Today, we will:

- Discuss elutriators, a practical example of air cleaning using the principle of laminar flow gravimetric settling in ducts
- Continue to discuss gravimetric settling in ducts – the well-mixed (turbulent) model
- Discuss inertial separation – particles in curved flow [Section 8.11]
- Discuss the analogy between gravimetric settling and inertial separation
- Discuss how to iterate to obtain the radial inertial separation speed in a curved duct

Practical Application – Horizontal Elutriator

A horizontal elutriator is a simple device that is sometimes used at the inlet to an APCS to remove some of the larger diameter particles from the air. It is often used in coal mines, for example. It consists of parallel horizontal plates, as sketched. Dirty air enters from the left at low air speed. As the air moves along, particles fall and settle on the plates. We assume that when the particles hit the plate, they stick there and remain on the plate – they are removed from the air. After some time, the device may get clogged, so it needs to be washed out. However, there are no moving parts, it does not require electricity, and it is simple and effective at removing particles from the air. Heavier and/or larger particles settle quickly, while lighter and/or smaller particles take longer to settle. Therefore we expect the removal efficiency of the device to depend on particle diameter and particle density.
GRADE EFFICIENCY

\[ \eta(D_p) = \text{removal efficiency of a cleaner as a fun. of } D_p \]

Eg. Horizontal elutriator

\[ L_c = \text{func}(D_p) \]

If \( L > L_c \) for a particular \( D_p \), \( \eta = 100\% \) for that \( D_p \)

If \( L \leq L_c \), \( \eta < 100\% \)

\[ \eta \propto \text{lin. in } L \]

\[ \eta(D_p) = \frac{L}{L_c} \]
Define $D_{p,\text{cut}}$ = dia at which $\eta(D_{p,\text{cut}}) = 50\%$

Thy way for laminar settling

- Well-mixed settling in ducts (turbulent)

$C_j(\text{in})$ increases w/ x but not w/ y

$C_j(x_i) < C_j(x_f)$
Assume particles stick to the wall when they hit the wall.

We expect:

1st-order ODE, exponential decay as previously.

WELL MIXED (one extreme)

LAMINAR (one extreme)

ACTUAL IS TYP. IN BETWEEN

SOLN FOR THE WELL-MIXED CASE

\[ \frac{C_j}{C_j(\text{in})} = \exp \left( -\frac{V_t}{UH} x \right) \]

(\text{at } x = L)

\[ L_c = \frac{UH}{V_t} \]

for laminar setting

So,

\[ \frac{C_j}{C_j(\text{in})} = \exp \left( -\frac{x}{L_c} \right) \]

\[ \eta = \text{removal efficiency} \]

\[ \eta = 1 - \frac{C_j}{C_j(\text{in})} \]

\[ \eta = 1 - \exp \left( -\frac{x}{L_c} \right) \]
Example: Removal Efficiency due to Well-Mixed Gravimetric Settling in a Duct

**Given:** Dusty air enters a horizontal duct of length $L = 14.4$ m and height $H = 6.0$ cm at average speed $U = 0.20$ m/s. Aerosol particles of a certain diameter $D_p$ under consideration have a terminal settling speed of $V_t = 0.00025$ m/s.

**To do:** Calculate the removal efficiency $\eta$ (sometimes use $E$) for these particles. Assume well-mixed (turbulent) settling, and assume that all particles that hit the floor of the duct remain there (they stick to the floor). Give your answer as a percentage to two significant digits.

**Solution:**

\[
\eta = 1 - \frac{C_f}{C_f(\text{in})} = 1 - \exp\left(\frac{-L}{L_c}\right)
\]

\[
L_c = \frac{HU}{V_t} = \text{some as previous problem}
\]

\[
L_c = \frac{(0.060 \text{ m}) (0.20 \text{ m/s})}{0.00025 \text{ m/s}} = 48.0 \text{ m} = L_c
\]

Here $L = 14.4$ m; $L < L_c$

Recall, for laminar case, $\eta = 30\%$ — **BEST CASE SCENARIO**

For well-mixed (turbulent)

\[
\eta = 1 - \exp\left(\frac{-14.4 \text{ m}}{48.0 \text{ m}}\right) = 0.25918
\]

$\eta \approx 26\%$ — **WORST CASE SCENARIO**
Duct $Re = \frac{\rho v U H}{\mu} = \frac{(1.184 \text{ kg/m}^3) (0.20 \text{ m})(0.060 \text{ m})}{1.849 \times 10^{-5} \text{ kg/s}}$

$Re = 768.4 < 2300$

Most likely laminar, but may be transitional.

**GRADE EFFICIENCY FOR THIS DUCT**

---

Graph showing efficiency $(\eta(D_p))$ vs. particle size $(D_p)$. The graph includes curves for laminar and turbulent flow, with a note indicating typical actual flow in between. The graph is on a log scale.
### INERTIAL SEPARATION

- e.g., curved duct

Assume:
- if particle hits outer wall, it sticks

\[ U_\theta = \text{speed of air (tangentially)} \]

\[ V \text{ is radial here} \]

Radial “settling” speed for inertial separation in a curved duct:

\[
V_r = \sqrt{\frac{4 \rho_p - \rho}{3 \rho} \frac{U_\theta^2}{D_p} \frac{C}{C_D}}
\]

Compare to gravimetric settling speed in quiescent air or in ducts (from a previous lecture):

\[
V_t = \sqrt{\frac{4 \rho_p - \rho}{3 \rho} \frac{g D_p C}{C_D}}
\]

- Same except
- replace \( V_t \) by \( V_r \)
- replace \( g \) by \( \frac{U_\theta^2}{r} \)
\textbf{DERIVATION}: We will skip all the details.

Bottom line → Same eqs as gravimetric settling, except replace $V_t$ with $V_r$.

\[ U_0^2 \] replace $g$ with $\frac{U_0^2}{L}$

(See eqs on previous pg)
Example: Comparison of Centrifugal and Gravitational Settling

Given: Dusty air enters a curved duct at average speed $U$. Aerosol particles of a certain diameter $D_p$ have a terminal settling speed of $V_t = 0.00025$ m/s in quiescent air. At the instant of time shown, a particle of diameter $D_p$ is at radius $r = 0.32$ m.

To do: Calculate the air speed $U$ such that the radial velocity $v_r$ of the particle is the same as its terminal settling velocity. Give your answer in m/s to three significant digits.

Solution:

Set $V_t = v_r$ when $\frac{U_0^2}{r} = g$

\[
U_0 = \sqrt{rg} = \sqrt{0.32 \text{ m} \times 9.80 \text{ m/s}^2}
\]

\[
U_0 = 1.77 \text{ m/s}
\]

Actual curved duct $U_0 \approx D_p$ much larger than this

e.g., $U_0 = 10$ m/s

\[
\frac{U_0^2}{r} = 312.5 \approx 32 \text{ g/s}
\]