An electronic instrument is to be isolated from a panel that vibrates at frequencies ranging from 35 Hz to 35 Hz. It is estimated that at least 80% vibration isolation must be achieved to prevent damage to the instrument. If the instrument weighs 85 N, find the necessary static deflection of the isolator.

\[ m = \frac{85 \text{ N}}{9.81 \text{ m/s}^2} = 8.66 \text{ kg} \]

Frequency range: \( 35 \text{ Hz} \leq \omega = \omega_n f \leq 157.08 \text{ rad/s} \)

\[ 35 \text{ Hz} \leq \omega = \omega_n f \leq 319.91 \text{ rad/s} \]

Governing Equation: \( x^2 + 2\omega_0 x + \omega_0^2 x = F_0 \sin(\omega t) \)

Response Amplitude: \( x = \frac{F_0}{\sqrt{(1 - r^2)^2 + (\omega_0^2 r^2)^2}} \)

\( \frac{x}{x_0} = T_R = \frac{[(1 + (2\pi f)^2)]^{1/2}}{[(1 - r^2)^2 + (2\pi f r^2)^2]^{1/2}} \]

- Range of magnitude of \( T_R = 0.8 \) (Safe)

For small \( \omega_0 \), \( TR = 0.2 \)

\[ \frac{1}{(1 - r^2)^2} \rightarrow 0.2 \quad \frac{1}{(1 - r^2)^2} \rightarrow \pm 5 \quad 1 - r^2 \quad r^2 = 0.4 \]

\[ r^2 = 0.4 \quad \text{or} \quad r = \frac{1}{\sqrt{0.4}} = 2.449 \]

For \( f = 35 \text{ Hz} = 157.08 \text{ rad/s} \), \( \frac{\omega}{\omega_n} = \frac{157.08}{35} = 4.443 \text{ rad/s} \)

\[ \frac{\omega}{\omega_n} \approx 4 \]
So, \( k = 35633.4 \text{ N/m} \) and \( \delta_{st} \geq 2.385 \text{ mm} \) will satisfy the design requirement.

The above results are obtained with \( \xi = 0 \). The same calculation procedures may be followed for \( \xi \neq 0 \). Some damping is always needed such as \( \xi = 0.1 \sim 0.5 \) depending on applications. You may try it with say \( \xi = 0.1 \).
A compressor of mass 120 kg has a rotating unbalance of 0.8 kg-m.

If an isolator of stiffness 0.5 MN/m and clamping ratio 0.01 is used, find the range of operating speeds at which the compressor will be least than 3500 N.

Given: \( m_e = 0.2 \text{ kg-m} \)

\( m_e = 130 \text{ kg} \)

\( k = 0.5 \times 10^6 \text{ N/m} \)

\( \varphi = 0.01 \)

\( F_e < 3500 \text{ N} \)

\[
TR = \frac{F_e}{m_e w^2} = \frac{1 + (2 \varphi w_n^2)^{\frac{1}{2}}}{(1 - r^2)^2 + (2 \varphi w_n^2)^{\frac{1}{2}}}
\]

\( F_e = m_e w_n^2 \)

\[
\frac{F_e}{m_e w_n^2} = \frac{1 + (2 \varphi w_n^2)^{\frac{1}{2}}}{(1 - r^2)^2 + (2 \varphi w_n^2)^{\frac{1}{2}}}
\]

\[
F_e = m_e w_n^2 \left[ \frac{1 + (2 \varphi w_n^2)^{\frac{1}{2}}}{(1 - r^2)^2 + (2 \varphi w_n^2)^{\frac{1}{2}}} \right]
\]

\[
F_e \left[ (1 - r^2)^2 + (2 \varphi w_n^2)^{\frac{1}{2}} \right] = m_e w_n^2 \left( 1 + (2 \varphi w_n^2)^{\frac{1}{2}} \right)
\]

\[
and \quad w_n^2 = \frac{k}{m} = 0.5 \times 10^6 \text{ N/m} \quad \Rightarrow \quad w_n = 4166.67 \text{ rad/s}\quad \Rightarrow \quad \omega_n = 64.35 \text{ rad/s}
\]

Setting \( F_e = m_e w_n^2 \)

\[
r^2 \geq \frac{1 + (2 \varphi w_n^2)^{\frac{1}{2}}}{(1 - r^2)^2 + (2 \varphi w_n^2)^{\frac{1}{2}}}
\]

\( F_e < 3500 \text{ N} \)

\[
(0.8 \text{ kg-m})(4166.67 \text{ rad/s})^2 \geq \frac{1 + (2 \times 0.01 \times 64.35)^{\frac{1}{2}}}{(1 - r^2)^2 + (2 \times 0.01 \times 64.35)^{\frac{1}{2}}}
\]

\[
r^2 \left[ 1 + 0.01 \times 141^2 \right]^{\frac{1}{2}} < 3
\]

\[
\left[ 1 - r^2 + r^4 + 0.01 \times 141^2 \right]^{\frac{1}{2}} < 3
\]

\[
F_e < 3500 \text{ N}
\]
The above plot gives the two ranges of operating speeds within which the force transmitted to foundation will be less than 2500 N.

Choose ranges: 0-\(\omega_1\) (0-55.96 rad/s) and \(\omega_2-\omega_3\) (79.72-1419.84 rad/s)
When a washing machine of mass 200 kg and an unbalance of 0.03 kgm is mounted on an isolator, the isolator deflects by 3 mm. Under the static load, find (a) the amplitude of the washing machine and (b) the force transmitted to the foundation at the operating speed of 1200 rpm.

Recall: Response amplitude $X = \frac{m \omega^2}{\left(1 - r^2\right)^2 + \left(3 \pi r\right)^2}$

Assume $r$ smaller or zero.

$X = \frac{me \omega^2}{m (1 - r^2)} = \frac{me \omega^2}{m (1 - r^2)}$ where $r = \frac{185,600 \text{ rad/s}}{44,39 \text{ rad/s}} = 3.687$

$X = \frac{me \omega^2}{m (1 - r^2)} = \frac{0.03 \text{ kgm} \times 503.537^2}{m (1 - 3.687^2)} = 1.1419 \times 10^{-4} \text{ m}$

$X = 1.1419 \times 10^{-4} \text{ m}$

Recall: $TR = \frac{F_t}{F_0} = \frac{1 + \left(3 \pi r\right)^2}{\left(1 - r^2\right)^2 + \left(3 \pi r\right)^2}$ assume $r = 0$ or small

$F_t = m \omega^2 = \frac{0.03 \text{ kgm} \times \left(44,39 \text{ rad/s}\right)^2}{m (1 - 3.687^2)} = 44.80 \text{ N}$
An air compressor of mass 300 kg, with an imbalance of 0.01 kg m, is found to have a large amplitude of vibration while running at 1800 rpm.

Determine the mass and spring constant of the absorber to be added if the natural frequencies of the system are to be at least 20% from the impressed frequency.

\[ m_c = 0.01 \text{ kg m} \]

\[ \omega_n = 125.664 \text{ rad/s} \]

\[ k_i = \frac{m_c \omega_n^2}{k_s} = 3153.73 \text{ N/m} \]

**Equation of Motion**

\[
\begin{bmatrix}
-m_k & 0 \\
0 & -k_s
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2
\end{bmatrix} + \begin{bmatrix}(k + k_s) & -k_s \\
-k_s & k_s
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2
\end{bmatrix} = \begin{bmatrix}f_0 \\ 0
\end{bmatrix} e^{i \omega t}
\]

**Design Objectives:**

1. Suppress vibration at 1800 rpm.
2. Natural frequencies 20% from impressed frequency.

**Amplitudes of Vibration**

\[
x_1 = \frac{k_s - \omega^2 m_s}{(k + k_s - \omega^2 m_s)k_s^2 - k_s^2} F_0
\]

\[
x_2 = \frac{k_s}{(k + k_s - \omega^2 m_s)k_s^2 - k_s^2} F_0
\]

To reduce \(x_1\), set \(k_s - \omega^2 m_s = 0\) in Eq. 1.

\[ m_s = \frac{k_s}{\omega^2} \rightarrow \frac{1}{2G} = \frac{k_s}{m_s} \rightarrow \frac{m_s}{k_s} = \frac{1}{2\pi^2 \omega_f^2} \]

Recall: \(\text{Det} \left[-\omega^2 M + k\right] = 0\)

\[ (k + k_s - \omega^2 m_s)(k_s - \omega^2 m_s) - k_s^2 = 0 \]

or \( (k + k_s - \omega^2 m_s)(k_s - \omega^2 m_s) - k_s^2 = 0 \)
\[ \omega > \frac{1}{2} \omega_r = 1.2 \times 125.166^4 = 150.79 \text{ rad/s} \]  
and \[ \omega < \frac{0.8 \omega_r}{2} = 0.8 \times 125.166^4 = 100.531 \text{ rad/s} \]

Use eq. (3) with eq. (5) and conditions (1) and (6)

\[ (k_1 + k_2 - \omega_r^2 m_1)(k_2 - \omega_r^2 m_2) - k_2^2 = 0 \]

\[ k_1 k_2 - k_2 m_1 \omega_r^2 = k_2^2 - k_1 m_1 \omega_r^2 = \kappa_2 m_1 \omega_r^2 + \kappa_1 \omega_r^4 \]

\[ \frac{k_1 - k_2 m_1 \omega_r^2 - m_1 \omega_r^2 - m_1 \omega_r^2 + m_2 \omega_r^4}{k_2} = 0 \]  

or \[ m_2 = 216.8 \text{ kg} \]

so \[ k_2 = 434.474 \text{ N/m} \]

\[ 315873 - 815873 \left( \frac{1}{125.166^4} \right)^2 (150.79)^2 - m_2 (150.79)^2 = 120.5150793^2 + 120.5150793 \left( \frac{1}{125.166^4} \right)^2 (150.79)^4 = 0 \]

or \[ m_2 = 40.5 \text{ kg} \]

\[ k_2 = 139553 \text{ N/m} \]

Since the values of \( m_2 \) and \( k_2 \) are larger for condition (6),

we must use these values: \[ m_2 = 40.5 \text{ kg} \quad k_2 = 139553 \text{ N/m} \]

Check

then \[ 315873 - 815873 \left( \frac{1}{125.166^4} \right)^2 (150.79)^2 - (40.5) \omega_r^2 - (120.5) \omega_r^4 + (120.5) \left( \frac{1}{125.166^4} \right)^2 \omega_r^4 = 0 \]

\[ \omega_r = 157.1 \text{ rad/s} \quad \omega_r = 100.531 \text{ rad/s} \]
In the previous problem, the true point is that one needs to examine results in both Part (a) and Part (b) to see which pair of $k_2$ and $m_2$ will satisfy the given design requirement of $\omega_n$ outside of $0.8\omega \sim 1.2\omega$.

Also, in the problem, because large vibration is developed at $\omega = 1200 \text{ rpm}$, it has been assumed to be the critical speed (natural frequency) of the original system before the absorber is added. Based on this assumption, $k_1$ is calculated as listed early in the problem.
Vibration Absorption

An electric motor, having an unbalance of 2 kg-cm, is mounted at the end of a steel cantilever beam. The beam is observed to vibrate with large amplitudes at the operating speed of 1500 rpm of the motor. It is proposed to add a vibration absorber to reduce the vibration of the beam. Determine the ratio of the absorber mass to the mass of the motor needed in order to have the lower frequency of the resulting system equal to 75% of the operating speed of the motor. If the mass of the motor is 300 kg, determine the stiffness and mass of the absorber. Also find the amplitude of vibration of the absorber mass.

\[
\begin{align*}
\text{motor} & \quad \frac{\omega_1}{m_1} \\
\text{absorber} & \quad \frac{\omega_2}{m_2}
\end{align*}
\]

Design Objectives: Design absorber to:
1) Suppress vibration at 1500 rpm
2) Lower critical frequency = 75% \( \omega = 0.75 \times 157.08 \approx 117.81 \text{ rad/s} \)

Amplitudes of Vibration

\[
X_1 = \frac{k_2 - \omega^2 m_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) + k_2^2} \left( F_{\omega} \right)
\]

\[
X_2 = \frac{k_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) + k_2^2} \left( F_{\omega} \right)
\]

To reduce \( X_1 \), set \( k_2 - \omega^2 m_2 = 0 \) in Eq. (1)

\[
\omega = \sqrt{\frac{k_2}{m_2}} \quad \text{or} \quad 157.08 = \sqrt{\frac{k_2}{m_2}} \Rightarrow m_2 = \left( \frac{1}{157.08} \right)^2 \quad (3)
\]

Then Eq. (2) \( X_2 = \)

\[
\frac{k_2}{(k_1 + k_2 - \omega^2 m_1)(k_2 - \omega^2 m_2) - k_2^2} \left( F_{\omega} \right) = \frac{F_{\omega}}{k_2} \quad (4)
\]
The problem asks to find the ratio of $m_2/m_1$. This may be done using Eq. (6) above without knowing the motor mass $m_1$. But it requires to assume that the critical speed is 1500 rpm of the motor system before an absorber is added. This assumption is reasonable as the problem states that large vibration is observed at this speed.