Today, we will:
- Do some review example problems – hypothesis testing (one sample)
- Review the pdf module: **Two Samples Hypothesis Testing** and do some examples

**Example: Hypothesis testing**

**Given:** A manufacturer claims that a plastic part is *at least* 6.00 cm long. You test the claim by performing a hypothesis test. You pick 30 parts at random from the assembly line, and carefully measure the length of each one. You calculate \( \overline{x} = 6.053 \) cm and \( S = 0.104 \) cm.

**To do:** To what confidence level (%) can we claim that the manufacturer’s claim is true?

**Solution:**

- **This is a one-sided hypothesis test since we see “at least”** (or one-tailed)
- Null hypotheses → set \( H_0 = \) manufacturer claim = 6.00 cm
- Set “side” to the least likely scenario
  
  **Here, since our \( \overline{x} > H_0 \) the least likely scenario is that \( M < H_0 \)**

  
  **[The alternative or research hypothesis is the opposite, i.e. that \( M > H_0 \)]**

- Calculate the critical t statistic:
  \[
  t_{crit} = \frac{\overline{x} - H_0}{S/\sqrt{n}} = \frac{6.053 - 6.00}{0.104/\sqrt{30}}
  \]
  \[
  t_{crit} = 2.79128
  \]

- Calculate the \( p \)-value @ \( t = 2.79128 \)

  
  df = 30 - 1 = 29
  
  \# tails = 1

- **Table (we df = 19 table)**
  \[ p = 0.004595 \] (after interpolation)

  
  OR, use Excel \( = T.DIST(2.79128, 29, 1) \) = \( p = 0.004595 \)

- **Interpret result:** We are 0.4595% confident that the null hyp. is true.

  
  **ACCEP**

  - We are 100 - 0.4595 = 99.5% confident that the manufacturer’s claim is true, i.e. that \( M > 6.053 \) cm.
"We would write this in a report as:

"We accept the manufacturer’s claim to 99.5% confidence"
or, "We accept the manufacturer’s claim (p = 0.0046)"

**ADDITIONAL QUESTION**: WHAT MANUFACTURER’S CLAIM WOULD GIVE US EXACTLY 95% CONFIDENCE?

**SOLUTION**: METHOD A → TRIAL & ERROR → KEEP GUESSING \( M_0 \)
until \( p = 0.0500 \)

METHOD B → DIRECT CALCULATION, WORKING BACKWARDS

For 95% confidence with **one tail**, \( \alpha/2 = 0.05 \), \( \Rightarrow p = 0.05 \)

\[ \text{90\% confidence} \quad \text{Area} = p\text{-value} = 0.05 \]

\[ \text{Set } \alpha = 0.1, \quad \alpha/2 = 0.05, \quad c = 0.90 \]

Table of critical \( t \) values → @ 90% conf., \( \Rightarrow df = 29 \), get \( t_{\alpha/2} = t_{\text{crit}} = 1.6991 \)

Now, since we know \( t_{\text{crit}} \) or \( t_{\alpha/2} \), we work "backward":

\[ t_{\alpha/2} = \frac{X - M_0}{S/\sqrt{n}} - M_0 = \bar{X} - t_{\alpha/2} \frac{S}{\sqrt{n}} \]

\[ = 6.053 - 1.6991 \frac{0.104}{\sqrt{30}} = 6.0207 \]

**BOTTOM LINE**:

A manufacturer’s claim of \( M_0 = 6.02 \) cm yields 95% confidence
or, we are 95% confident that \( \mu > 6.02 \) cm
Example: Hypothesis testing

Given: We buy a gadget that is supposed to increase the gas mileage of our car. We take 6 trips without the gadget and 6 (nearly identical) trips with the gadget. The results:

<table>
<thead>
<tr>
<th>$x_A$ (mpg without gadget)</th>
<th>$x_B$ (mpg with gadget)</th>
<th>$\bar{\delta} = x_B - x_A$ (mpg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.6</td>
<td>26.2</td>
<td>0.6</td>
</tr>
<tr>
<td>27.3</td>
<td>27.1</td>
<td>-0.2</td>
</tr>
<tr>
<td>24.2</td>
<td>24.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>28.7</td>
<td>29.2</td>
<td>0.5</td>
</tr>
<tr>
<td>23.6</td>
<td>24.5</td>
<td>0.9</td>
</tr>
<tr>
<td>25.1</td>
<td>24.9</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

To do: Determine if there is a statistically significant improvement (increase) in gas mileage.

Solution:

- This is a paired samples hypothesis test ($n=6$ for both A & B)

  - Calculate sample means $\bar{x}_A = 25.75$ mpg $\bar{x}_B = 26.00$ mpg

  - Calculate $\bar{\delta} = x_B - x_A$ for each run (see table)

  - Now use $\bar{\delta}$ in our hypothesis as a one-tail hypothesis test, but with $\delta$ as our variable instead of $x$.

  - Calculate $\bar{\delta} = 0.25$ (same as $x_B - x_A$ by the way = Sample mean of $\delta$)

  - Sample standard deviation of the six $\delta$ values $\bar{\delta}_S = 0.4764$

  - Null hypothesis $\rightarrow M_0 = 0$, i.e. the "side" is set to the least likely scenario. Since our $\bar{\delta} > 0$ (improvement in mpg)

    - We set null hyp. to $M < M_0$ or $M = 0$ here

    - Our null hypothesis is set to the least likely scenario, which is that gas mileage decreases rather than increases (thinking like a statistician)

  - $t$ statistic $\rightarrow t = \frac{\bar{\delta} - M_0}{\bar{\delta}_S/\sqrt{n}} = \frac{0.25 - 0}{0.4764/\sqrt{6}} = 1.285 = t$ statistic
[Note, we use \( S \) instead of \( X \), but the procedure is otherwise the same as before]

1. **p-value** → \[
\text{We do not have a table for } df = 5, \text{ so either look up in a table in a statistics book or we Excel}
\]

\[
\text{Excel} \rightarrow p = TDIST \left( |t\text{-statistic}|, df, \# \text{ tails} \right)
\]

\[
= TDIST \left( 1.285, 5, 1 \right)
\]

[absolute value since \( t\text{-pdf} \) is symmetric and Excel does not work if \( t \) is negative]

We get \( p\text{-value} = 0.1275 \approx 12.75\% \)

2. **INTERPRETATION**:

   There is a 12.75\% probability that the null hypothesis (the least likely scenario that mpg get worse) is true OR We are 87.25\% confident that the gadget improves mpg

3. **CONCLUSION**:

   Since 87.25\% < 95\% (i.e. since \( p = 0.1275 > 0.05 \)),

   \[\text{We cannot accept the claim that this gadget improves gas mileage to at least 95\% confidence}\]

**Bottom line** → I would not waste my money on this gadget

**Note**: Statistically speaking, we cannot reject the claim, but we also cannot accept it. We should do more testing
Example: Hypothesis testing

Given: [Continuation of previous example] We buy a gadget that is supposed to increase the gas mileage of our car. We take 6 trips without the gadget and 8 trips with the gadget. We do not attempt to pair up the tests. The results:

<table>
<thead>
<tr>
<th>$x_A$ (mpg without gadget)</th>
<th>$x_B$ (mpg with gadget)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.6</td>
<td>26.2</td>
</tr>
<tr>
<td>27.3</td>
<td>27.1</td>
</tr>
<tr>
<td>24.2</td>
<td>24.1</td>
</tr>
<tr>
<td>28.7</td>
<td>29.2</td>
</tr>
<tr>
<td>23.6</td>
<td>24.5</td>
</tr>
<tr>
<td>25.1</td>
<td>24.9</td>
</tr>
<tr>
<td>26.5</td>
<td>25.8</td>
</tr>
</tbody>
</table>

To do: Determine if there is a statistically significant improvement (increase) in gas mileage.

Solution: Now since $n_A \neq n_B$, this is not a paired-sample test. This is a hypothesis test with two independent samples.

Procedure:

1. Calculate statistics $\rightarrow n_A = 6, n_B = 8$
   $\overline{x}_A = 25.75, \overline{x}_B = 26.0375$
   $s_A = 1.9274, s_B = 1.6405$

2. Null hypothesis $\rightarrow$ Since $\overline{x}_B > \overline{x}_A$, we set the null hypothesis to
   The least likely scenario, i.e.,
   $H_0: \overline{x}_B \geq \overline{x}_A$

3. Critical t statistic $\rightarrow t = \frac{\overline{x}_A - \overline{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} = \frac{-0.2941}{10} = -0.2941 = t$

4. Use Welch's eq. to get df $\rightarrow df = 10$

5. Calculate p-value @ $|t| = 0.2941$ in Excel,
   $p = TDIST (0.2941, 10, 1) = 0.3873$
For \( p = 0.3873 \), we interpret:

- There is a 38.7% probability that the null hypothesis is true. In other words, there is a 38.7% probability that the gas mileage actually decreases due to this gadget.

OR

- We are 61.3% confident that the manufacturer's claim is true (i.e., that the gadget improves mpg).

**Conclusion** - Since 61.3% < 95%, we cannot accept the manufacturer's claim.

[Again, we cannot reject the claim, but we cannot accept it.]

In class, I will show how to do this problem in Excel, using the macro called "t-test: Two Sample Assuming Unequal Variances"