Today, we will:

- Discuss **half-life** in first-order dynamic systems
- Talk about first-order dynamic systems with a **ramp function input**
- Finish reviewing the pdf module: **Dynamic System Response (2nd-order systems)**
- Do some more example problems – dynamic system response

Definition of half-life: *Half-life* is the time required for a variable to go half-way from its present value to its final value. Half-life can also be thought of as the 50% response time.
Example: Dynamic system response (first-order and half-life)

**Given:** Output variable $y$ responds like a first-order dynamic system when exposed to a sudden change of input variable $x$. The half-life of the system is 2.00 s. When $x$ is suddenly increased, $y$ grows from an initial value of 100 to a final value of 132.

**To do:** Calculate the time (in seconds) required for $y$ to reach a value of 124.

**Solution:**

![Graph showing the response of $y$ over time]
Example: Dynamic system response (second-order)

Given: The following second-order ODE:

\[ 5\ddt^2 y + \dt y + 1000y = x(t) \]  

(1)

The forcing function is a step function (sudden jump):

\[ x(t) = 0 \] for \( t < 0 \)

\[ x(t) = 25 \] for \( t > 0 \)

(a) To do: Calculate the natural frequency and damping ratio of this system.

(b) To do: Calculate the equilibrium response (as \( t \to \infty \), what is \( y \)?).

Solution:
**Example: Dynamic system response**

**Given:** A spring-mass-damper system is set up with the following properties: mass $m = 22.8$ g, spring constant $k = 51.6$ N/cm, and damping coefficient $c = 3.49$ N·s/m ($c$ is also called $\lambda$ in some textbooks). The forcing function is a step function (sudden jump).

**To do:**

(a) Calculate the damping ratio of this system. Will it oscillate?

(b) If the system will oscillate, calculate the oscillation frequency in hertz. [*Note: Calculate the physical frequency, not the radian frequency.*] Compare the actual oscillation frequency to the undamped natural frequency of the system.

**Solution:**