Today, we will:

- Finish discussing the filling of tanks with VOCs
- Discuss gradient diffusion and the Reynolds analogy
- Do Candy Questions for Candy Friday

![Diagram of filling methods]

**Figure 4.3** Methods to fill vessels with liquids; (a) splash filling, (b) submerged filling, and (c) bottom filling (redrawn from AWMA Handbook on Air Pollution Control, 2000).
Re-filling a tank with a different liquid

\[ P_k = P_{v,k} \quad f = 1 \text{ for species } k \]

All of the vapors from species k get emitted

\[ \dot{m}_k = \frac{P_{v,k} M_k Q}{R_n T} \]

since \( f = 1 \) (displacement filling)

If we add a different chemical (j)

\[ \text{total emission} = \dot{m}_j + \dot{m}_k \]

depends on how we fill - determining f
Diffusion - movement of a gas from a high concentration to a low concentration.

Experiment:
- Wave of concentration
- Liquid VOC
- High conc.
- Low conc.

- **Gradient Diffusion**
  - Let $a =$ concentration of a property
  - One-D analysis $\rightarrow a = a(z)$

- Graph:
  - lower conc. of $a$
  - $\frac{da}{dz} = \text{gradient}$
  - $t = 0$
  - Gradient Diffusion
  - High concentration of $a$
As we diffuse \((t \uparrow)\), the gradient decreases.

Mathematically, let \(J_A\) = net amount of property \(A\) transported (diffused) per unit time per unit area in the \(t\) direction.

The one-dimensional diffusion Eq. for any property \(A\) \((a = \frac{A}{L^2})\):

\[
J_A = -b \frac{da}{dt}
\]

Dimension:

\[
\{J\} = \left\{ \frac{A}{t \cdot L^2} \right\}
\]

\[
\{a\} = \left\{ \frac{A}{L^2} \right\}
\]

\[
\{b\} = \left\{ \frac{L^3}{t} \right\}
\]

\[
\{b\} \approx \left\{ \frac{L^3}{t} \right\} \text{ for any property } A
\]

\[
b = \text{Diffusion coefficient} = \left[ \frac{m^2}{s} \right] \text{ for any property } A
\]
**Examples of the One-Dimensional Diffusion Equation**

<table>
<thead>
<tr>
<th>Property with a gradient</th>
<th>Amount diffused per area per time</th>
<th>Diffusion coefficient</th>
<th>1-D gradient diffusion equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A and a</strong> (the property with a gradient)</td>
<td><strong>(J_A)</strong> (the amount of (A) diffused per unit area per unit time)</td>
<td><strong>(b)</strong> (diffusion coefficient)</td>
<td><strong>(J_A = -b \frac{da}{dz})</strong></td>
</tr>
<tr>
<td><strong>A</strong> = energy = (mC_pT)</td>
<td><strong>(J_A = q = \text{heat flux})</strong></td>
<td><strong>(\kappa = \text{thermal diffusivity})</strong></td>
<td><strong>(q = -\kappa \frac{d}{dz}(\rho C_pT))</strong></td>
</tr>
<tr>
<td>(a = \frac{A}{V} = mC_pT = \rho C_pT)</td>
<td>(q = \text{rate of heat (energy) transfer per unit area})</td>
<td>(\kappa = \frac{k}{\rho C_p})</td>
<td>or <strong>(q = -k \frac{dT}{dz})</strong></td>
</tr>
<tr>
<td>So, we have a gradient of <strong>temperature</strong></td>
<td>{(q)} = {\text{energy} \over \text{area} \cdot \text{time}}</td>
<td>{(\kappa)} = {\text{length}^2 \over \text{time}}</td>
<td><strong>Heat diffusion equation</strong></td>
</tr>
<tr>
<td><strong>A</strong> = momentum = (mU)</td>
<td><strong>(J_A = -\tau = \text{shear stress})</strong></td>
<td><strong>(\nu = \text{kinematic viscosity})</strong></td>
<td><strong>(-\tau = -\nu \frac{d}{dz} (\rho U))</strong></td>
</tr>
<tr>
<td>(a = \frac{A}{V} = \rho U)</td>
<td>(-\tau = \text{rate of momentum transfer per unit area})</td>
<td>(\nu = \frac{\mu}{\rho})</td>
<td>or <strong>(-\tau = -\mu \frac{dU}{dz})</strong></td>
</tr>
<tr>
<td>So, we have a gradient of <strong>velocity</strong></td>
<td>{-(\tau)} = {\text{momentum} \over \text{area} \cdot \text{time}}</td>
<td>{(\nu)} = {\text{length}^2 \over \text{time}}</td>
<td><strong>Equation for 1-D shear stress</strong></td>
</tr>
<tr>
<td><strong>A</strong> = number of mols of species (j = n_j)</td>
<td><strong>(J_A = J_j = \text{molar flux})</strong></td>
<td><strong>(D_{aj} = \text{binary diffusion coefficient between air and species} j)</strong></td>
<td><strong>(J_j = -D_{aj} \frac{d}{dz} (c_{molar,j}))</strong></td>
</tr>
<tr>
<td>(a = \frac{n_j}{V} = c_{\text{molar},j})</td>
<td>(J_j = \text{rate of transfer of mols of species} j \text{ per unit area})</td>
<td>(D_{aj} = \text{binary diffusion coefficient between air and species} j)</td>
<td><strong>Fick’s law</strong></td>
</tr>
<tr>
<td>So, we have a gradient of <strong>species</strong> (mols, but we can also think of it as a gradient of species mass)</td>
<td>{(J_j)} = {\text{mols} \over \text{area} \cdot \text{time}}</td>
<td>{(D_{aj})} = {\text{length}^2 \over \text{time}}</td>
<td></td>
</tr>
</tbody>
</table>

**Multi step side by side**

\[
M_j J_j = -D_{aj} \frac{d c_j}{dz} \quad \text{Fick’s Law in terms of mol conc.}
\]
Nondimensional ratio of diffusion coefficients

For all 3 example above, \( \{b^3\} = \{L^2/T\} \)

\[
\begin{align*}
\text{property} & \quad \text{ratio} \quad \text{form ratio of above} \\
\text{energy} & \quad \frac{b}{K} \quad \text{from ratio of above} \\\n\text{momentum} & \quad \frac{L}{u} \\
\text{species} & \quad D_{ij}
\end{align*}
\]

\( S_c = \text{Schmidt} \# = \frac{\nu}{D_{ij}} \quad \text{ratio of momentum diffusion to species diffusion} \)

\( P_r = \text{Prandtl} \# = \frac{\nu}{K} \quad \text{ratio of momentum diff. to heat energy diff.} \)

\( Le = \text{Lewy} \# = \frac{K}{D_{ij}} \quad \text{ratio of heat energy diff. to species diff.} \)
**Reynolds Analogy** – Energy, momentum, and mass, all diffuse in similar fashion. Compare:

**Suddenly heated wall** \[ T = T_0 = 0^\circ C \text{ everywhere, then suddenly } T = T_i \text{ at the wall.} \]

\[
\begin{array}{c}
T_0 \quad T_i \quad T
\end{array}
\]

- Energy is diffused
  - Rate of diff depends on \( K \)

**Suddenly moving wall** \[ U = U_0 = 0 \text{ m/s everywhere, then suddenly } U = U_i \text{ at the wall.} \]

\[
\begin{array}{c}
U_0 \quad U_i \quad U
\end{array}
\]

- Momentum is diffused
  - Rate of diff depends on \( U \)

**Sudden removal of a membrane** \[ c_{\text{molar}} = c_{\text{molar},0} = 0 \text{ mol/m}^3 \text{ everywhere, then suddenly } c_{\text{molar}} = c_{\text{molar},i} \text{ at the location of the membrane, and the membrane disappears suddenly}. \]

\[
\begin{array}{c}
c_{\text{molar},0} \quad c_{\text{molar},i} \quad c_{\text{molar}}
\end{array}
\]

- Mass (or species) is diffused
  - Rate depends on \( D_{ij} \)
Reynolds Analogy → all 3 properties diffuse in similar fashion

**Laminar Diffusion**

- Slow diffusion
- Depend on $\frac{L^2}{\nu} \text{ or } K$

**Turbulent Diffusion**

- Fast diffusion

Reynolds → in turbulent diffusion, all properties diffuse at approx. the same rate

Turb. diffusion → define $K_t \gg K$

\[
\begin{align*}
K_t \sim U_t \sim D_{ij, t} \\
\nu_t \gg \nu \\
D_{ij, t} \gg D_{ij}
\end{align*}
\]

$Pr_t \approx Sc_t \approx Le_t \approx 1$ → Exam 1 material ends here