Operational Amplifiers (Op-Amps)

Introduction
- An operational amplifier (abbreviated op-amp) is an integrated circuit (IC) that amplifies the signal across its input terminals. Op-amps are analog, not digital, devices, but they are also used in digital instruments.
- Op-amps are widely used in the electronics industry, and are thus rather inexpensive – the ones used in the lab are about $0.25 each!
- In this learning module, no details are given about the internal structure of the op-amp. Rather, we summarize many useful applications of op-amps.

Description of op-amps
- A triangle is used as the universal symbol for an op-amp (in schematic circuit diagrams), as shown to the right.
- The supply voltage terminals are at the top and bottom of the schematic diagram. Supply voltage is necessary because the op-amp requires power to run its internal circuitry. Both a positive and negative supply voltage are required, typically +/− 15 V. In other words, $V^+_{\text{supply}} = 15$ V, and $V^-_{\text{supply}} = -15$ V.
- In applications, any + and – voltage between about 10 to 20 V can be used for the supply voltage, depending on the manufacturer’s specifications. (Note: We don’t usually draw the supply voltages on circuit diagrams, but they must be connected or the op-amp will not work!)
- The signal input terminals are on the left – a positive input terminal $V_p$, and a negative input terminal, $V_n$. However, the actual input voltages do not need to be positive and negative for inputs $V_p$ and $V_n$, respectively.
- In fact, the $V_p$ input is usually referred to as the noninverting input instead of the positive input, and the $V_n$ input as the inverting input instead of the negative input, respectively.
- The connection for the output voltage $V_o$ is on the right (pointed) side of the op-amp, as sketched above.

Ideal versus actual op-amps
- An ideal op-amp has infinite input impedance, and therefore it has no effect on the input voltage. This is called no input loading.
- An actual op-amp has very high, though not infinite, input impedance (typically millions of ohms), so that it has little effect on the input voltage. This is called minimal input loading.
- A direct result of the high input impedance is that we may assume negligible current flowing into (or out of) either op-amp input, $V_p$ or $V_n$. This result helps us to analyze op-amp circuits, as discussed below.
- An ideal op-amp has zero output impedance, so that whatever is done to the output signal farther downstream in the circuit has no affect on the output voltage $V_o$. This is called no output loading.
- An actual op-amp has very low, though not zero, output impedance (typically a few Ω), so that what is done downstream of the op-amp has little effect on the output voltage. This is called minimal output loading.
- An ideal op-amp has infinite gain, $g$ (Note that a lower case $g$ is used here for the op-amp gain so as not to be confused with $G$, the gain of amplifier or filter circuits.) Op-amp gain $g$ is called open loop gain.
- An actual op-amp has a very high, though not infinite, open loop gain; $g$ is typically in the $10^5$ to $10^6$ range for the off-the-shelf op-amps that are commonly used in circuits.
- In the analysis of most of the circuits discussed below, the op-amps are approximated as ideal. The performance of an actual (real) op-amp is similar, but not exactly the same as that of an ideal op-amp.

Open-loop versus closed-loop configurations
- In an open-loop configuration, as in the above schematic diagram, $V_o = g(V_p - V_n)$.
- In other words, the output voltage $V_o$ is a factor of $g$ times the input voltage difference, $V_p - V_n$. This might be useful if the incoming signal is extremely small (microvolts), and in need of high amplification.
- In practice, however, circuits are most often built with a feedback loop (closed-loop configuration), which forces the noninverting input and the inverting input to be nearly equal to each other, $V_p \approx V_n$.
- Without a feedback loop, the op-amp can easily saturate since $g$ is large. Saturation means that the output voltage clips at some maximum value, typically a volt or so lower than supply voltage $V^+_{\text{supply}}$.
- Likewise, saturation can occur at the low end as well, when the output voltage clips at some minimum value, typically a volt or so greater than supply voltage $V^-_{\text{supply}}$.
Example:

**Given:** The gain of an op-amp is 1 million \((g = 1 \times 10^6)\). The high supply voltage \(V_{\text{supply}}^+\) is 15.0 V. The op-amp saturates at 13.9 V.

**To do:** Calculate the input voltage difference \((V_p - V_n)\) that will cause saturation when the op-amp is operated in an open-loop configuration.

**Solution:** From the open-loop gain equation, \(\frac{V_p - V_n}{g} = \frac{13.9 \text{ V}}{1 \times 10^6} = 1.39 \times 10^{-5} \text{ V} = 13.9 \mu\text{V}\).

Thus, **any voltage difference greater than 13.9 \mu\text{V} will saturate the op-amp.**

**Discussion:** Note how easily an op-amp is saturated. This is why the open-loop configuration is rarely used.

Closed-loop op-amp circuits

- Almost all practical circuits utilize op-amps in a **closed-loop configuration** to avoid op-amp saturation.
- In all of the examples below, there is **feedback** from the output terminal of the op-amp to one of the input terminals. (This is what makes the configuration a **closed-loop** configuration.)
- In the analyses to follow, it is assumed that \(V_n \approx V_p\) because of the feedback loop, and because we approximate the op-amps as nearly ideal. This will be better understood as these circuits are analyzed.
- **Note:** In all the schematic diagrams to follow, the supply voltages \(V_{\text{supply}}^+\) and \(V_{\text{supply}}^-\) are not shown, but you must remember to wire these to the op-amp, or it will not work!

**Buffer** (also called a **voltage follower**)

- **Purpose:** To provide high impedance for a voltage signal.
- **Schematic:**
  - A circuit diagram or schematic of a buffer is shown to the right.
  - Note the symbol for ground at the bottom of the schematic diagram.
  - In a buffer, the output is fed directly back into the inverting input terminal to provide the feedback loop.
  - The voltage signal is fed directly into the noninverting input terminal.
- **Analysis:**
  - Since the op-amp output is connected directly to the inverting input, \(V_{\text{out}} = V_o = V_n\).
  - If the op-amp were **ideal**, \(V_n = V_p = V_{\text{in}}\) and, \(V_{\text{out}} = V_{\text{in}}\). For a real op-amp, \(V_n \approx V_p = V_{\text{in}}\), and \(V_{\text{out}} \approx V_{\text{in}}\).
- **Discussion:**
  - What have we accomplished? At first it appears that **nothing has been accomplished at all**, since the output voltage is simply equal to the input voltage! Why do we even need a buffer in the first place?
  - Actually, a buffer is very important when the input signal needs to be **isolated** from the rest of the circuit.
  - Because of the high input impedance of the op-amp, the buffer causes \(V_{\text{in}}\) to be insensitive to anything that happens downstream in the circuit.
  - For example, suppose the signal must be connected to something that draws a current. Without the buffer, \(V_{\text{in}}\) would be affected by the current draw. With the op-amp buffer in place, however, the current draw has no effect on input signal \(V_{\text{in}}\).
  - Because of the low output impedance of the op-amp, the output voltage \(V_{\text{out}}\) is likewise not affected significantly by the current draw; \(V_{\text{out}}\) remains nearly equal to \(V_{\text{in}}\) regardless of the downstream circuit.

**Inverting amplifier**

- **Purpose:** To multiply (and invert) a voltage signal by some factor.
- **Schematic:**
  - A schematic of an inverting amplifier is shown to the right.
  - The input signal comes into the negative (inverting) input of the op-amp, after first passing through resistor \(R_1\).
  - Since current flows from the input (at voltage \(V_{\text{in}}\)) through resistor \(R_1\), and then to the op-amp input (at zero voltage), the input impedance of this op-amp circuit is about the same as \(R_1\).
  - The resistors are typically around 10 k\(\Omega\) to 100 k\(\Omega\).
  - The output impedance of this op-amp circuit is on the order of one ohm.
- **Analysis:**
  - Since negligible current flows into the op-amp at input \(V_n\), the same current (\(I\)) flowing through the first resistor also flows through the second resistor, as shown.
o Since we assume that $V_{out}$ is measured by a voltmeter, oscilloscope, or data acquisition system with very high input impedance, all of this current ($I$) must flow into the output terminal of the op-amp, as shown.

o The sign of the current is assumed, as sketched in the diagram. If $V_{in}$ is positive, this assumption is correct. If $V_{in}$ is negative, the current is of opposite sign to that shown.

o Using Ohm’s law, $V_{in} - IR_1 = V_n$.

o But, for ideal op-amp behavior, $V_o = V_p = 0$, since $V_p$ is grounded. [For a real op-amp, $V_n \approx V_p = 0$.]

o Thus, solving for the current, $I = V_{in}/R_1$.

o In the feedback portion of the circuit, $V_o = V_n - IR_2 = 0 - IR_2 = -(V_{in}/R_1)R_2$.

o Or, finally, $V_{out} = V_o = -\frac{R_2}{R_1}V_{in} = GV_{in}$, where the gain $G$ of any amplifier is defined as $G = \frac{V_{out}}{V_{in}}$.

o The gain of our inverting amplifier turns out to be the negative ratio of $R_2$ to $R_1$, i.e.,

**Discussion:**

o Resistors $R_1$ and $R_2$ can be chosen to achieve any desired gain. If $R_2$ is greater than $R_1$, the signal is amplified (and inverted)

o The magnitude of the gain does not necessarily have to be greater than unity. An inverting amplifier can actually attenuate (and invert) a signal if $R_2$ is less than $R_1$.

### Inverter

**Purpose:** Invert (change the sign) of a signal without amplification or attenuation.

**Schematic:**

- A schematic diagram of an inverter is shown to the right.
- The schematic diagram is identical to that above for the inverting amplifier. The only difference is that $R_1$ and $R_2$ have the same value, and are indicated simply as $R$.

**Analysis:**

- The analysis is the same as above.
- For identical $R_1$ and $R_2$, the gain is $G = -R_2/R_1 = -1$. Thus, $V_{out} = V_o = -V_{in}$

**Discussion:**

- An inverter simply changes the sign of a signal. It also acts as a buffer, so all of the advantages of the buffer listed above apply here as well. We could instead call the inverter an inverting buffer.

### Inverting summer

**Purpose:** To add (and invert) two voltage signals.

**Schematic:**

- A schematic diagram is shown to the right.
- All three resistors in this circuit have the same value, typically around 10 kΩ to 100 kΩ.
- The actual value of $R$ is not critical. However, in typical circuits, resistors lower than 1 kΩ are generally not used because they waste power unnecessarily.
- Likewise, resistors higher than 1 MΩ are generally not used because they can lead to stray capacitance effects, details of which are beyond the scope of the present discussion.

**Analysis:**

- We examine the input (left) side of the op-amp first (using Ohm’s law): $V_1 - I_1R = V_2 - I_2R = V_n$.
- But, if the op-amp is nearly ideal, $V_n \approx V_p = 0$ (since $V_p$ is grounded). Thus, $V_1 = I_1R$ and $V_2 = I_2R$.
- Now we examine the output (right) side, again using Ohm’s law, and utilizing the fact that negligible current flows into the op-amp at input $V_n$ (hence, the current flowing through the top right resistor is approximately equal to $I_1 + I_2$). Therefore, $V_o = V_n - (I_1 + I_2)R = 0 - I_1R - I_2R = -V_1 - V_2$.
- Or, finally, $V_{out} = V_o = -(V_1 + V_2)$

**Discussion:**

- The two input voltages $V_1$ and $V_2$ have been added, but the output is the negative of the sum – hence the name inverting summer.
This is a very effective and safe way to perform a summation of two voltages. The main advantage is due again to the high input impedance of the op-amp – voltages $V_1$ and $V_2$ are isolated in this summing circuit. That is, they are not affected by what happens downstream of the op-amp or by each other. The inversion (negative sign) of the signal is not a problem. If the negative sign needs to be removed, it can be done so with a simple inverter, as discussed above.

- **Example:**
  **Given:** Some op-amps and a bunch of 20-kohm resistors are available in the lab.
  **To do:** Show how these components can be used to *double the voltage of an input signal*. Use *inverting* circuits, and draw the circuit diagram.

  **Solution:**
  - We use an inverting amplifier to produce the gain. Then we use an inverter in series to remove the negative sign produced by the inverting amplifier.
  - If $R_1$ is selected as 20 k$\Omega$, $R_2$ must be 40 k$\Omega$ to achieve amplification by a factor of two. Two resistors in series will do the job nicely.
  - The available resistors can also be used for the inverter.
  - The schematic diagram is shown to the right.

  - The left half of the circuit is the *inverting amplifier*, with a gain of $G = \frac{R_2}{R_1} = \frac{40 \text{ k}\Omega}{20 \text{ k}\Omega} = 2$.
  - The output voltage from the first op-amp is labeled $V_1$, and $V_1 = GV_{in} = -2V_{in}$.
  - The right half of the circuit is the *inverter*, with $V_{out} = -V_1$. Thus $V_{out} = 2V_{in}$ as desired.

  **Discussion:** The above circuit requires 2 op-amps and 5 resistors. If we use a noninverting amplifier instead, as discussed next, we can achieve the same result with just 1 op-amp and 2 resistors.

**Noninverting amplifier**

- **Purpose:** To amplify a voltage signal without inverting.
- **Schematic:**
  - A schematic of a noninverting amplifier is shown to the right.
  - As seen, the noninverting amplifier is similar to the inverting amplifier except that the input signal comes into the positive (noninverting) input of the op-amp instead of into the negative (inverting) input.
  - The input impedance of this op-amp circuit is on the order of hundreds of megaohms (the input impedance of a noninverting amplifier is equal to the internal impedance of the op-amp itself).
  - The output impedance of this op-amp circuit is on the order of one ohm.
- **Analysis:**
  - Since negligible current flows into the op-amp at input $V_n$, the same current ($I$) flowing through the first resistor also flows through the second resistor, as shown.
  - Assuming that $V_{out}$ is measured by a voltmeter, oscilloscope, data acquisition system, etc. with very high input impedance, all the current ($I$) must flow out of the output terminal of the op-amp, as shown.
  - The sign of the current is assumed, as sketched in the diagram. If $V_{in}$ is positive, this assumption is correct. If $V_{in}$ is negative, the current is of opposite sign to that shown.
  - For ideal op-amp behavior, $V_n = V_p = V_{in}$ (since $V_{in}$ is wired to $V_p$). [For a real op-amp, $V_n \approx V_p = V_{in}$.]
  - Using Ohm’s law, $V_n - IR_1 = V_{in} - IR_1 = 0$ since the left-most wire of the circuit is grounded.
  - Thus, solving for the current, $I = V_{in}/R_1$.
  - In the feedback portion of the circuit, $V_o = V_n + IR_2 = V_{in} + IR_2 = V_{in} + (V_{in}/R_1)R_2$.
  - Finally, $V_{out} = GV_{in} = (1 + \frac{R_2}{R_1})V_{in} = GV_{in}$, where the gain of a noninverting amplifier is $G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$. 

- The inversion (negative sign) of the signal is not a problem. If the negative sign needs to be removed, it can be done so with a simple inverter, as discussed above.

- **Discussion:** The above circuit requires 2 op-amps and 5 resistors. If we use a noninverting amplifier instead, as discussed next, we can achieve the same result with just 1 op-amp and 2 resistors.

**Noninverting amplifier**

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- **Schematic:**
  - A schematic of a noninverting amplifier is shown to the right.
  - As seen, the noninverting amplifier is similar to the inverting amplifier except that the input signal comes into the positive (noninverting) input of the op-amp instead of into the negative (inverting) input.
  - The input impedance of this op-amp circuit is on the order of hundreds of megaohms (the input impedance of a noninverting amplifier is equal to the internal impedance of the op-amp itself).
  - The output impedance of this op-amp circuit is on the order of one ohm.
- **Analysis:**
  - Since negligible current flows into the op-amp at input $V_n$, the same current ($I$) flowing through the first resistor also flows through the second resistor, as shown.
  - Assuming that $V_{out}$ is measured by a voltmeter, oscilloscope, data acquisition system, etc. with very high input impedance, all the current ($I$) must flow out of the output terminal of the op-amp, as shown.
  - The sign of the current is assumed, as sketched in the diagram. If $V_{in}$ is positive, this assumption is correct. If $V_{in}$ is negative, the current is of opposite sign to that shown.
  - For ideal op-amp behavior, $V_n = V_p = V_{in}$ (since $V_{in}$ is wired to $V_p$). [For a real op-amp, $V_n \approx V_p = V_{in}$.]
  - Using Ohm’s law, $V_n - IR_1 = V_{in} - IR_1 = 0$ since the left-most wire of the circuit is grounded.
  - Thus, solving for the current, $I = V_{in}/R_1$.
  - In the feedback portion of the circuit, $V_o = V_n + IR_2 = V_{in} + IR_2 = V_{in} + (V_{in}/R_1)R_2$.
  - Finally, $V_{out} = GV_{in} = (1 + \frac{R_2}{R_1})V_{in} = GV_{in}$, where the gain of a noninverting amplifier is $G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$. 

- The inversion (negative sign) of the signal is not a problem. If the negative sign needs to be removed, it can be done so with a simple inverter, as discussed above.

- **Discussion:** The above circuit requires 2 op-amps and 5 resistors. If we use a noninverting amplifier instead, as discussed next, we can achieve the same result with just 1 op-amp and 2 resistors.
**Example:**

*Given:* Some op-amps and a bunch of 20-kohm resistors are available in the lab.

*To do:* Show how these components can be used to *double the voltage of an input signal*. Use noninverting circuits, and draw the circuit diagram.

**Solution:**

- We use a noninverting amplifier to produce the gain.
- If $R_1$ is selected as 20 kΩ, $R_2$ must also be 20 kΩ to achieve amplification by a factor of two.
- The schematic diagram is shown to the right.
- The gain of this noninverting amplifier is
  
  $$G = 1 + \frac{R_2}{R_1} = 1 + \frac{20 \text{kΩ}}{20 \text{kΩ}} = 2.$$  

- Thus, $V_{out} = GV_{in}$, or $V_{out} = 2V_{in}$ as desired.

**Discussion:** The above circuit requires only 1 op-amp and 2 resistors. Compare this to the previous example using inverting circuits, where we needed 2 op-amps and 5 resistors to achieve the same result.

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**First-order, active, low-pass, inverting filter**

*Purpose:* To low-pass filter (and invert) a voltage signal.

*Schematic:*

- The schematic is shown to the right.
- As seen, a first-order, active, low-pass, inverting filter is simply an inverting amplifier with a capacitor added in parallel to the resistor in the feedback loop.
- You are invited to compare this filter circuit with that of a simple passive first-order RC low-pass filter circuit.

*Analysis:*

- We consider the simplest case in which the resistor values are chosen to be identical ($R_1 = R_2 = R$); there is no amplification.
- At low frequencies, the capacitor acts like an open switch, and thus does not contribute anything. The circuit is then the same as an inverter, with $V_{out} \approx -V_{in}$, and low frequencies pass unaffected (except that the signal is inverted).
- At high frequencies, the capacitor acts like a closed switch, rendering $R_2$ useless, and forcing $V_o$ to nearly equal $V_p$. However, $V_p$ is grounded, so $V_{out} = V_o \approx V_p \approx V_p = 0$. In other words, high frequencies are attenuated significantly – the output voltage is nearly zero.
- It turns out that at intermediate frequencies, this circuit behaves as a first-order Butterworth low-pass filter. The equation for the gain $G$ of the filter is the same as that defined previously for a simple RC filter, except for the negative sign. The phase shift is also the same as that defined previously.
- The cutoff frequency of this active filter is in fact the same as that for a simple, passive, first-order, low-pass RC filter, $f_{cutoff} = \frac{1}{2\pi R_2 C}$. Notice that $R_2$ is used here rather than $R_1$.

*Discussion:*

- If a simple RC circuit is used for filtering, the filter is called a passive filter.
- When an op-amp with feedback is used, as in the present case, the filter is called an active filter.
- As with an inverting amplifier, values of $R_2$ and $R_1$ can be selected such that there is amplification (or attenuation) of the signal as well.
- If the negative sign is a problem, we can add a simple inverter in series. Or, we can use noninverting circuitry instead.

**First-order, active, high-pass, inverting filter**

*Purpose:* To high-pass filter (and invert) a voltage signal.

*Schematic:*

- The schematic is shown to the right.
- The first-order, active, high-pass, inverting filter is similar to the first-order, active, low-pass, inverting filter discussed above, except that the capacitor is
placed in series with the input resistor instead of in parallel with the feedback resistor.

- **Analysis:**
  - If the resistor values are chosen to be identical \((R_1 = R_2 = R)\), there is no amplification.
  - At low frequencies, the capacitor acts like an open switch, so \(V_{in}\) is effectively isolated from the output.
  - Another way to look at this case is that at low frequencies, no current can flow through the capacitor or through resistor \(R_1\). Thus, \(V_{out} = V_n \approx V_p = 0\) since \(V_n\) is grounded; low frequencies are attenuated.
  - At high frequencies, the capacitor acts like a closed switch, having no effect on the circuit. The circuit then becomes the same as an inverter, with \(V_{out} = -V_{in}\); high frequencies pass unaffected (except that the signal is inverted).
  - It turns out that at intermediate frequencies, this circuit behaves as a first-order Butterworth high-pass filter. The equation for the gain \(G\) of the filter is the same as that defined previously for a simple RC filter, except for the negative sign. The phase shift is also the same as that defined previously.
  - The cutoff frequency of this active filter is in fact the same as that for a simple, passive, first-order, low-pass RC filter, \(f_{cutoff} = \frac{1}{2\pi R_1 C}\). Notice that \(R_1\) is used here rather than \(R_2\).

- **Discussion:**
  - Since an op-amp with a feedback loop is used in the present case, the filter is an active filter.
  - As with an inverting amplifier, values of \(R_2\) and \(R_1\) can be selected such that there is amplification (or attenuation) of the signal as well.
  - If the negative sign is a problem, we can add a simple inverter in series. Or, we can use noninverting circuitry instead.

*Low-voltage clipping circuit*

- **Purpose:** To clip voltages below some reference voltage, \(V_{ref}\).
- **Schematic:**
  - The schematic is shown to the right.
  - The noninverting input of the op-amp is connected to the reference voltage \(V_{ref}\).
  - The component just to the right of the op-amp is called a switching diode.
  - A switching diode allows current to flow *in one direction only* (in the direction of the small triangle – to the right in the orientation shown here), and blocks currents from flowing the opposite way.

- **Analysis:**
  - If \(V_{in} > V_{ref}\)
    - Current tries to flow through resistor \(R\) in the direction indicated on the above diagram.
    - No current can flow into the left side (input side) of the op-amp, because of its high input impedance.
    - Likewise, no current can flow downstream from (to the right of) \(V_{out}\) because it is assumed that \(V_{out}\) either goes into another high-impedance component, such as another op-amp, or is being measured with a high impedance voltmeter, oscilloscope, digital data acquisition system, etc.
    - Thus, the only path for the current to flow is through the feedback loop and back into the output (right side) of the op-amp as shown.
    - However, current cannot flow through the switching diode in the direction indicated on the above diagram! Hence, no current can flow at all. Since \(I = 0\), \(V_n = V_{in}\) by Ohm’s law when \(I = 0\).
  - But \(V_{out} = V_{ref}\). So, finally, \(V_{out} = V_{in}\) when \(V_{in} > V_{ref}\).
  - In other words, the low-voltage clipping circuit has no effect on voltages greater than \(V_{ref}\).
  - Note that for \(V_{in} > V_{ref}\), \(V_p = V_{ref}\) while \(V_n = V_{in}\). Thus, \(V_n \neq V_p\) for this case, and the op-amp is saturated. (This is because the feedback loop, though present, is interrupted by the switching diode.)
  - If \(V_{in} < V_{ref}\)
    - \(V_p = V_{ref}\), \(V_n = V_p = V_{ref} > V_{in}\), and current tries to flow through resistor \(R\) in the direction opposite of that indicated on the above diagram.
    - Current tries to flow from the output of the op-amp, through the switching diode, and through the feedback loop.
In this case, current *can* flow through the switching diode in the direction indicated in the diagram to the right.

Ideally, there is no voltage drop across the diode; thus, \( V_o = V_n \).

Since the current cannot flow into the inverting input terminal \( V_n \) of the op-amp, current *must* flow through the resistor in the direction shown here.

Hence, by Ohm's law, \( V_n - IR = V_{in} \). (Since \( V_n > V_{in} \), current flows to the left through resistor \( R \) as sketched in the circuit diagram to the right.)

But \( V_n \approx V_p = V_{ref} \), and \( V_{out} = V_{ref} \). So, finally, \( V_{out} = V_{ref} \) when \( V_{in} < V_{ref} \).

In other words, *the low-voltage clipping circuit clips (to \( V_{ref} \)) voltages that are less than \( V_{ref} \).*

- **Discussion:**
  - \( V_{out} = V_{in} \) as long as \( V_{in} \) is greater than \( V_{ref} \). However, if \( V_{in} \) drops below \( V_{ref} \), \( V_{out} = V_{ref} \). This is what is meant by low-voltage clipping.
  - The sketch to the right indicates the clipping.
  - Notice that the output follows the input exactly, but every time the input signal drops below \( V_{ref} \), it gets clipped to \( V_{ref} \).

**High-voltage clipping circuit**

- **Purpose:** To clip voltages above some reference voltage, \( V_{ref} \).

- **Schematic:**
  - The schematic is shown to the right.
  - The high-voltage clipping circuit is identical to the low-voltage clipping circuit, except the switching diode is turned around the opposite way.

- **Analysis:**
  - The analysis is similar, but opposite to that above, and we leave out the details.
  - Results: \( V_{out} = V_{ref} \) when \( V_{in} > V_{ref} \); current *can* flow through the switching diode in the direction indicated on the schematic. *The high-voltage clipping circuit clips (to \( V_{ref} \)) voltages greater than \( V_{ref} \).*
  - \( V_{out} = V_{in} \) when \( V_{in} < V_{ref} \); current tries to flow the wrong way through the switching diode, but cannot. *The high-voltage clipping circuit has no effect on voltages smaller than \( V_{ref} \).*

- **Discussion:**
  - High-voltage clipping is sometimes necessary to protect electronic components from voltages that are too high.
  - For the same input voltage signal as above, the sketch to the right indicates the clipping.
  - Every time the input signal rises above \( V_{ref} \), it gets clipped.
  - To construct a circuit that clips both high and low voltages, we connect a low-voltage clipping circuit and a high-voltage clipping circuit *in series*. In such a case, the reference voltage for the low-voltage clip must be smaller than that for the high-voltage clip.

**Miscellaneous properties of op-amp circuits**

- It is not the intent of this learning module to discuss the internal circuitry of an op-amp.

- However, there are several aspects of actual op-amps that cause problems in circuits – problems that would not exist for ideal op-amps – due to limitations of the internal circuitry of real op-amps. Among these are:
  - Saturation effects (as previously mentioned)
  - Input loading effects (as also previously mentioned)
  - Common-mode rejection ratio (CMRR) effects
  - Gain-bandwidth product (GBP) effects

- The latter two are discussed in detail below.
Common-mode rejection ratio (CMRR)

- Definition: Common-mode rejection ratio (CMRR) is defined as:
  \[ \text{CMRR} = 20 \log_{10} \left( \frac{g}{G_{CM}} \right) \]

  - \( g \) is the open-loop voltage gain of the op-amp as discussed previously. Another name for \( g \) is differential voltage gain. In other words, \( g \) is the gain in voltage when the input to \( V_p \) and \( V_n \) are different.
  - \( G_{CM} \) is the common-mode voltage gain. In other words, \( G_{CM} \) is the gain in voltage when the input to \( V_p \) and \( V_n \) are the same.
  - The units of CMRR are decibels (dB). A large value of CMRR implies that \( G_{CM} \) is small compared to \( g \), which is desirable for rejection of common-mode noise. Modern op-amps have values of CMRR greater than 100 dB.

- Analysis:
  - Common-mode gain occurs when the same noise (typically high frequency, small amplitude noise) is present in both \( V_p \) and \( V_n \).
  - For example, consider identical noise at both input terminals of the op-amp, as sketched to the right.
  - Since \( V_o = g(V_p - V_n) \), we expect \( V_o \) to be exactly zero since \( V_p = V_n \) at any instant in time.
  - However, it turns out that the op-amp (due to its internal circuitry – beyond the scope of the present discussion) actually amplifies the common-mode noise by factor \( G_{CM} \) as sketched.
  - Since noise typically appears in both inputs (common-mode input), while the signal typically is wired to only one input (differential-mode input), CMRR needs to be large to ensure a large signal-to-noise ratio at the output of the op-amp.

- Application:
  - Which is better – inverting or noninverting op-amp amplifiers?
  - It turns out that there are two competing effects to consider:
    - Input loading: Recall that input loading problems occur when the input impedance \( R_i \) of an electronic device is not high enough.
      - For a noninverting op-amp amplifier, \( R_i \) is determined by the internal circuitry of the op-amp, and is generally quite large (of order 1 M\( \Omega \)).
      - For an inverting op-amp amplifier, \( R_i \) is largely independent of the internal circuitry of the op-amp, and is instead approximately equal to \( R_i \) (typically of order 10 k\( \Omega \) to 100 k\( \Omega \)).
      - Thus, if input loading is of primary concern, noninverting amplifiers should be used.
    - Noise rejection: As mentioned above, noise generally appears in both input terminals of the op-amp.
      - For a noninverting op-amp amplifier, it turns out that in many applications, the noise amplitude is nearly the same in the \( V_p \) and \( V_n \) input terminals, and thus common-mode gain is more of a problem in these circuits.
      - For an inverting op-amp amplifier, it turns out that the noise amplitude is generally greater in the \( V_n \) input terminal than in the \( V_p \) input terminal (since the \( V_p \) input terminal is grounded), and thus common-mode gain is less of a problem in these circuits.
      - Thus, if noise reduction and signal-to-noise issues are of primary concern, inverting amplifiers should be used.

Gain-bandwidth product (GBP)

- Definition: Gain-bandwidth product (GBP), also called simply bandwidth is defined as:
  \[ \text{GBP} = G_{\text{theory}} \cdot f_c \]

  - \( G_{\text{theory}} \) is the theoretical gain of the op-amp amplifier as discussed previously, and given the symbol \( G \).
  - \( f_c \) is the internal cutoff frequency of the op-amp itself.
  - For a given op-amp, GBP is a constant – it is one of the specifications supplied by the op-amp manufacturer. The GBP of modern op-amps is typically on the order of 1.0 MHz, and is often listed as simply bandwidth.
  - The units of GBP are the same as those of frequency.
  - Without going into details, the inner circuitry of an op-amp acts like a first-order low-pass filter (with cutoff frequency \( f_c \)) when very high frequency inputs are applied.
So, the actual gain of the op-amp amplifier drops off at high frequencies, just like a low-pass filter, as sketched above for the case in which $G_{\text{theory}}$ is 1.

The range of frequencies from 0 Hz (DC) to internal cutoff frequency $f_c$ is called the bandwidth. In some electronics literature, the bandwidth is defined simply as $f_c$ itself, $\text{bandwidth} = f_c$ when $G_{\text{theory}} = 1$.

Equations for the internal low-pass filtering effects of the op-amp are the same as those for first-order low-pass Butterworth filters except that $f_c$ rather than $f_{\text{cutoff}}$ is the cutoff frequency. E.g., the gain $G_{\text{GBP}}$ due to internal low-pass filtering effects is $G_{\text{GBP}} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$, where $f$ is the input signal frequency.

GBP effects are related to the op-amp’s slew rate, defined as the rate of change in output voltage when the input is a step change of voltage. The units of slew rate are typically volts per microsecond (V/µs).

**Analysis:**
- We now compare GBP effects for inverting and noninverting op-amp amplifiers.

**Noninverting op-amp amplifier:**
- The theoretical gain (no GBP effects) of a noninverting op-amp amplifier is $G_{\text{theory}} = \frac{1 + R_2}{R_1}$.
- We define $GBP_{\text{noninverting}}$ as the noninverting gain-bandwidth product of the op-amp. Note: This is the same as the GBP value supplied by the op-amp manufacturer in their list of specifications.
- The equation for GBP is written as $GBP_{\text{noninverting}} = G_{\text{theory}} \cdot f_c$, from which the internal cutoff frequency can be calculated (for known values of $GBP_{\text{noninverting}}$ and $G_{\text{theory}}$).
- The actual gain of a noninverting op-amp amplifier is thus less than the theoretical gain, namely,
  $$G = G_{\text{theory}} \cdot G_{\text{GBP, noninverting}} = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$
  where $f_c = \frac{GBP_{\text{noninverting}}}{G_{\text{theory}}}$.

  Consider for example a 741 op-amp with $GBP_{\text{noninverting}} = GBP_{\text{manufacturer spec}} = 1.0$ MHz. For a theoretical gain of 10, the internal cutoff frequency of the op-amp is thus $f_c = \frac{GBP_{\text{noninverting}}}{G_{\text{theory}}} = \frac{(1.0 \text{ MHz})/10}{0.10 \text{ MHz}} = 100,000$ Hz. But $f_c$ is not a constant – it depends on $G_{\text{theory}}$.

  As we increase the theoretical gain, $f_c$ must decrease. For this example op-amp, when $G_{\text{theory}} = 100$, $f_c = 10,000$ Hz, and when $G_{\text{theory}} = 1000$, $f_c = 1,000$ Hz. In other words, the larger the theoretical gain of the amplifier, the lower the internal cutoff frequency (bandwidth) of the amplifier.

**Inverting op-amp amplifier:**
- The theoretical gain (no GBP effects) of an inverting op-amp amplifier is $G_{\text{theory}} = -\frac{R_2}{R_1}$.
- We define $GBP_{\text{inverting}}$ as the inverting gain-bandwidth product of the op-amp (a negative quantity), $GBP_{\text{inverting}} = -\frac{R_2}{R_1 + R_2} \cdot GBP_{\text{noninverting}}$. Note: This is not the same as the GBP value supplied by the op-amp manufacturer in their list of specifications.
- The equation for GBP is written as $GBP_{\text{inverting}} = G_{\text{theory}} \cdot f_c$, from which the internal cutoff frequency is determined (for known values of $GBP_{\text{inverting}}$ and $G_{\text{theory}}$).
- The actual gain of an inverting op-amp amplifier is thus less than the theoretical gain, namely,
  $$G = G_{\text{theory}} \cdot G_{\text{GBP, inverting}} = -\frac{R_2}{R_1} \cdot \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}}$$
  where $f_c = \frac{GBP_{\text{inverting}}}{G_{\text{theory}}}$.

  As with a noninverting op-amp amplifier, when we increase the theoretical gain of an inverting op-amp amplifier, $f_c$ must decrease. Again, as above, the larger the theoretical gain of the amplifier, the lower the internal cutoff frequency (bandwidth) of the amplifier.

- For either case (inverting or noninverting amplifier), we see, therefore, that it is futile to try to amplify a very high frequency signal by a very large gain using only one op-amp – the internal low-pass filtering effects quickly overpower the theoretical gain of the amplifier.
- Solution? Use two (or more) op-amp amplifiers in series. Some examples will be shown in class.