Today, we will:

- Discuss additional items about strain gages that are not in the pdf notes: **temperature compensation, transverse strain, strain on arbitrary surfaces**
- Discuss the learning module **strain gage rosettes**
- Do some review example problems – stress, strain, and strain gages

**Example: Strain gages and temperature compensation**

**Given:** A quarter-bridge strain gage circuit is constructed using $R_3$ as the strain gage, as sketched. The beam being measured is located a short distance from the bridge circuit. Unfortunately, the temperature at the location of the experiment fluctuates a lot, and the resistance of the strain gage is also quite sensitive to temperature. **We get a false reading when the temperature changes** – a change in temperature appears (falsely) like a change in strain, even if the actual strain on the beam is not changing.

\[
\frac{V_o}{V_s} \approx \left( \frac{R_{2,\text{initial}} + R_{3,\text{initial}}}{R_{2,\text{initial}} + R_{3,\text{initial}}} \right)^2 \left( \frac{\delta R_1}{R_{1,\text{initial}}} - \frac{\delta R_2}{R_{2,\text{initial}}} + \frac{\delta R_3}{R_{3,\text{initial}}} - \frac{\delta R_4}{R_{4,\text{initial}}} \right)
\]

**To do:** Figure out a way to eliminate the temperature effect.

**Solution:** Use a “dummy” gage, of opposite sign, as a temperature compensating gage.

Recall the approximate equation for $V_o$ when all four resistors of the Wheatstone bridge have small changes in resistance:

\[
V_o \approx \frac{R_{2,\text{initial}} R_{3,\text{initial}}}{(R_{2,\text{initial}} + R_{3,\text{initial}})^2} \left( \frac{\delta R_1}{R_{1,\text{initial}}} - \frac{\delta R_2}{R_{2,\text{initial}}} + \frac{\delta R_3}{R_{3,\text{initial}}} - \frac{\delta R_4}{R_{4,\text{initial}}} \right)
\]

Pick $R_2$ (or $R_4$) for temperature compensation, since $\delta R_2$ and $\delta R_4$ have opposite signs.

**Mount strain gage for $R_2$ on a part of the beam or structure that is exposed to the same temperature as $R_3$, but is not exposed to strain.**

It is a “dummy” strain gage.

Since $\delta R_2$ and $\delta R_3$ have opposite signs, changes in $T$ cancel out! Clever!!
Example: Strain gages

Given: A Wheatstone bridge circuit is constructed to measure strain in a component of a truss beam on a bridge.
- All resistors and strain gages are nominally 120 ohms.
- The strain gage factor is 2.05.
- The supply voltage to the bridge is 6.00 V.
- With no load, the bridge is balanced \( V_o = 0 \).
- An axial strain of 350 \( \mu \)strain is applied such that the strain gage is in tension.

(a) To do: Calculate the output voltage in mV when resistor 2 is the strain gage.

(b) To do: Calculate the output voltage in mV when resistors 1 and 2 are the strain gages, and both strain gages are in tension with \( \varepsilon_a = 350 \mu \)strain.

(c) To do: Calculate the output voltage in mV when resistors 1 and 3 are the strain gages, and both strain gages are in tension with \( \varepsilon_a = 350 \mu \)strain.

Solution:

(a) Recall, \( R_1 \) and \( R_3 \) are the “positive” resistors

\( R_2 \) and \( R_4 \) are the “negative” resistors

Here, \( R_2 \) is the strain gage

\[ \text{So, } \varepsilon_a = - \frac{V_o}{V_s} \frac{4}{S} \quad n=1 \text{ for a quarter bridge} \]

\[ V_o = - \frac{V_s \varepsilon_a S}{4} \]

Number:

\[ V_o = - \left( 6.00V \times 350 \times 10^{-6} \right) \left( 2.05 \right) \left( \frac{1000 \text{ mV}}{V} \right) = -1.07625 \text{ mV} \]

\[ V_o \approx -1.08 \text{ mV} \]

(b) \( R_1 \) (+) and \( R_2 \) (-) are of opposite signs \( \iff \)

They cancel each other out!

\[ V_o = 0 \]

(c) \( R_1 \) (+) and \( R_3 \) (+) are of the same sign, so the voltage adds up

(If identical strain gages both exposed to the same strain \( V_o \) doubles)

Also, \( R_1 \) and \( R_3 \) are positive, so

\[ \varepsilon_a = + \frac{V_o}{V_s} \frac{4}{S} \quad n=2 \text{ for half bridge} \]

\[ V_o = \frac{V_s \varepsilon_a S}{2} \]

We have doubled the sensitivity \( \iff \)

\[ V_o = 2.15 \text{ mV} \]

(Sensitivity = \frac{\Delta \text{output}}{\Delta \text{input}})
Tranverse Strain: Consider a hanging beam, of dimensions $L$ and $W$, with thickness $t$.

Equations: \[ \varepsilon_a = \frac{\delta L}{L} = \text{axial strain} \]

- Axial strain $\varepsilon_a$ is in one direction.
- However, strain in one direction leads to strain of the opposite sign in other directions.
- We call this transverse strain.

\[ \varepsilon_t = \frac{\delta W}{W} \quad (<0) \]

- Here, $\varepsilon_a > 0$, so $\varepsilon_t < 0$

- Define transverse strain $= 90^\circ$ to axial strain $\rightarrow \varepsilon_t = \text{transverse strain}$

- Define Poisson's Ratio $\nu = \frac{-\varepsilon_t}{\varepsilon_a}$

Poisson's Ratio is a property of the material.

\[ \nu = \frac{1}{\varepsilon} \rightarrow \text{we define it with a sign so that } \nu \text{ is positive} \]

- For most metals, $\nu = \frac{1}{4}$ to $\frac{1}{3}$ is dimensionless

\[ \nu = \frac{1}{4} \text{ for most isotropic materials (no preferred direction)} \]

E.g., wood has grain and is not isotropic.
Example: Strain gages and transverse strain

Given: A 2.8 cm × 5.0 cm rectangular rod is stretched from its initial length of 0.4000 m to a length of 0.4005 m.

- The modulus of elasticity of the rod material is 95.0 GPa (gigapascals).
- Poisson’s ratio of the rod material is 0.333.

(a) To do: Calculate the axial stress in units of MPa.
(b) To do: Calculate the transverse strain in units of microstrain.
(c) To do: A strain gage with a strain gage factor of 2.10 is glued to the rod before it is stretched, aligned with the direction of stretching. A quarter bridge Wheatstone bridge circuit is constructed, with the strain gage as resistor $R_1$. The strain gage is balanced before the rod is stretched. The bridge supply voltage is 7.50 V. Calculate the output voltage (in mV) of the bridge after the rod is stretched.

Solution:

(a) By definition, $\varepsilon_a$ = axial strain $= \frac{\Delta L}{L} = \frac{(0.4005 - 0.4000) m}{0.4000 m} = \frac{0.0005}{0.4000} = 0.00125 = \varepsilon_a$ in [1250 μstrain]

Axial Stress $\sigma_a = E \varepsilon_a = (95.0 \times 10^9 \text{ Pa}) (0.00125) (\frac{1 \text{ MPa}}{10^6 \text{ Pa}}) = 118.75 \text{ MPa}$

Answer: $\sigma_a = 118 \text{ MPa}$

(b) $\varepsilon_t$ = transverse strain $= -\nu \varepsilon_a = -0.333 (0.00125) = -0.00041625 = \varepsilon_t$

or, $\times 10^6 \Rightarrow \varepsilon_t = -416 \mu\text{strain}$

[Note: $\sigma_t = 0$ since we are not applying any stress in the transverse direction]

(c) The strain gage is in tension (stretched) when F is applied as sketched.

$R_1$ is one of the Gage resistors in the bridge circuit [recall, $R_1$, $R_3$ are $\Theta$ve, $R_2$ and $R_4$ are $\Theta$ve]

$V_o = \frac{n}{4} \varepsilon_a S V_S = \frac{1}{4} (0.00125) (2.10) (7.50 V) (\frac{1000 \text{ mV}}{V}) = 4.9219 \text{ mV}$

Answer: $V_o = 4.92 \text{ mV}$
**STRESS AND STRAIN ON A SURFACE**

- Consider an element on the surface of a material that has two normal stresses applied \( \sigma_x \) and \( \sigma_y \) [Principal Stresses]

![Diagram of stress elements]

- How do we calculate the strains \( \varepsilon_x \) and \( \varepsilon_y \)?

At first glance, we might say \( \varepsilon_x = \frac{\sigma_x}{E} \) and \( \varepsilon_y = \frac{\sigma_y}{E} \), but no! There are also transverse strains, so the solution is more involved than this.

**ANALYSIS:** Let’s pretend that stress \( \sigma_x \) occurs first by itself.

**STAGE 1:**

- We analyze for stage 1 (subscript 1) in the \( x \) direction (positive for tension as shown).

  \[ \varepsilon_{x1} = \frac{\sigma_x}{E} \]

- But there will also be transverse strain \( \varepsilon_{y1} = -\nu \frac{\sigma_x}{E} \) (negative because it shrinks in the \( y \) direction).

Recall, \( E = \) Young's modulus, \( \nu = \) Poisson's ratio. Assume there are known material properties.

**STAGE 2:** Now add stress \( \sigma_y \) in the \( y \) direction.

- Here, \( \varepsilon_{y2} = \frac{\sigma_y}{E} \) (positive in \( y \)-direction).

- Transverse: \( \varepsilon_{x2} = -\nu \frac{\sigma_y}{E} \) (negative in \( x \)-direction).
Combining these (since strain adds linearly),

\[ \varepsilon_x = \varepsilon_{x1} + \varepsilon_{x2} = \frac{S_x}{E} - \nu \frac{S_y}{E} \rightarrow \varepsilon_x = \frac{1}{E} (S_x - \nu S_y) \]  

\[ \varepsilon_y = \varepsilon_{y1} + \varepsilon_{y2} = \frac{S_y}{E} - \nu \frac{S_x}{E} \rightarrow \varepsilon_y = \frac{1}{E} (S_y - \nu S_x) \]  

Bottom line:
- \( \varepsilon_x \) is smaller than \( \frac{S_x}{E} \) because of the effect of \( \varepsilon_y \).
- \( \varepsilon_y \) is smaller than \( \frac{S_y}{E} \) because of the effect of \( \varepsilon_x \).

(assuming both \( S_x \) and \( S_y \) are positive, as sketched in our analysis)

What about an arbitrary surface in which we do not know beforehand which are the directions of principal stresses or principal strains?

Here \( x \) and \( y \) are chosen for convenience, and may have nothing to do with the directions of principal stress or strain.

Note: For simple axial strain problems, we need only one strain gage, and we align it in the direction of the axial (principal) strain.

Align the strain gage in the direction of the applied stress.

This is automatically in the direction of principal strain, \( \varepsilon_a \).
But, for the general case, we don't know the directions of principal strain. It turns out that we need to measure a minimum of 3 strains in 3 directions along the surface in order to determine the strain field.

We use a strain gage rosette.

Example:

- Measures $\varepsilon_y$.
- Measures $\varepsilon_{45^\circ}$.
- Measures $\varepsilon_x$.

Three strain gages, mounted close together:

- In x direction
- In y direction
- @ 45° angle

[You can buy strain gage rosettes already made with all 3 on one backing]

To analyze this, use Mohr's circle and equations you learned in E. Mech. class — can determine principal strains & their directions.

[See learning module on website for pictures of strain gage rosettes]