Today, we will:

- Discuss **Dilution Ventilation with Unsteady Properties**
- Do some example problems – unsteady dilution ventilation
- Discuss **Removal by Solid Surfaces (Adsorption)**
- Do Candy Questions for Candy Friday

\* **WELL-MIXED MODEL WITH NON-CONSTANT COEFFICIENTS**

\( (\text{Sec. 5.4}) \)

\[ \frac{dC(t)}{dt} = Q(t) C_a(t) + S(t) - Q(t) C(t) \]  

\( Q, C_a, S, \) are functions of time, not constant

In general, cannot find an analytical solution for 1st order ODE with non-constant coefficients
Two types of problem:

1. If the parameters change in piece-wise fashion

   We can solve analytically for each piece.

   ![Graph showing piece-wise function](image)

   This is same as we did for $mbb$

2. If parameters are general function of time

   ![Graph showing oscillatory function](image)

   Solve numerically, e.g., **Runge-Kutta**
Example (Example 5.4 in text – Conference Room with 100% Make-up Air)

**Given:** A conference room of volume 33.3 m$^3$ contains 6 people who smoke at irregular times as follows, over a 90 minute period:

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>number of smokers ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; t &lt; 10$</td>
<td>2</td>
</tr>
<tr>
<td>$10 &lt; t &lt; 20$</td>
<td>4</td>
</tr>
<tr>
<td>$20 &lt; t &lt; 30$</td>
<td>0</td>
</tr>
<tr>
<td>$30 &lt; t &lt; 40$</td>
<td>6</td>
</tr>
<tr>
<td>$40 &lt; t &lt; 50$</td>
<td>4</td>
</tr>
<tr>
<td>$50 &lt; t &lt; 90$</td>
<td>0</td>
</tr>
</tbody>
</table>

Assume that the smoke particles are emitted at a rate of 1100 µg/min while each cigarette is smoked (Repase and Lowery, 1980). A ventilation system removes air from the room at a rate of 6.3 m$^3$/min and replaces it with ambient air at the same rate. Such a condition corresponds to 100% make-up air and 0% recirculated air. The smoke concentration in the ambient air and the initial smoke concentration in the room are both 20. µg/m$^3$.

**To do:** Assuming well-mixed conditions, compute and plot smoke concentration as a function of time for the 90-minute period. Also compute the maximum concentration and the average concentration. Compare the average concentration to the ACGIH standard of 3 mg/m$^3$ (3000 µg/m$^3$) for “not otherwise classified” (NOC) respirable particles (particles with diameter $D_p$ less than about 2 or 3 µm).

**Solution:** I used Excel, splitting the time into six different time periods as per the given table (number of smokers). The Excel file for this problem is also available on the course website. The maximum smoke concentration is around 920 µg/m$^3$, and the average smoke concentration over the period is around 330 µg/m$^3$, both of which are within the ACGIH standard.
we expect:

(very analogous to m_{lb} problem previously)

See Excel HC_Example-5.4.Smoker.xls

Type (2) problem → I use R-K see Excel, Matlab files

Consider a "tee" in a duct

(2)

\[ Q_2 = 1 \text{ m}^3/\text{min} \]

\[ C_2 = 15 \text{ mg/m}^3 \]

(1) \[ Q_1 = 2 \text{ m}^3/\text{min} \]

\[ C_1 = 10 \text{ mg/m}^3 \]

(3) \[ Q_3 = Q_1 + Q_2 = 3 \text{ m}^3/\text{min} \]

\[ C_3 = ? \]

Solu: For the specie: \[ \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = C_1 Q_1 + C_2 Q_2 \]

(10)(2) + (1)(11) = 3f \[ \frac{\text{mg}}{\text{min}} \]
\[ C_2 = \frac{\dot{m}_3}{Q_2} = \frac{35 \text{ mg/min}}{3 \text{ m}^2/\text{min}} = 11.67 \frac{\text{mg}}{\text{m}^3} \]
Example (Example 5.5 in text – the Clever Outdoorsman)

**Given:** A man sleeps overnight in a cabin with volume $V = 32.65 \text{ m}^3$. The rate of “fresh” air entering the cabin is 0.30 air changes per hour. The concentration of CO in the outside “fresh” air is $c_a = 10 \text{ PPM (11.4 mg/m}^3)$. A kerosene space heater emits CO according to $S(t) = 1500 \left[ 1 + \sin(0.80 \pi t) \right]$ mg CO per hour, where $t$ is in hours.

**To do:** Calculate the concentration of CO in the cabin as a function of time.

**Solution:**

- **Type of Unsteady problem.** Use R-K

\[ Q = \frac{Q}{2h} = N \cdot \left( \frac{0.3 \text{ hr}^{-1}}{\text{air changes/hr}} \right) (32.65 \text{ m}^3) = 9.795 \text{ m}^3/\text{hr} \]

- $c(t=0) = c_a = 11.4 \text{ mg/m}^3$

- Integrate with R-K in Matlab & Excel (See website for file)

**Figure 2.41** Response to carbon monoxide as a function of concentration (PPM) and exposure time (hr). The OSHA 8-hr PEL is 35 PPM and the EPA Primary Air Quality Standard is 9 PPM (redrawn from Seinfeld, 1986).
Removal by solid surface (Sec 5.5)

- Wall, curtain, keys, etc. adsorb contaminants (e.g., cigarette smoke)

Terminology = “plate out” or “wall loss”

Sink (reduction) of the contaminant

Modified form

Cigarette smoke adsorbs into material (curtains, rugs, walls)

Desorption when no smoking occurs

Desorption is typically much slower than adsorption

(can take months)
Model for wall adsorption:

\[ \dot{M}_{\text{wall loss}} = k_w A_s C \]

- Mass of contaminant
- Initial contaminant concentration
- Wall area

or deposition velocity
or adsorption coeff.

For instance:

\[ k_w = \left( \frac{t}{L} \right) \quad [k_w] = \left[ \frac{M}{s} \right] \]

See text for empirical eqs for \( k_w \)

I will give \( k_w \) on exam, HW, etc.

Room eq. becomes [mass flow rate of species]

\[ \dot{A} \frac{dc}{dt} = Qc_a + \dot{S} - Qc - k_w A_s c \]

Put (1) into standard 1st order ODE form:
\[
\frac{dc}{dt} = \frac{S + QC_a}{A} - \frac{Q + kW A_s}{A} \\
\frac{dc}{dt} = \frac{B}{A} - A
\]

Same sol'n as before. If all coeff. are const.
\[
\frac{C_{sf} - C}{C_{sf} - C(0)} = e^{\exp(-At)}
\]
An. sol'n $A$
\[
C_{sf} = \frac{B}{A}
\]
\[
C_{sf} = \frac{S + QC_a}{Q + kW A_s}
\]

If $kw, Q, S, C_a$ are only of time, can use $R-K$ to solve.