r_c^M\}$ as there are $M$ sensors in the network) and it can be mapped to the lattice $\mathbb{Z}^M$ in the Euclidean space $\mathbb{R}^M$. Formally, there exists a map defined as

$$f : r_c \mapsto (I_1, I_2, \ldots, I_M),$$

where $I_k, k \in \{1, 2, \ldots, M\}$ is an integer and $(I_1, I_2, \ldots, I_M) \in \mathbb{Z}^M$. It is noted that the map $f$ is bijective, implying that the vector $r_c$ can be uniquely mapped to $M$-tuple of integers. Probability of successful search of targets over the surveillance region, which serves as the performance measure for the network, is calculated in terms of a continuous Euclidean space $\mathbb{R}^M$. Under the relation shown in Equation (13) and the facts that $W_T$ is fixed and that $W_c$ can attain only discrete values, the vector $W_d$ is constrained to belong to a discrete set; consequently, the sensing radius vector $r_d$ belongs to a discrete set. As discussed earlier, the objective function $\pi_{ss}(W_d, W_c)$ is expressed as an implicit function $\pi_{ss}(W_d, W_c) = \pi_{ss}(W_c)$ of the communication power vector $W_c$ when $W_T = W_d + W_c$ (see Equation (13)). In this setting, the objective function $\pi_{ss}$ becomes an implicit function of the communication radius vector $r_c$, because $W_c$ is determined by $r_c$ on a fixed time-horizon.

**Remark 4.1:** In view of the above arguments, the optimisation problem is now reduced to identification of the vector $r_c^*$ that uniquely determines $W_c^*$ and $W_d^*$ on a fixed time-horizon. The optimal $r_c^*$ keeps the network connected and simultaneously maximises the probability $\pi_{ss}$ of successful search over the surveillance region. Since a feasible operating point belongs to an $M$-dimensional space, finding the optimal solution is a combinatorial optimisation problem. An exhaustive search for finding the optimal combination could be NP-complete (Sipser, 2012).

Next a two-stage optimisation process is introduced to solve the discrete optimisation problem.

1. The optimisation process is initiated by a global search of the $M$-dimensional solution space by using GA-based meta-heuristic search (Bishop, 2006). This step is expected to provide a solution to a close vicinity of the global optimal solution.
2. Solutions obtained by GA are fed as initial conditions into gradient-free local search algorithm, known as PS (Bishop, 2006). This step leads to a local refinement of the solution by making small perturbations in the solutions obtained from the GA search.

### 4.1 Genetic algorithm

GA belong to a class of computationally efficient meta-heuristic tools for searching optima in large parameter spaces with a limited structure (Michalewicz & Fogel, 2004). Examples of other directed random search techniques, which could be used to obtain solutions close to the global optima, are the following:

- cultural algorithms that add a macro-evolutionary belief function to the value of each $r_c$;
- particle swarm algorithms in which iterative variations of $r_c$ are imposed by comparison to the best value of $r_c$ obtained to that point; and
- ant colony optimisation that involves adding artificial pheromone levels to good values of $r_c$ to draw further iterations toward that direction.

Since all of the above alternative techniques involve problem-specific tuning, this paper focuses on GA as a method that can be applied generically to the problem at hand. In the setting of GA, a feasible solution $r_c^*$ is an $M$-dimensional vector representing the communication radius for every sensing node, for which the set of all possible neighbours (excluding itself) is created. Such a set is sorted in the ascending order relative to the respective distances from that node. A bijective map $f$ (see Equation (15)) from the sorted set to the integer lattice $\mathbb{Z}^M$ is then created. Thus, $r_c^*$ can be represented as an $M$-tuple of integers, which is represented as a binary string and fed into GA to initiate the optimisation process. Formally,

$$f(r_c) \mapsto (I_1, I_2, \ldots, I_M) \mapsto 000100 \ldots .01.$$

A sufficient number of runs of the GA would lead to a small neighbourhood of the optimal solution. The numerical details of GA are presented in the next section. The solutions obtained by GA are used as initial conditions for PS, which performs a local greedy perturbation and a local improvement of the solution is obtained. The PS optimisation method is described next.

### 4.2 Pattern search optimisation

The idea is to be able to get closer to the optimal solution by doing a finer gradient-free search around the solution obtained from GA using a direct search method. The PS algorithm starts with an initial guess to evaluate the objective function and, after initialisation, the algorithm searches for a set of points around the initial point. The objective function is calculated at each of the points around the initial
point and the best point is searched in that neighbourhood. The initial point is then moved to this ‘best neighbour’ and the process is terminated if either the tolerance constraints are satisfied, or a maximum number of iterations is reached.

The neighbourhood of a point is constructed in the solution space as a set of points in which one of the indices of the communication vector have been perturbed by one. Therefore, every point in the solution space will have 2 of the communication vector have been perturbed by one.

Further details of the algorithms are provided in Section 5.

Algorithm 1

**Algorithm 1** Two-stage optimisation for node-level energy management

**Output:** $W^*_c, W^*_d$

1. Fix FAR, $T^*_{\text{goal}}$, E and $W_N$.
2. Assume a model for communication and a communication protocol for the sensor network.
3. Calculate $W_{TR}$ and $W_c$ using the specified design variables.
4. Calculate the average power left for detection as an implicit function of $W_c$, and thus, $r_c$.
5. Find the radius of detection of sensors using the power for detection available at individual sensors.
6. Obtain probability of search, $\pi_{ss}$ as an implicit function of $r_c$.
7. Represent $r_c$ as unique $M$-tuple of integers and thus, create binary representations for the same.
8. Use $\pi_{ss}$ as an implicit function of $r_c$ as the objective function. Initiate a GA search by using a feasible $r_c$ in binary representation as the initial point. (This begins the first stage of optimisation.)
9. Use the solution obtained by genetic algorithm as an initial point for the PS optimisation. (This finishes the optimisation process.)
10. Using the optimised vector $r^*_c$, and the network lifetime, compute $W^*_d$.
11. Using $W^*_c$, $W_T$ and $W_N$, calculate $W^*_d$.

5. Results and interpretation of numerical simulation for example problems

This section presents the results of the proposed method of network performance optimisation. Examples are presented to demonstrate the operations of the optimisation algorithms and the results are generated via simulation of the following two sensor networks.

1. Network#1 that consists of eight sensing nodes (i.e., $M = 8$) and a sink node.
2. Network#2 that consists of 32 sensing nodes (i.e., $M = 32$) and a sink node.

As stated earlier, the sink node receives all information for processing, but it transmits no messages except when it has to broadcast the planned or re-planned topology of the network. The network is required to remain active for a fixed time-horizon $T^*_{\text{goal}}$ and the batteries of all sensing nodes are initialised to be at the same energy level. To find the optimal operating conditions, the design space of optimisation is first explored with a GA that is executed for a predefined number of generations (i.e., iterations). Then, the best solution of GA is used as the starting point in the next stage for execution of a PS algorithm. The optimal solution of PS is then compared with the average behaviour obtained by using Monte-Carlo simulation for performance evaluation of the optimisation scheme.

Figure 7 shows the results derived from simulation of Network#1, where node 1 through 8 serve as the sensing nodes, each having the probability of detection, $\pi_{\text{det}} = 0.6$, while the node 9 is the sink. The sensors have been randomly placed within a $1500m \times 1500m$ surveillance region. A feasible solution is obtained by GA by execution over 30 generations, where each generation contains 100 members. Each member of a generation is represented as a combination of discrete communication levels for the sensing nodes, which is a binary string of length $L=24$. Only the fittest member of a population is carried over to the next generation and the mutation probability is taken to be $1/L$. The PS is terminated when there is no further improvement in the performance of the network with a local change in the solution point. The same termination condition is used for PS in all the simulations presented in this section.

Figure 7(a) shows the topology of Network#1, generated by GA, where the achieved probability of successful search $\pi_{ss}$ is $\sim 0.86$. The operating point obtained by GA is used as an initial condition for local PS optimisation that eliminates the insignificant links in the network to make more power available for target detection. Figure 7(b) shows the corresponding network topology, where three out of the eight sensing nodes in Network#1 use single-hop links to communicate to the sink (i.e., node 9). Consequently, $\pi_{ss}$ is improved to $\sim 0.89$. 

---

*Downloaded by [Pennsylvania State University] at 11:14 03 April 2015*
Figure 7. Optimised performance of Network#1 (The arrows denote the directions of information flow). (a) Topology generated by the genetic algorithm (GA). (b) Topology generated by pattern search (PS) with additional local optimisation.

Figure 8(a) presents the convergence of GA used in Figure 7(a) for Network#1. Being a relatively small network, GA efficiently explores the solution space as seen from the trend of convergence over the generations as seen in Figure 8(a). The results of Monte-Carlo simulation in Figure 8(b) are approximately fitted with Gaussian distribution by the regular χ² goodness of fit (Brunk, 1975). The mean performance is ∼0.54 and the standard deviation is ∼0.24. Three out the eight nodes in the network use single link communication with the sink in the optimised scenario.

A sensor network is expected to adapt to the variations of energy availability at different sensor nodes. Along this line, Figure 9 presents typical results of adaptation to energy variations across Network#1. Since the GA parameters are the same as in the previous cases of Figures 7 and 8, the structure of the sensor network is essentially unchanged with the exception of assigning different sensor locations in

Figure 8. Monte-Carlo simulation for convergence and stability analysis and performance evaluation of Network#1. (a) Convergence and stability of genetic algorithm (GA). (b) Performance evaluation by Monte-Carlo simulation; GA = genetic algorithm, PS = pattern search.

Figure 9. Energy availability across Network#1.
the surveillance region. After a certain period of operation, node 5 in Network#1 happens to have maximum battery energy left, as seen in Figure 9. The optimal network topology is achieved when all sensors have equal energy as seen in Figure 10(a). Apparently, node 1 becomes as the potential bottleneck, because a vast majority of the remaining nodes transmit their data packets via node 1. By making use of the information on variations in energy availability across the sensor network, the optimal network topology is presented Figure 10(b), where node 5 becomes the new bottleneck instead of node 1 as all neighbouring nodes now communicate through node 5. Figure 10(c) shows the results of Monte-Carlo simulation for variable energy availability. The results are approximated with Gaussian distribution by the regular $\chi^2$ goodness of fit (Brunk, 1975). The mean performance is $\sim 0.51$ and the standard deviation is $\sim 0.21$. The GA optimisation is able to achieve a $\pi_{ss}$ of 0.76, which improves to 0.79 via the local direct search by PS.

Figure 11 shows the results of GA for Network#2, where node 1 through 32 serve as sensing nodes, each having probability of detection, $\pi_{det} = 0.6$, while node 33 is the sink. The sensors have been randomly placed in the 1500m $\times$ 1500m surveillance region, which is similar to that for Network#1. The communication noise for Network#2 is increased to maintain a comparable level of performance with a larger number of sensors, which would require augmentation of the energy level of the transmitted signal.

Similar to what was done for Network#1, GA is run for 200 generations, where each generation contains 500 members, to arrive at a near-optimal solution for the topology of Network#2. Each member in GA is represented as a combination of discrete communication energy level for the sensing nodes, which is a binary string with length, $L=160$. The best fitting member of a population is carried over to the next generation. Taking the mutation probability to be $\frac{1}{L}$, GA converges to the near-global optimal solution where probability of successful search $\pi_{ss}$ is approximately 0.81. The operating point, obtained by GA, is used as an initial condition for PS optimisation, where the local optimisation removes some of the redundant communication links in the network. This makes more power available for detection with an improved $\pi_{ss} \approx 0.83$, where about a third of the nodes use single-link communication with the sink. Consequently, no single node is overloaded with packets, which makes the network robust to an unanticipated node failure.

Since the solution space of Network#2 is larger than that of Network#1, a larger number of GA generations is required before the search converges; however, the search is reasonably stable and does converge to a neighbourhood of the optimal solution, as seen in Figure 11(a). The near-global optimality of GA is shown by the convergence of mean and standard deviation of the objective function over the generations of GA as seen in Figure 11(b); after the transient phase of search is over, the population of the successive generations slightly fluctuates around the

![Figure 11](image_url)
optimal solution. The results of the Monte-Carlo simulation in Figure 12 are approximated with a Gaussian distribution by using the regular $\chi^2$ goodness of fit (Brunk, 1975). The mean performance is $\sim 0.41$ and the standard deviation is $\sim 0.10$. For clarity of presentation, all results are also listed in Table 1.

Using a combination of GA and PS algorithms, Figure 13 shows a Pareto-optimal surface obtained as a trade-off between network lifetime and performance for Network#2 under the constraints of CFAR, constant total energy, and fixed (communication) SNR. As the statistics of GA runs suggest, this Pareto-optimal surface (Miettinen, 1998) is expected to be in a close vicinity of the true global optimal surface.

6. Conclusions and future work

This paper presents optimisation of energy-efficient sensor networks for persistent surveillance in an underwater environment. The proposed optimisation algorithm allocates the available energy between sensing and communication at individual nodes, both of which are required for an active sensor network. The problem is posed as optimal identification of the power requirements for data-packet transmission for each node in the sensor network. It is shown by simulation on two networks of different size that the

<table>
<thead>
<tr>
<th>Network</th>
<th>GA</th>
<th>GA + PS</th>
<th>MC</th>
<th>MC standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.86</td>
<td>0.89</td>
<td>0.54</td>
<td>0.24</td>
</tr>
<tr>
<td>1 (Case 2)</td>
<td>0.76</td>
<td>0.79</td>
<td>0.51</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>0.81</td>
<td>0.83</td>
<td>0.41</td>
<td>0.10</td>
</tr>
</tbody>
</table>
The proposed algorithm adapts to changes in the energy availability across the sensor network, which might occur due to non-uniform power requirements in different parts of the network; a Pareto-optimal surface shows the trade-off between performance and network lifetime.

The proposed algorithm is validated by using standard statistical tools on simulated surveillance scenarios. Although this paper uses models relevant to underwater communication and detection, the framework of energy management could be very well-suited for other types of sensor network.

Future research is recommended in the following areas for the enhancement of the proposed method in the following areas.

1. Improvement in computational efficiency of network optimisation for distributed execution on large sensor networks: this research area would require identification of the critical parameters in an abstract model, which can be optimised locally by each node with local information. For example, depending on the relative position of the neighbouring nodes, each sensor node will decide to choose the link that uses minimum energy. Simultaneously, in order to make the network more robust to link failures, each sensor may attempt to increase the number of connections to its neighbouring nodes.

2. Optimal sensor placement for energy efficiency: in view of the fact that optimal network topology is a function of sensor location, energy-efficient sensor placement will tend to maximise the probability of successful search with fixed energy availability constraints.

Disclosure statement

Any opinions, findings and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the sponsoring agencies.

Funding

This work has been supported in part by the US Office of Naval Research [grant number N00014-14-1-0545]; the US Army Research Laboratory [grant number W911NF-14-2-0068].

References


