Today, we will:

- Finish the pdf module: **Linear Velocity Measurement** and do some example problems
- Additional notes: **hydraulic jacks, tire gages, strain gage pressure cells**

### Pressure Measurement in Moving Fluids

We define three types of pressure used in moving fluids:

- **Stagnation pressure** $P_{stag} = \text{pressure at a stagnation point where the velocity is slowed down to zero nearly isentropically. This is the pressure at the nose (stagnation point) of a probe in the flow.**}

- **Static pressure** $P = \text{pressure that would be measured by an infinitesimal pressure sensor moving with the flow. This is the pressure upstream of a probe in the flow.**}

It turns out that the pressure at point 2 in the sketch below is approximately equal to the static pressure, since the velocity at point 2 is approximately equal to $V$ and the streamlines are straight (not curved, which leads to pressure changes) at point 2.

- **Dynamic pressure** $\rho V^2/2 = \text{difference between stagnation and static pressure} = P_{stag} - P$. This is the “extra” pressure that is felt at the stagnation point at the nose of a probe in the flow.

![Sketch of pressure measurement in moving fluids](image)

We combine these definitions in a practical application – measurement of velocity.

**Comments:**

- $P$ (static pressure) is the same $P$ we use in thermodynamics = the thermodynamic pressure of the fluid (the pressure moving with the fluid)
- “Static” is confusing → means that the probe is static with respect to the fluid, i.e., it is moving with the fluid.
- How to measure static pressure? — Imagine a small neutrally buoyant pressure sensor moving with the fluid, with a wireless transmitter that we monitor remotely [easy to imagine, but extremely hard to actually make]
- Fortunately, it turns out that the pressure at point 2 in the above sketch, after the flow has become parallel to the probe, is approximately equal to the static pressure since $V_2 \approx V$

$$V_1 = 0 \quad V_2 \approx V$$

$$P_1 = P_{stag} \quad P_2 \approx P_{static}$$

$$\rho V^2/2 = P_{stag} - P$$
Equations: \[ P_{\text{stagnation}} = P_{\text{static}} + \frac{1}{2} \rho V^2 \] (from Bernoulli eq.)

\[ P_{\text{stagnation}} = P_1 \text{ in our sketch} \]
\[ P_{\text{static}} = P_2 \text{ in our sketch} \]
\[ \frac{1}{2} \rho V^2 = \text{dynamic pressure} \]

So, \[ \frac{1}{2} \rho V^2 = P_{\text{stagnation}} - P \]
\[ = P_1 - P_2 \]

or, solving for \( V \),
\[ V = \sqrt{\frac{2(P_{\text{stagnation}} - P)}{\rho}} = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \]

\[ \text{Pitot formula} \]

So, if we can measure \( P_{\text{stagnation}} \) and \( P_{\text{static}} \) at \( \rho \) the same point in the flow, we can calculate \( V \)

* Pitot probe — measures only stagnation pressure
* Static probe — measures only static pressure
* Pitot-static probe — measures both stagnation and static pressure

Most useful
Example: Velocity measurement

Given: A Pitot-static probe is placed in an air jet to measure the air speed. The differential pressure is measured with a U-tube manometer that uses mercury ($\rho = 13,600$ kg/m$^3$) as the manometer fluid. The difference in column height between the two legs of the manometer is $h = 1.20$ cm. The air density is 1.204 kg/m$^3$.

To do: Calculate the air speed at the location of the Pitot-static probe.

Solution:

* $P_1 = P_{str}$
* $P_2 = P$ (static pressure)
* Manometer (hydraulic):
  $$P_1 - P_2 = P_{str} - P = (\rho_{Hg} - \rho_{air}) gh$$

* Pitot formula:
  $$V = \sqrt{\frac{2 \left( P_{str} - P \right)}{\rho_{air}}}$$
  **Use $\rho$ of the fluid, not $\rho$ manometer!!**
  (a common mistake)

or
  $$V = \sqrt{\frac{2 g \left( \rho_{Hg} - \rho_{air} \right) h}{\rho_{air}}}$$

$$= \sqrt{\frac{2 \left( 9.80 \text{ m/s}^2 \right) \left( 13600 - 1.204 \right) \text{ kg/m}^3 \cdot 0.0120 \text{ m}}{1.204 \text{ kg/m}^3}}$$

$$V = 51.6 \text{ m/s}$$
Example: Pressure measurement and hydrostatics

**Given:** A hydraulic jack is constructed with the large piston diameter equal to 25.4 cm (10 inch) and the small piston diameter equal to 0.635 cm (1/4 inch).

**To do:** How much weight (in lbf) can a person lift with the jack if he exerts a force of 20.0 lbf on the small piston? *Give your answer to three significant digits.*

**Solution:**

\[
\frac{F_2}{F_1} = \frac{A_1}{A_2} = \frac{\pi D_1^2}{\pi D_2^2} = \left(\frac{D_1}{D_2}\right)^2
\]

\[
F_2 = F_1 \left(\frac{D_1}{D_2}\right)^2
\]

**Numbers:**

\[
F_2 = (20.0 \text{ lbf}) \left(\frac{10 \text{ in}}{\frac{1}{4} \text{ in}}\right)^2 = 32,000 \text{ lbf}
\]

\[
F_2 = 32,000 \text{ lbf} \quad !! \quad \text{Huge!}
\]

**Comments:**

- Oil density does not matter.

- To conserve mass, piston 2 moves up much less than piston 1 moves down. \([\text{factor of} \, A_2/A_1]\)

\[\text{That is why you have to pump the jack handle so many times to move the car up by a small amount.}\]
Tire Pressure Gauge – How does it work?

- **Piston, area \( A \)**
- **Spring, with spring constant \( k \)**
- **Calibrated rod**

\[ P_{\text{in}} \]  
\[ \rightarrow \]  
use gage pressure

The key to understanding this is that the rod is **loose** (it is not attached to the piston).

- When \( P_{\text{in}} \) is applied, the piston moves out, forcing the rod to also move out.
- But since the rod is loose, it does not move back in when \( P_{\text{in}} \) is removed — it stays where it was and we read \( P_{\text{in}} \).

**Equation** — spring: \( F = kx = \text{linear} \)

**Piston:** \( F = P_{\text{in}, \text{gage}} \cdot A \) 

\[
P_{\text{in}, \text{gage}} \cdot A = kx \quad \Rightarrow \quad P_{\text{in}, \text{gage}} = \frac{kx}{A} = \text{linear}
\]

Then, after reading, you have to push the rod in manually.
Strain Gage Pressure Cell – How does it work?

- Applied gage pressure $P_g$
- Wall thickness $t$
- Outer diameter $D$
- Strain gage

$$ P_g = P_{in} - P_{atm} $$

*This is similar to the soda can / strain gage experiment we did in the lab*

*Recall from E. Mech. class, for a thin-walled cylinder under internal gage pressure $P_g$,

$$ \sigma_H = \text{hoop stress} = \frac{P_g D}{2t} $$

$$ \sigma_L = \text{longitudinal stress} = \frac{P_g D}{4t} $$

*Recall, at a surface,

$$ \sigma_x \quad \sigma_y \quad \sigma_z $$

*If $\sigma_x$ i. $\sigma_y$ are principal stresses then we had previously derived these relationships for principal strains:*

$$ \varepsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) $$

$$ \varepsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) $$

*Here, let’s define the directions as $X = L$ (longitudinal)*

$$ Y = H \quad (\text{hoop}) $$
Thus, \[ \varepsilon_H = \frac{1}{E} (\sigma_H - \nu \sigma_L) \] and \[ \varepsilon_L = \frac{1}{E} (\sigma_L - \nu \sigma_H) \]

- Plug in the equations for \( \sigma_H \) and \( \sigma_L \) above:

\[
\varepsilon_H = \frac{1}{E} \frac{P_g D}{t} \left( \frac{1}{2} - \frac{1}{4} \nu \right) = \frac{P_g D}{2E t} \left( 1 - \frac{1}{2} \right) = \varepsilon_H
\]

- To use this device, we measure \( \varepsilon_H \) (hoop strain), then we use Eq. (1) to calculate \( P_g \) (gage pressure being measured).

**Comments:**
- The above eq's are for one strain gage. We may use two strain gages to double the sensitivity (half bridge instead of quarter bridge).
- It is also common to add two "dummy" temperature-compensating strain gages that are mounted on the thin end plate(s) (negligible strain)
- [Now use a full bridge!]

\[ R_1 \parallel R_4 = \text{hoop strain gages} \]
- \( R_2 \parallel R_3 = \text{temperature-compensating strain gages (dummy gages)} \]
Finally, let's discuss the time response of a cylindrical pressure cell.

**Problem:** If we suddenly increase $P_{in}$, it can take some time to fill up the cylindrical can with air. *(must wait until the whole volume of air comes to equilibrium pressure inside the can)*

So → the time response is poor *(too slow)*

**Solution:** Add a "filler" → a solid plug (another smaller cylinder that fits inside the pressure cylinder, but does not touch the walls of the cylinder *(thus it does not affect the pressure or the strain gage reading)*).

This annular volume is much smaller than the volume of the whole cylinder. → Therefore the time response is greatly improved!

Another option is to flatten the cylinder, but then it must be calibrated since our hoop stress equations are no longer valid *(not round anymore)*.