Today, we will:
- Do some example problems with Lapple reverse flow cyclones
- Discuss how cyclone performance varies with size, flow rate, etc.
- Discuss particle collectors in series and parallel

Example: Lapple Cyclone

**Given:** A standard reverse flow Lapple cyclone is used to clean up a dusty air flow exhausted by a sanding machine in a wood shop. The main body diameter of the cyclone is \( D_2 = 45.0 \) cm (0.450 m).
- particle density \( \rho_p = 730 \text{ kg/m}^3 \)
- bulk volume flow rate of air \( Q = 0.55 \text{ m}^3/\text{s} \)
- Air is at STP: \( \rho = 1.184 \text{ kg/m}^3, \mu = 1.849 \times 10^{-5} \text{ kg/(m s)} \).

**To do:** Calculate the grade efficiency \( \eta(D_p) \) for 10-\( \mu \)m particles. Give your answer as a percentage to 3 significant digits.

**Solution:** Some equations:

\[
D_{p,\text{cut}} = \sqrt[3]{\frac{3\mu D_2^3}{128\pi Q(\rho_p - \rho)}}
\]

\[
\eta(D_p) = \frac{1}{1 + \left(\frac{D_p}{D_{p,\text{cut}}}\right)^2}
\]

For \( D_{p,\text{cut}} \) (where \( \eta(D_{p,\text{cut}}) = 50\% \)):

\[
D_{p,\text{cut}} = \sqrt[3]{\frac{3\times1.849 \times 10^{-5} \times 0.450^3}{128\pi \times 0.55 \times (730 - 1.184)}} = 5.6 \times 10^{-6} \text{ m} = 5.6 \mu\text{m}
\]

At \( D_p = 10 \mu\text{m} \):

\[
\eta(10 \mu\text{m}) = \frac{1}{1 + \left(\frac{10 \mu\text{m}}{5.6 \mu\text{m}}\right)^2} = 76.1\%
\]

I repeated for a range of \( D_p \):

grade efficiency curve \( \eta(D_p) \) vs \( D_p \).
Q: What if we increase $Q$?
   - $N$ $\uparrow$, $U_0$ $\uparrow$
   - $A$: $U_0$ $\uparrow$, accel. $\frac{U_0^2}{2}$ $\uparrow$

A: Smaller $D_{cut}$ "better"

Q: Why is this not always the best solution? (increase $Q$)
A: Pressure drop goes way up as $Q$ $\uparrow$

$\Delta P \propto Q^2$

$\Delta P = 40.96 \rho \left(\frac{Q}{Wh}\right)^2$

$W_{fan} = \text{power to run this cyclone}$

$W_{fan} \propto Q^3$  \(\Rightarrow\)  higher $Q$ costs a lot more to operate

$W_{fan} = (\frac{1}{\eta_{blower}}) \Delta P$
What happens if cyclone is larger or smaller?

For a given $U_b$, if $r_m \uparrow$, $\text{Accel} \uparrow$  

"better" performance if $r_m$ is smaller

But, $\Delta \tau \propto (\frac{Q}{WH})^2$

eg. if $W, H$ both decrease by a factor of 2

$\Delta \tau \uparrow$ by factor of $4^2 = 16$

$W_{fan} \propto \Delta \tau \Delta P$  

$W_{fan} \uparrow$ by a factor of 16  

@ same $Q$
Example: Design of a Lapple Cyclone

**Given:** Dusty air from a manufacturing plant needs to be cleaned before being exhausted to the environment. Here is what we know about the dusty air:
- the air is polydisperse, with a wide variety of particle sizes
- particle density $\rho_p = 1500 \text{ kg/m}^3$
- bulk volume flow rate of air $Q = 0.111 \text{ m}^3/\text{s}$
- the air is at STP: $\rho = 1.184 \text{ kg/m}^3, \mu = 1.849 \times 10^{-5} \text{ kg/(m s)}$

**To do:** Design a standard reverse flow Lapple cyclone to clean the air such that the removal efficiency of 2.5-μm particles is 80%. In particular, calculate dimension $D_2$, the diameter of the Lapple cyclone canister. Give your answer in meters to 3 significant digits.

**Solution:** Some equations:

\[
D_{p,cut} = \frac{3\mu D_2^3}{128\pi Q(\rho_p - \rho)}
\]

\[
\eta(D_p) = \frac{1}{1 + \left(\frac{D_p}{D_{p,cut}}\right)^2}
\]

1. **Equation for $D_{p,cut}$**

\[
D_{p,cut} = D_p \sqrt{\frac{1-\eta}{\eta}}
\]

2. **Equation for $D_2$**

\[
D_2 = \sqrt[3]{\frac{D_{p,cut}^2}{128\pi Q(\rho_p - \rho)} \frac{128\pi Q(\rho_p - \rho)}{3\mu}}
\]

How much dust will the cyclone remove in a year?

Assume $\eta_{\text{blow}} = 75\%$
Power required = \( \dot{W}_{\text{blower}} = \frac{1}{\eta_{\text{blower}}} \frac{Q \Delta P}{\dot{V}} \)

For Lapple, we know \( W = \frac{D_2}{y} \)

\( H = \frac{D_2}{2} \)

\( \dot{W}_{\text{blower}} = 2621.44 \ \text{W} \)

For air:

\[
\dot{W}_{\text{blower}} = 2621.44 \left( 1.184 \ \frac{\text{kJ}}{\text{m}^3} \right) \frac{1}{0.75} \left( 0.111 \ \frac{\text{m}^3}{\text{s}} \right)^3 \left( \frac{\text{N} \cdot \text{m}^2}{\text{kJ} \cdot \text{m}} \right) \left( \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} \right)
\]

\( = 24313.7 \ \text{W} \)

\( \dot{W}_{\text{blower}} = 24.3 \ \text{kW} \)

@ \( 10^4 \text{$/kW \cdot hr$} \) per yr ->

\[
\text{Cost} = \left( \frac{3600}{\text{kW \cdot hr}} \right) (24.3 \text{ kW}) (365.25 \text{ day}) \left( \frac{24 \text{ hr}}{\text{day}} \right) = 21,300 \ \text{per year}
\]

Let's try same problem, but with 4 cyclings in parallel
Same \( Q \) small, but \( Q = 0.111 \text{ m}^3/\text{s} \),

\[ \div 4 \text{ here, } Q \text{ per unit } = 0.02775 \text{ m}^3/\text{s} \]

Keep efficiency requirement the same — 80% \( e \) \( D_p = 2.5 \text{ \mu m} \)

\[ \text{redo calculation } \rightarrow D_2 = 0.0778 \text{ m} \]

This will have same performance
\[ W_{\text{blower per unit}} = 2621.44 \left( \frac{1}{n_{\text{blower}}} \right) \frac{Q^3}{D^4} \]
\[ = 2621.44 \left( \frac{1}{0.75} \right) \frac{(0.0277)^3}{(0.07781)^4} \]
\[ = 2412.3 \, \text{W} = 2.412 \, \text{kW} \]
for one unit

So total power = \( 4 \times 2.412 \, \text{kW} \)

Total cost = \$8460 per yr

compares to \$21,700 \ldots \ldots \!

For the same performance!

We often see cyclones in parallel!
We see cyclones in parallel in many applications.

Why:
1) Reduce operating cost for same $N$
2) Can get better $N$ for same operating cost
Example: Lapple Cyclones in Series and Parallel

**Given:** Dusty air is cleaned by one large Lapple cyclone in series with four smaller Lapple cyclones in parallel. Details:
- particle density \( \rho_p = 1500 \, \text{kg/m}^3 \)
- bulk flow rate of air \( Q = 0.111 \, \text{m}^3/\text{s} \)
- air at STP: \( \rho = 1.184 \, \text{kg/m}^3, \mu = 1.849 \times 10^{-5} \, \text{kg/(m s)} \)
- \( D_{p,\text{cut,1}} = 10 \, \text{microns}; \) \( D_{p,\text{cut,2}} = 2.5 \, \text{microns} \)

**To do:** Calculate the overall removal efficiency of 2.0-\( \mu \)m particles. Give your answer in percentage to 3 significant digits. Some equations are provided here for convenience.

Parallel:
\[
\eta(D_p)_{\text{overall}} = 1 - \sum_{j=1}^{m} f_j \left[1 - \eta(D_p)_j\right], \quad f_j = \frac{Q_j}{Q_{\text{total}}}
\]

Lapple:
\[
\eta(D_p) = \frac{1}{1 + \left(\frac{D_p}{D_{p,\text{cut}}}ight)^2}
\]

Series:
\[
\eta(D_p)_{\text{overall}} = 1 - \prod_{j=1}^{m} \left[1 - \eta(D_p)_j\right]
\]

**Solution:**

Same eqn for series + parallel is generally true for any \( D_{p,\text{cut}} \) but now they are grade efficiency \( \eta = \eta(D_p) \)

For unit:

\[
\begin{align*}
\eta_1(D_p) &= \frac{1}{1 + \left(\frac{2}{10}\right)^2} \\
&= 0.3846 \\
\eta_2(D_p) &= 0.3902
\end{align*}
\]

Overall:
\[
\eta(D_p)_{\text{overall}} = 1 - (1 - \eta_1(D_p))(1 - \eta_2(D_p)) = 41.4 \%
\]
Repeat for range of $D_p$

Grade Efficiency

41% @ 2 μm

Notice: Cleaner 2 does most of the cleaning! (it has a much smaller $D_p$ cut)