Chapter 7 Dislocations

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7.1 The Onset of Permanent Deformation

Like many new scientific ideas, the linear defect called a dislocation came about in order to explain the yawning discrepancy between then-current theory of the stress threshold for permanent deformation in solids and experimental observation. In common with a few other scientific phenomena, dislocations were conceived many years before they were actually observed.

Permanent deformation of a solid, also called plastic deformation, involves the slippage of one block of atoms over another block by a distance of roughly one atomic spacing. Movement occurs preferentially on certain atomic planes called slip planes and in particular directions called slip directions. Figure 7.1 illustrates the fundamental distinction between elastic and plastic deformation caused by an applied shear stress.

![Fig. 7.1 The distinction between elastic and plastic deformation due to an applied shear stress – for the simple cubic lattice structure](image)

Elastic strain tilts the entire array of vertical atom planes without altering the relative positions of the atoms. Plastic deformation occurs at a higher shear stress and causes a portion of the solid to move, or slip, from the normal array to another normal configuration offset by approximately one lattice constant. Except for the steps at the sides, the atomic structure in the plastically-deformed solid is perfect. An important distinction between the two types of deformation is their reversibility: when the stress is removed, the elastically-deformed body returns to its original shape; a plastically-deformed body does not – the deformation is irreversible.

Suppose that movement from left to right in Fig. 7.1 occurs by displacement of the entire block of atoms above the slip plane over the lower block in a single step. Such motion requires first an increasing system energy, reaching maximum when the vertical atomic planes in the upper block are exactly midway between planes of the lower block. Continued motion from this position to the final position on the right in Fig. 7.1 is accompanied by reduction in system energy, which ultimately returns to the value in the unstressed solid. Since force is the negative of the derivative of the energy, and stress is force per unit area, this variation in energy implies a critical value of the shear stress.
stress at which the deformation switches from elastic to plastic. The critical shear stress according to this mechanism, also called the theoretical shear strength, is readily derived (see Ref. 1, p. 210; Ref. 2, p. 226, Ref. 3, p. 83, Ref. 4, p. 175, Ref. 5, p 11, Ref. 6, p. 1); it is predicted to be on the order of 0.1G, where G is the shear modulus of the solid (Eq (6.26)). This prediction is obeyed only for very thin vapor-grown specimens called whiskers; for all other solids, including carefully-grown bulk single crystals, the "en bloc" model prediction is roughly four orders of magnitude larger than the value observed in, say, the standard tensile test. A discrepancy of this size naturally excites the imagination of theorists, three of whom independently invented the dislocation to rectify the disaccord (see Ref. 6, pp. 1 – 4 for this interesting bit of scientific history).

7.2 Dislocations

The Edge Dislocation

The theoretical notion of the microscopic entity called an edge dislocation has a macroscopic analog in the way a rug is moved. The upper left hand sketch of Fig. 7.2 suggests the considerable effort required to pull the entire rug over the floor; the next sketch shows that pushing a small wave-like section from one end to the other accomplishes the same displacement with a substantially reduced effort. The left hand rug-pulling picture is the analog of the en bloc model of the theoretical shear strength described in the preceding paragraph; the right hand counterpart is the analog of the edge dislocation depicted in the bottom part of the figure.

Fig. 7.2 The edge dislocation in the simple cubic lattice and the rug analogy

The applied shear stress causes the first column of atoms on the left and above the dashed horizontal plane to shift alignment from the first column below the plane to the second lower column, leaving a step on the left-hand face. This one-at-a-time switch
in column alignment between the upper and lower atomic rows continues until the entire upper portion of the crystal has moved to the right by one lattice constant. The net deformation of the body is the right hand structure in Fig. 7.2, which is identical to that in Fig. 7.1. The horizontal plane separating the intact, stationary lower block of atoms from the displaced upper block is called the *slip plane*. The movement of the dislocation along this plane is termed *slip* or *glide*.

The middle image in Fig. 7.2 shows the structural configuration midway in the realignment process. An extra half-plane of atoms appears to have been inserted in the crystal, with its bottom edge terminating on the slip plane. Of course, the apparent extra half plane is just one of the existing upper planes pushed to the right. The termination of the half plane is the edge dislocation line on the slip plane. It is represented as an upside-down tee, a convenient shorthand that eliminates the need for drawing all of the atoms in the structure when indicating the presence of the dislocation. If the extra half-plane had been inserted from the bottom of the diagram in Fig. 7.2, the tee would be drawn right side up. Such an edge dislocation would be of opposite sign as the one in Fig. 7.2; one is termed *positive* and the other *negative*.

The shear stress to maintain motion of an isolated dislocation in an absolutely pure crystal with no nearby dislocations is called the *Peierls stress*. Its value is estimated at ~ $10^{-8} \text{G}$ (Ref. 5, Sect. 10.2), which is several orders of magnitude lower than the measured stress need to sustain slip in annealed metals. The presence of other dislocations and of impurities ranging from atoms to large agglomerates is responsible for the actual stress required to cause slip in real crystals (Sect. 7.12).

**The Screw Dislocation**

The deformation produced by the edge dislocation in Fig. 7.2 can be reproduced by a fundamentally different mechanism termed the *screw dislocation*. This line defect is depicted in fig. 7.3, again for the simple cubic lattice but with the atoms represented by small cubes instead of spheres. If the sketches in Fig. 7.3 are rotated by $90^\circ$ and viewed on end instead of in perspective, the direction of the shear stress, the plane on which slip occurs, and the final deformed shape of the crystal are seen to be identical to those in Fig. 7.2. There is no satisfactory macroscopic analog of the shape change due to a screw dislocation; the closest would be halving a piece of paper by tearing instead of yanking apart.

The screw dislocation is shown as the dashed line in Fig. 7.3. Its name derives from the descending spiral experienced by tracing a circuit round the dislocation line following the original atom planes. When viewed end on, the screw dislocation is symbolized by $\mathbf{S}$. Like the edge dislocation, screw dislocations come in two types, right-hand and left-hand, depending on the direction of the spiral.

**The Burgers Vector**

A useful characterization of atomic distortions around a dislocation line is termed a *Burgers vector*. As its name suggests, it has both direction and magnitude. A simple
method of determining the Burgers vector is indicated by the circuits around the
dislocation lines indicated by the bold-line traces in Figs. 7.2 and 7.3. Starting at an
arbitrary atom and moving around the dislocation line by the same number of atoms left
and right and up and down does not end on the starting atom. The arrow joining the
starting and ending atoms following this circuit is the Burgers vector, denoted by \( \vec{b} \). Two
other directional quantities characterize a dislocation: the direction of movement \( \vec{M} \) and
the direction of the line proper, \( \vec{L} \). The two basic dislocation types can be succinctly
characterized by:

- **edge:** \( \vec{b} \perp \vec{L}; \ \vec{M} \parallel \vec{b}; \ \vec{M} \perp \vec{L}; \)

- **screw:** \( \vec{b} \parallel \vec{L}; \ \vec{M} \perp \vec{b}; \ \vec{M} \perp \vec{L}; \)

The important distinction is the first: the Burgers vector of an edge dislocation is
perpendicular to the line while that of a screw dislocation is parallel to the line.

The magnitude of the Burgers vector, \( b \), for the simple cubic lattice, whether
screw or edge, is equal to the lattice parameter \( a_0 \), the direction is [100], and the slip
plane in which it moves is (100). The notation for this Burgers vector is:

\[
\vec{b}_{sc} = a_0 [100](100)
\]

where the first term on the right is the magnitude of \( b \), the second is the direction of \( \vec{b} \),
and the last term designates the slip plane. The generalization of this shorthand for
other lattice structures will be covered shortly.
Mixed Dislocations – Loops

Figures 7.2 and 7.3 suggest that dislocations are straight lines running through the entire length of the crystal. While the latter inference is correct, the former is not; dislocations can be of any shape, and need not lie in a single slip plane. However, they cannot start or stop inside the crystal. The transmission electron microscope (TEM) image in Fig. 7.4 shows the complexity of the dislocation structure in a real metal.

Fig. 7.4 TEM image of the dislocation structure in a titanium alloy magnified 50,000 times. From Ref. 7

The dislocations (lines) are not straight, and appear to begin and/or end within the specimen. The latter is an artifact of the TEM; what appears to be a termination is in reality a change in the direction of the line out of the plane of focus of the TEM.

If a dislocation line is curved, or a straight segment is neither edge nor screw, the dislocation line is said to be mixed. This means that it is part edge and part screw. Atomic representations of a curved dislocation are not particularly informative, but are given in Ref. 1, p. 105, Ref. 2, p. 83, and Ref. 7, p. 130. The important feature of the mixed dislocation is that they possess a unique Burgers vector, as will be shown below for the loop.

Loops are closed curves whose periphery consists of a dislocation. The loop may or may not be planar, but here we consider only the plane version. They come in two varieties. In the first type, the interior of the loop consists of atoms that have been shifted by one Burgers vector, as in Figs. 7.2 and 7.3. The crystal structure at the interior plane of the loop is perfect; the distortion caused by the shift of atoms is accommodated by the peripheral dislocation. In the second type, the loop is formed by removal of a disk of atoms from a close-packed atomic plane or by addition of a disk of atoms between close-packed atomic planes. The periphery of these loops are also dislocations, although of a different type than those of the first type of loop. The first type of loop is called a shear loop because it is produced by the action of a shear stress on the slip plane.
The dislocation that forms the periphery of the shear loop is most easily understood by considering the loop to be a square rather than a circle on the slip plane. Figure 7.5 shows this square designated by the corners abcd and the shift of atoms inside the square indicated by the arrows above and below the square loop. The magnitude of the shift is one atomic spacing in a simple cubic structure or the magnitude of the Burgers vector for other crystal types.

The middle sketch shows the atomic structure where plane 1 cuts the segments ab and cd of the square. These are seen to be edge dislocations of opposite sign. Similarly, plane 2 cuts the loop along segments ad and bc. The right hand sketch shows that these segments represent screw dislocations, one right-handed, and the other left-handed. Even if the loop is circular, as in the bottom sketch, the pure edge and pure screw configurations are present only at points on the circle 90° apart. In between, the dislocation is mixed edge/screw, with the preponderant type changing from edge to screw and back as the circle is traversed.

A remarkable feature of the loop, whether it be square or circular, planar or nonplanar, is that it is characterized by a single Burgers vector. The Burgers vector is perpendicular to the pure edge portions of the loop and parallel to the pure screw components. Only the directions of $\vec{b}$ are reversed between the negative and positive edge portions and the right-hand and left-hand parts. When a shear stress is applied to the slip plane in the sense of the pair of arrows in the square loop of Fig. 7.5, the loop...
expands outward on all sides (Sect. 7.8) until it leaves the crystal. The resulting permanent deformation is identical to the final block shift shown in Figs. 7.2 and 7.3.

The second type of loop is called a prismatic loop. This type is fundamentally different from the shear loop; the only features the two types have in common is their circular shape and their ability to expand or contract radially. The more numerous differences between shear and prismatic loops are summarized in Table 7.1

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Shear loop</th>
<th>Prismatic loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>On slip plane</td>
<td>Between close-packed planes</td>
</tr>
<tr>
<td>Peripheral dislocation type</td>
<td>Mixed</td>
<td>Edge</td>
</tr>
<tr>
<td>Central portion of loop</td>
<td>Perfect crystal lattice</td>
<td>Stacking fault</td>
</tr>
<tr>
<td>Mechanism of growth</td>
<td>Shear stress</td>
<td>Absorption of point defects</td>
</tr>
<tr>
<td>Orientation of Burgers vector</td>
<td>Parallel to loop</td>
<td>Perpendicular to loop</td>
</tr>
</tbody>
</table>

Corresponding to the two types of point defects, vacancies and interstitials, there are two types of prismatic loops. These are shown in Fig. 7.7. The interstitial loop consists of a disk-shaped layer (seen end-on in the figure) of atoms formed by assembling free interstitial atoms from the bulk solid. The atom-layer agglomeration is thermodynamically more stable than the same number of atoms dispersed in the lattice as self-interstitials. In structures other than the simple cubic type depicted in Fig. 7.6, formation of the interstitial loop disrupts the regular ordering of planes in the perfect lattice, creating a stacking fault. Interstitial loops form only in solids bombarded by high-energy radiation (e.g., neutrons) because only this environment produces sufficient quantities of self-interstitials.

The right-hand sketch in Fig. 7.6 shows a prismatic loop formed by the collapse of a disk of vacancies on a close-packed plane. In common with the interstitial loop, the periphery of the vacancy loop is a circular edge dislocation with a Burgers vector perpendicular to the plane of the loop. However, the Burgers vectors of the two types are of opposite sign. Vacancy loops are formed during irradiation. The interaction of irradiation-produced point defects with these two types of loops provides a mechanism for the phenomenon of irradiation growth in fuel elements containing uranium metal or zirconium alloys (see Chap. 19). In addition, vacancy loops can be produced by rapid quenching of metals. The substantial equilibrium concentration of vacancies at high
temperatures (Chap. 3, Sect. 3.2) provides an adequate supply of this type of point defect for nonequilibrium production of loops to compete with point-defect absorption by microstructural sinks as the metal is rapidly cooled.

7.3 Dislocations in Real Crystals

The preceding two sections used a simple cubic crystal structure to illustrate the basic features of dislocations. This particular lattice type is of no practical interest in elemental solids, since only polonium exhibits this structure. The simple cubic structure often appears as a sublattice in two-component ionic solids. For example, CsCl consists of two interlaced sc sublattices of Cs⁺ and Cl⁻ ions. In this work, the dislocation structure in the fcc elements is emphasized, with lesser attention paid to the bcc and hcp lattice types. Some information on dislocations in ionic solids can be obtained elsewhere (p 120, Ref. 5, p. 152, Ref. 8).

The lattice structures of the more common metals are fcc, bcc, or hcp. In these structures, the slip planes, slip directions, and dislocation atomic configurations are more complex than in the simple cubic crystal. Pioneering tensile tests on oriented single crystals conducted in the 1930s revealed one or perhaps two combinations of slip direction and slip plane characteristic of each lattice type. Figure 7.7 shows a single crystal of an fcc metal oriented and pulled along a [100] direction.

![Fig. 7.7 Slip characteristics in an oriented single crystal (fcc)](image)

Failure by slipping (i.e., dislocation motion) always occurs on a (111) plane and in a [110] direction. As shown in Fig. 7.8, there are four orientations of (111) planes in the fcc unit cell, each of which has three [110] directions. Of these $4 \times 3 = 12$ combinations, which are collectively termed the slip system, dislocations first move on the combination that experience the largest resolved shear stress. According to Fig. 7.7, the tensile force acts on the shaded area $A/\cos \phi$, and the shear component of the

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* Although there are 8 unit cells in the figure, (111) planes in the pairs connected by the two-ended arrows are parallel, and so do not constitute planes of different orientation.

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tensile force along the slip direction is \( F \cos \lambda \). The resolved shear stress on the slip plane in the slip direction is:

\[
\sigma_S = (F/A) \cos \phi \cos \lambda
\]  

(7.1)

Fig. 7.8 Slip systems in the fcc structure

The value of the applied tensile stress \( F/A \) at which slippage first appears corresponds to a critical resolved shear stress, \( \sigma_S^{\text{crit}} \). If the orientation of the specimen is changed (e.g., from [100] along the tensile axis to, say, [211]), slip initiation occurs at different values of \( F/A, \phi, \) and \( \lambda \); however, \( \sigma_S^{\text{crit}} \) calculated from Eq (7.1) is unchanged. This observation implies two features of slip in crystals:

1. The values of \( \phi \) and \( \lambda \) correspond to the member of the slip system that experiences the largest resolved shear stress
2. The orientation-independent value of \( \sigma_S^{\text{crit}} \) represents the minimum shear stress to move an edge or screw dislocation in a particular lattice type. It is of order of magnitude \( 10^{-3} \) to \( 10^{-4} \) times smaller than the threshold shear stress computed from the “en bloc” model of permanent deformation described in Sect. 7.1.
Experiments of the type depicted in Fig. 7.7 performed on other lattice types show roughly the same ratio of $\sigma^\text{crit}_S$ to the shear modulus $G$ found for the fcc lattice. However, the preferred slip directions and slip planes are distinctly different. In the bcc lattice, for example, slip occurs on (110) planes in [111] directions. Since there are six of the former and two of the latter, the slip system of the bcc structure has $6 \times 2 = 12$ members. The slip systems for the fcc and bcc lattice types suggests a generalization: slip occurs most easily on the closest-packed plane and in the closest-packed direction of the particular crystal structure. Both the bcc and bcc types follow this rule.

Compact Notation for the Burgers Vector

In order to completely characterize the Burgers vector, its direction and length need to be specified. The former is usually the Miller indices of the close-packed direction in the particular lattice type. The magnitude is determined by this direction and the requirement that the minimum unit of slip reproduce the perfect lattice. As shown in the upper right-hand unit cell in Fig. 7.8, the magnitude of the Burgers vector for fcc slip is one-half of a face diagonal, or $b = a_o/\sqrt{2}$. The Burgers vector is denoted by: $\vec{b} = c[ijk]$, where $i$, $j$, and $k$ are the Miller indices of the direction. The constant $c$ is determined by the known magnitude of $\vec{b}$ according to: $c = b/\sqrt{i^2 + j^2 + k^2}$. Thus for the fcc Burgers vector, substitution of $a_o/\sqrt{2}$ for $b$ and $i = 1, j = 1, k = 0$ yields $c = (a_o/\sqrt{2})/\sqrt{2} = a_o/2$. The Burgers vector is designated as $a_o/2[110]$. Even though the three arrows in the upper right hand cube of Fig. 7.8 have different directions, they are all the same kind of Burgers vector denoted by $a_o/2[110]$. For completeness, the plane in which the dislocation (and hence the Burgers vector) lies is appended to the preceding notation. For the fcc structure, the Burgers vector of both screw and edge dislocations is:

$$\vec{b}_{\text{fcc}} = \frac{a_o}{2}[110](111) \quad (7.2a)$$

In the bcc structure, the close-packed plane in which the Burgers vector lies is (110), and the close-packed direction in which it points is [111]. The length of the Burgers vector in this lattice is the distance between the central atom in the unit cell and a corner atom, or $b_{\text{bcc}} = (\sqrt{3}/2)a_o$ and the value of $c = (\sqrt{3}/2)a_o/\sqrt{1^2 + 1^2 + 1^2} = a_o/2$. The Burgers vector is:

$$\vec{b}_{\text{bcc}} = \frac{a_o}{2}[111](110) \quad (7.2b)$$

Two dislocations of comparable energy are found in the hexagonal lattice structure. Dislocations in the basal plane (the (0001) plane, see Fig. 2.3) are the most common. As in the fcc structure, which the hcp type closely resembles, the Burgers

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* Table 8-2 of Ref. 4 shows the slip systems for the common lattice structures

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vector points in the directions along the six sides of the hexagon, one of which is the \([\overline{1}1\overline{2}0]\) direction. The designation of this dislocation is:

\[
b_{\text{basal}}^{\text{hcp}} = \frac{a}{3}[\overline{1}1\overline{2}0](0001) \tag{7.2c}
\]

Dislocations of the same direction lie in prism \((10\overline{1}0)\) planes as well. These dislocations are designated as

\[
b_{\text{prism}}^{\text{hcp}} = \frac{a}{\sqrt{2}}[\overline{1}1\overline{2}0](10\overline{1}0) \tag{7.2d}
\]

The method of determining the c factor is different than it is for the usual 3-digit technique described above for the cubic crystals. The 4-digit method is described in Ref. p. 107 of Ref. 6 and p. 108 of Ref. 7. The slip system of each of these dislocations contains only 3 members (1 plane, 3 directions).

The dislocations denoted by Eqs (7.2a) – (7.2d) are called perfect dislocations because the atom movement they represent reproduces the crystal structure, without leaving a trace of its passing. However, the mechanism of this motion is not as straightforward as the simple shift of a half plane of atoms, as indicated in Fig. 7.2 for the sc lattice. The mechanism of atom shifting during slip in the fcc structure is shown in Fig. 7.9. As shown in Fig. 3.5, the fcc structure consists of stacking of \((111)\) planes in an ABCABC... sequence. In Fig. 7.9, an A layer is drawn as circles in the familiar hexagonal pattern of the close-packed plane. The B and C layers above the A layer are not shown as circles; instead, the positions of the centers of these layers in the interstices of the A layer are merely indicated by letters. Slip consists of movement of the B layer (and all layers above) relative to the A layer (and all layers below) in a manner that reproduces the original perfect stacking sequence. The overall displacement is shown in Fig. 7.9 as the heavy arrow connecting the two B sites. However, the actual movement is the two-step path from B site to a C interstice and then to the next B site. This path is favored because the energy barrier of a direct B-to-B movement over the A layer is higher than the path following the two saddles between B and C and C and B. This can be appreciated by trying the two movements with ball models of close-packed \((111)\) planes.

![Fig. 7.9 The slip mechanism in fcc crystals](image)

ABCABC... sequence. In Fig. 7.9, an A layer is drawn as circles in the familiar hexagonal pattern of the close-packed plane. The B and C layers above the A layer are not shown as circles; instead, the positions of the centers of these layers in the interstices of the A layer are merely indicated by letters. Slip consists of movement of the B layer (and all layers above) relative to the A layer (and all layers below) in a manner that reproduces the original perfect stacking sequence. The overall displacement is shown in Fig. 7.9 as the heavy arrow connecting the two B sites. However, the actual movement is the two-step path from B site to a C interstice and then to the next B site. This path is favored because the energy barrier of a direct B-to-B movement over the A layer is higher than the path following the two saddles between B and C and C and B. This can be appreciated by trying the two movements with ball models of close-packed \((111)\) planes.
In other terms, the perfect dislocation $b_{\text{fcc}}$ splits into two partial dislocations labeled $b_s$ in Fig. 7.9. This process is called dissociation, and is written as a reaction:

$$\frac{a_0}{2} [110][111] \rightarrow \frac{a_0}{6} [211][111] + \frac{a_0}{6} [12\bar{1}][111] \quad (7.3)$$

The two partial dislocations on the right hand side of this reaction are called Shockley partial dislocations. They represent the displacements from B to C and C to the next B.

The perfect dislocations for the bcc and hcp structures given by Eqs (7.2b) – (7.2d) also undergo similar multiple-step atom displacements.

A useful view of edge dislocations is along the direction perpendicular to the slip plane. What has been loosely referred to as an extra “half plane” of atoms is an accurate description of the actual structure only for the sc lattice. This is so because this lattice consists of layered (100) planes. The top diagram in Fig. 7.10 shows the simple cubic lattice viewed in this manner. The atoms marked with an X represent the termination of the half plane, which is the row of atoms just above the inverted T in Fig. 7.2.
The stacking sequences are different for the other cubic structures. Additionally, the repetitive nature of the stacking of planes for a particular lattice type depends upon which planes are involved. In the fcc lattice, for example, the common view is of a series of (111) planes placed above each other in an ABCABC… sequence (Fig. 3.5). However, the fcc lattice can also be reproduced by stacking (110) planes in an ABAB… pattern. Since the (110) planes are perpendicular to (111) planes, they constitute the extra half planes whose termination is the edge dislocation with the Burgers vector of Eq (7.2a). The middle sketch in Fig. 7.10 shows the two (110) half planes that constitute the edge dislocation. The terminating atoms are connected by a heavy line, indicating that the “half plane” is corrugated rather than flat.

**Fig. 7.10 Extra half planes constituting the edge dislocation in cubic structures**
The bcc structure can be constructed by stacking (111) planes in an ABCABC… sequence. Thus three (111) half planes constitute the edge dislocation in this lattice. The corrugation of the “half-plane” is even more pronounced than in the fcc structure.

Climb and Cross Slip

So far in this chapter, the only motion permitted of a dislocation was along its slip plane. Solids contain a variety of obstacles that effectively halt dislocation glide (or slip). Among these are precipitate particles, bubbles or voids, grain boundaries, or other dislocations that are immobile because part of them lie out of their slip planes. If a mobile dislocation is pushed by a shear stress into one of these obstructions, one of three events occur:

1. The mobile dislocation is stopped
2. The mobile dislocation literally “cuts through” the obstacle
3. The mobile dislocation moves to another slip plane that is parallel to the original slip plane, on which it can continue to glide

The ability of a dislocation to evade the obstacles in its path is crucial to plastic deformation; if dislocations did not possess this flexibility, they would quickly become immobile.

For an edge dislocation, movement from one slip plane to a parallel one requires movement call climb. In this mechanism, the extra half plane of atoms that constitutes the dislocation literally grows or shrinks. It does so by absorbing vacancies or interstitials from the bulk solid. The former case is shown in Fig. 7.11.

**Fig. 7.11 Mechanism of edge dislocation climb by vacancy absorption**
The first panel shows a vacancy making a random jump into a lattice site adjacent to the bottom of the half plane of atoms constituting the dislocation. Once in this location, exchange of the vacancy with the bottom atom of the dislocation is certain because of the favorable energetics. The third panel shows the end result – the dislocation has “climbed” to a new slip plane parallel to the original one.

Climb of an edge dislocation perpendicular to its slip plane is a form of motion fundamentally different from the slip or glide motion shown in Fig. 7.2. Because climb requires mobility of vacancies in the lattice, it is active only at high temperatures (say
1/3 of the melting point and higher) while slip has no temperature dependence and is the sole mechanism of edge dislocation movement at room temperature.

A process comparable to the upward climb of edge dislocations also occurs with interstitials. In this case, absorption of interstitials at the bottom of the half plane of atoms causes the edge dislocation to move downward instead of upward. This process is of no significance in unirradiated metals because of the very low concentration of interstitials at thermodynamic equilibrium (Chap. 3, p. 8). However, irradiation by high-energy neutrons in a nuclear reactor displaces atoms from their normal lattice positions, creating equal numbers of interstitials and vacancies (Chap. 12).

Screw dislocations also absorb or emit point defects. However, rather than moving away from the slip plane, screw dislocations develop a helical shape but otherwise remain stationary.

Edge dislocations are constrained to move on the slip plane perpendicular to the termination of the extra half plane of atoms. Although there are a number of distinct slip planes of different orientation – 4 (111) in the fcc lattice, Fig. 7.8, and 6 (110) in the bcc structure – an edge dislocation on one of them cannot change to another. This restriction does not apply to screw dislocations. Because the slip planes in cubic lattices intersect, the screw dislocation can move from one to another, provided that the line of intersection of the slip planes is parallel to the screw dislocation.

**Example:** With reference to the upper right cube in Fig. 7.8, a screw dislocation with its line in the [1 1 0] direction is moving on the (111) plane. Of the remaining nonparallel close-packed planes, only the (1 1 1) (lower right cube) intersects the (111) at a line in the [1 1 0] direction. This is best seen by examining the (1 1 0) plane labeled ABCD in the lower right cube. In Fig. 7.12, this dislocation line is perpendicular to the page and moves in the direction of the arrows. If an obstacle is encountered on the (111) plane, the dislocation can switch to the (1 1 1) plane at their intersection. It will move with equal ease on both planes because resolved shear stress responsible for its movement is parallel to the dislocation line (see Fig. 7.3), and is therefore the same on the (111) and (1 1 1) slip planes. The dislocation can switch back to a (111) plane, as shown in Fig. 7.12, or continue moving along the (1 1 1) plane. This process is called **cross slip.**
7.4 Dislocation Density

Defining a measure of the “concentrations” of dislocations in a solid is not as straightforward as it is for discrete objects such as precipitates or bubbles. For the latter, the number of objects per unit volume is a perfectly satisfactory gauge of their concentration, or number density as it is often called.

Since most dislocations are lines (not necessarily straight) that start at one surface of a body and terminate at another surface, counting their number per unit volume not an option. Instead, the measure of dislocation concentration is defined as the total length of dislocation lines (including loops) per unit volume. This unit is called the dislocation density. This quantity is denoted by $\rho$ and has units of cm/cm$^3$, or cm$^{-2}$. This quantity can be determined by measuring the length of the dislocations in TEM images, such as the one in Fig. 7.4.

There are a number of theoretical models of processes involving dislocations, including diffusion of point defects to them, creep mechanisms involving them, and the dependence of plastic yielding on them. For modeling purposes, the chaotic distribution of the dislocations seen in Fig. 7.4 must be greatly simplified.

The most common method of converting the randomness of the actual dislocation structure is shown in Fig. 7.13. On the left, the dislocations are treated as straight lines extending from one face of a unit cube to the opposite face. All dislocations between a particular pair of opposing faces are parallel. There are equal numbers of lines joining the other two faces of the unit cube. Both edge and screw dislocations are included, but there is no attempt to distinguish between them in reporting the dislocation density. Each dislocation segment in the unit cube is 1 cm in length, so the total number of segments is equal to the dislocation density.

Fig 7.13 Simplified model of dislocation lines in a solid

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The right hand sketch in Fig. 7.13 shows the dislocations emerging at one face of the cube. Instead of random intersections, the termination of the dislocations are arranged on a uniform square lattice. The circles around each dislocation emergence represent the unit cell; that is, a structure that is replicated throughout the solid. The circles are terminations of a cylinders of radius \( R \) with the dislocation at the axis. The number of unit cells per cm\(^2\) is \( \rho/3 \), the remaining 2/3 of the dislocation belonging to the two other orthogonal directions. The unit cell radius is chosen so that the sum of the areas of the circles just fills the unit square. Since there are \( \rho/3 \) dislocations/cm\(^2\), the reciprocal is the area associated with each dislocation, or the area of the unit cell is:

\[
\frac{1}{\rho/3} = \pi R^2.
\]

Solving for the unit cell radius yields:

\[
R = \sqrt[3]{\frac{3}{\pi \rho}}.
\] (7.4)

Interaction between the cylindrical unit cells from one pair of unit cube faces with the unit cells from the other orthogonal faces is neglected, although in reality these must intersect each other. Without the very severe simplifications contained in Eq (7.4), it would be impossible to analyze many important processes involving point defects and dislocation lines.

Complete elimination of dislocations from a metal is neither possible nor desirable; dislocations are responsible for the strength of structural metals, and without them, bridges, skyscrapers, airplanes, ships, etc. would not exist. Dislocations are produced in a metal during fabrication and by plastic deformation in subsequent processing or in service. The dislocation density can be changed (within limits) by post-fabrication treatments such as cold-working, which increases \( \rho \) by plastic deformation, or annealing, which decreases \( \rho \) by permitting the thermodynamic tendency to eliminate any defect to take place.

The wide variety of processing methods available allows for control of the dislocation density over a 12-order of magnitude range. Table 7.1 shows typical values of \( \rho \) for various materials.

**Example:** How far apart are the dislocations in a metal with \( \rho = 10^{10} \) cm\(^{-2}\)?

Using this value of \( \rho \) in Eq (7.4) gives \( R = 10^{-5} \) cm, or the dislocations are separated by roughly 0.2 \( \mu \)m.

### Table 7.2 Dislocation Densities of Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Dislocation density, cm(^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vapor-grown thin whiskers</td>
<td>&lt;1</td>
</tr>
<tr>
<td>High purity silicon single crystals</td>
<td>10(^3)</td>
</tr>
<tr>
<td>Annealed metals</td>
<td>10(^8)</td>
</tr>
<tr>
<td>As-fabricated metals</td>
<td>10(^{10})</td>
</tr>
<tr>
<td>Cold-worked metals</td>
<td>10(^{12})</td>
</tr>
</tbody>
</table>
7.5 Stress Fields Around Dislocations

The crystal lattice in the vicinity of a dislocation is distorted (or strained). The strains produce stresses that can be calculated by elasticity theory beginning from a radial distance about 5b, or ~ 15 Å from the axis of the dislocation (Ref. 6, p. 36). Within this zone, the strains are too large for elasticity theory to be valid. Methods utilizing interatomic force calculation are required here. Fortunately, this dislocation core where elasticity theory fails contributes only minimally to all phenomena involving the atomic distortions due to the presence of the dislocation. The dislocation core is universally ignored in calculating the consequences of the stresses around dislocations.

The stress field around a dislocation is responsible for several important interactions with the environment. These include:
1. An applied shear stress on the slip plane exerts a force on the dislocation line, which responds by moving or changing shape.
2. Interaction of the stress fields of dislocations in close proximity to one another results in forces on both which are either repulsive or attractive.
3. Edge dislocations attract and collect interstitial impurity atoms dispersed in the lattice. This phenomenon is especially important for carbon in iron alloys and for irradiation-produced self-interstitials in metals exposed to high-energy neutron fields.

Screw Dislocation

We start with the screw dislocation because it has a much simpler stress field around it than the edge dislocation. The circuit shown in the diagram in the upper left hand corner of Fig. 7.3 shows that a displacement equal to the magnitude of b, the Burgers vector, is attained in a complete circular path around the z axis of the dislocation line. If one imagines unrolling an annular ring of radius r, thickness Δr and unit length in the z direction, the resulting sheet resembles the dotted figure in Fig. 6.2. The displacement v in Fig. 6.2 is replaced by b and the perpendicular length L₀ is replaced by the length of the unrolled annular ring, which is its original circumference 2πr. The shear strain is the displacement divided by the perpendicular length, or ε₀z = b/2πr. The corresponding shear stress is (see Eq (6.23)):

\[
\sigma_{\theta z} = \frac{Gb}{2\pi r} 
\]

(7.5)

All other stress components are zero. The stress field of Eq (7.5) is axisymmetric about the dislocation line and falls off as r⁻³. In Cartesian coordinates, two shear stress components are nonzero:

\[
\sigma_{xz} = -\frac{Gb \sin \theta}{2\pi r} \quad \sigma_{yz} = \frac{Gb \cos \theta}{2\pi r} 
\]

(7.5a)

Edge Dislocation
The stresses around an edge dislocation are derived in Ref. 7, p. 37. In cylindrical coordinates, they are:

\[
\begin{align*}
\sigma_{rr} &= \sigma_{\theta\theta} = -\Gamma \frac{\sin \theta}{r} \\
\sigma_{\theta r} &= -2\Gamma \nu \frac{\sin \theta}{r} \\
\sigma_{zz} &= \frac{G \Gamma}{2(1-\nu)} \\
\sigma_{\theta \theta} &= \frac{G \Gamma}{2(1-\nu)} \cos \theta \\
\sigma_{rr} &= \frac{G \Gamma}{2(1-\nu)} \sin \theta \\
\sigma_{\theta \theta} &= \frac{G \Gamma}{2(1-\nu)} \cos 2 \theta + \sin 2 \theta \\
\sigma_{yy} &= -\Gamma \sin (\cos 2 \theta - \sin 2 \theta) \\
\sigma_{yx} &= \Gamma \frac{\cos (\cos 2 \theta - \sin 2 \theta)}{r}
\end{align*}
\] (7.6)

The two remaining shear stress components are zero. The coefficient $\Gamma$ is:

\[
\Gamma = \frac{G \nu}{2(1-\nu)}
\] (7.8)

In rectangular coordinates, Eqs (7.6) and (7.7) are (\(\sigma_{zz}\) remains unchanged):

\[
\begin{align*}
\sigma_{xx} &= -\Gamma \frac{\sin \theta (1 + 2 \cos^2 \theta)}{r} \\
\sigma_{yy} &= -\Gamma \frac{\sin \theta (\cos^2 \theta - \sin^2 \theta)}{r} \\
\sigma_{yx} &= \Gamma \frac{\cos \theta (\cos^2 \theta - \sin^2 \theta)}{r}
\end{align*}
\] (7.6a)

The cylindrical coordinate system and the directions of the various stress components of the edge dislocations are shown in Fig. 7.14.

**Fig 7.14 Active stress components in the vicinity of an edge dislocation**

### 7.6 Elastic Strain Energy

The stress field around a dislocation produces elastic strain energy (see Eq (5.31)). This energy, which is referenced to that of the perfect crystal, is the reason that the dislocation is a nonequilibrium object, not unlike point defects.

The elastic strain energy of dislocations is of interest for two reasons: first, it determines the type of dislocation in a particular crystal type; and second, it determines...
the response of a pinned, or immobile, dislocation to forces exerted on it by applied stresses.

The elastic strain energy around a screw dislocation is easy to derive because only one stress component is generated in its vicinity. Letting $2 = \theta$ and $3 = z$ in Eq (5.31), all terms except the very last are zero. Substituting Eq (7.5) into this term gives:

$$E_{el} = \frac{Gb^2}{8\pi r^2} \text{ energy per unit volume at distance } r$$

A volume element $2\pi rdr$ is chosen and the above equation is integrated to give the total energy per unit length of dislocation:

$$E_d = E_{core} + \int_{r_o}^{R} 2\pi r E_{el} dr = E_{core} + \frac{Gb^2}{4\pi} \left[ \ln\left(\frac{R}{r_o}\right) \right]$$

where $E_{core}$ is the energy of the core of the dislocation ($0 \leq r \leq r_o$). The limits on the integral are the radial extremes of the unit cell shown in the diagram at the right in Fig. 7.13. The lower limit is the core radius of the dislocation, and the upper limit is the radius of the unit cell given by Eq (7.4)*. Taking $r_o \sim 15$ Å and $R = 0.1 \mu m$ ($\rho = 10^{10}$ cm$^{-2}$), the bracketed term in the above equation is $1/3$. The choices of $R$ and $r_o$ are somewhat arbitrary; with $r_o = 6$ Å and $R = 1 \mu m$ ($\rho = 10^8$ cm$^{-2}$), the bracketed term is $2/3$. Assuming the bracketed term to be unity is in part justified by neglecting the contribution of $E_{core}$. This gives the dislocation’s self energy per unit length as:

$$E_d = \frac{Gb^2}{3}$$

$E_d$ for an edge dislocation is obtained by substituting Eqs (7.6) and (7.7) into Eq (5.31). The result differs from $E_d$ for a screw dislocation by the term $1 - \nu$ in the denominator of the bracketed term. Since $1 - \nu \sim 2/3$, the coefficient of $Gb^2$ is increased by $\sim 50\%$ over that of a screw dislocation. This change is within the uncertainty of the coefficient of $Gb^2$, so Eq (7.9) is applied to both edge and screw dislocations.

Just as the energy per unit area of a surface acts as a surface tension, so the energy per unit length of a dislocation ($E_d$) can be treated as a line tension. This aspect of the dislocation self-energy comes into play when a curved section of a dislocation is acted upon by an applied stress. The loop shown in Fig 7.5, for example, is maintained by a dynamic balance between the shrinkage tendency of the line tension and the expansion tendency of the applied shear stress. If the latter were removed, the line tension would cause the loop to shrink until the dislocation segments of opposite sign at the extremities of a diameter met and annihilated each other. The loop would simply vanish.

* The stress fields of adjacent dislocations tend to cancel each other.

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The other important feature of Eq (7.9) is the dependence of $E_d$ on $b$. The smaller the Burgers vector, the lower the dislocation self energy. Consequently, the energetically most favorable dislocation is the one that points in the direction of the closest-packed atomic row of the crystal structure. The magnitude of this energetically-favored dislocation is the distance between adjacent atoms in this direction. This “rule” can be seen in the Burgers vectors drawn on the fcc lattice in the upper right hand cube in Fig. 7.8. The Burgers vector for the bcc structure (Eq 7.2b) and for the hcp lattice (Eqs (7.2c) and (7.2d)) also obey the smallest-$b$ “rule”.

### 7.7 Stresses to Initiate Dislocation Movement

In order to start a dislocation moving on its slip plane, a sufficiently large shear stress must be applied. The magnitude of this stress depends upon the microstructure of the solid. The resistance to initiation of dislocation movement is due to: i) impurity atoms near to or attached to the dislocation; ii) the presence of nearby or intersecting dislocations; and iii) pinning of dislocations at points where they leave their slip planes. Despite the inherent variability of these impediments to dislocation motion, high-purity single crystals exhibit a critical resolved shear stress $\sigma_{\text{crit}}$ (Eq (7.1) et seq) that is sufficiently reproducible to be considered a property of the metal+.

It is of obvious interest to understand the truly intrinsic resistance to dislocation slip of an absolutely perfect crystal containing a single perfectly straight dislocation line. Such a stress cannot be measured because specimens satisfying the above criteria cannot be made. However, its origin can be inferred from the motion of the plane of atoms illustrated in Fig. 7.2 for an edge dislocation. The upper half-plane of atoms appears to move to the right because the atomic plane below the slip plane and to the right of the dislocation switches alignment to the upper half-plane. In so doing, a small energy barrier must be surmounted. In effect, this periodic energy barrier to slip is a miniature version of the much larger periodic energy barrier required for slip by the *en bloc* mechanism (Sect. 7.1). The stress required to move the dislocation by the mechanism of Fig. 7.2 in the hypothetical crystal of complete microstructural and compositional perfection is called the Peierls stress, $\sigma_P$. Although there are no experimental measurements of $\sigma_P$, there is no shortage of theoretical estimates of its magnitude. Unfortunately, there is no agreement as to the correct model+.

At the opposite extreme of the Peierls stress is the applied stress required for initiation of plastic deformation is commercial structural alloys. This is the yield stress $\sigma_Y$.

Table 7.3 summarizes the three characteristic stresses for initiation of dislocation motion in materials of varying purity.

---

+ A list of $\sigma_{\text{crit}}$ for various materials is given on p. 228 of Ref. 4
+ These theories are reviewed in Ref. 7, p. 147 et seq; see also Sect. 6.7.2
Table 7.3 Characteristic Stresses for Initiation of Dislocation glide
(in order if increasing magnitude)

<table>
<thead>
<tr>
<th>Name of stress</th>
<th>Symbol</th>
<th>Material Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peierls</td>
<td>$\sigma_p$</td>
<td>Perfect crystal; single straight dislocation</td>
</tr>
<tr>
<td>Critical resolved</td>
<td>$\sigma_{crit}$</td>
<td>High-purity single crystal</td>
</tr>
<tr>
<td>Yield*</td>
<td>$\sigma_Y$</td>
<td>Commercial product</td>
</tr>
</tbody>
</table>

* see Sect. 7.12 for an explanation of the causes of yielding

Because of their dependence on the material and its microstructural condition, no values of the critical stresses are given in the table.

7.8 Forces exerted by applied stresses on dislocations

The critical stresses summarized in Table 7.3 are produced by a shear stress on the slip plane containing the dislocation arising either directly from an applied shear force or indirectly by resolution of an applied normal stress. As shown in Fig. 7.2, the dislocation moves to the right if the applied shear stress $\sigma_S$ above the slip plane points in the direction of the Burgers vector. This situation is shown in detail suitable for analysis in Fig. 7.15.

![Fig. 7.15 Diagrams for deriving forces on dislocations due to applied stresses](image)

Consider first the edge dislocation acted upon by an applied shear stress on the slip plane and in the direction indicated. This shear stress can be converted to a force per unit length of dislocation. The reason for this conversion is that other forces may be exerted on the dislocation; having the force due to applied shear stress in hand permits force balances to be made on the dislocation. To determine the force $F_x$, the work to move the dislocation by a distance $\Delta x$ is calculated in two ways. Considering a unit length of dislocation in the $z$ direction, the force applied to the area $1 \times \Delta x$ by the shear stress is $\sigma_{yx} \Delta x$. The distance over which this force acts is the Burgers vector $b$, because this is the actual length that the atomic planes above and below the slip plane have moved as the dislocation passes them. Thus, the work done by the shear stress in moving the dislocation a distance $\Delta x$ is:
\[ W(\text{due to shear stress}) = \sigma_{yx} \Delta x \ b \]

Alternatively, we can imagine a hypothetical force \( F_x \) acting on the unit length of dislocation line and moving it a distance \( \Delta x \). The work done from this viewpoint is:

\[ W(\text{due to force}) = F_x \Delta x \]

These two descriptions of the work must be equal, which leads to the result:

\[ F_x = \sigma_{yx} \ b \quad (7.10) \]

If the direction of the applied shear stress \( \sigma_{yx} \) were reversed, the force would act in the negative \( x \) direction; similarly, if the orientation of the edge dislocation were reversed (i.e., the extra half plane of atoms below the slip plane), \( F_x \) would act in the negative \( x \) direction with the shear stress directed as shown in the diagram of Fig. 7.15.

For the screw dislocation, Fig. 7.3 shows that the only component of the shear stress that exerts a force is the \( \sigma_{yz} \). This shear stress component is depicted as two circles above and below the slip plane, with the filled circle acting in the positive \( z \) direction. By an analysis identical to the one given above for the edge dislocation, the \( x \) and \( y \) forces per unit length exerted on the screw dislocation are:

\[ F_x = \sigma_{yz} \ b \quad (7.11a) \quad F_y = \sigma_{xz} \ b \quad (7.11b) \]

In this case, the stress component parallel to the dislocation line is the source of the force perpendicular to the line. This may appear counterintuitive, but examination of Fig. 7.3 suggests a simple analogy; when you tear a sheet of paper, you pull up and down perpendicular to the sheet and the paper rips in the direction perpendicular to the forces you exert on it (try it).

The climb of an edge dislocation (Fig. 7.11) requires a normal stress acting in the direction shown in the right hand sketch of Fig. 7.15+. The work analysis employed for the glide force on an edge dislocation due to a shear stress can also be applied to the climb process. In this case a positive normal stress (i.e., tensile) produces a force per unit length of dislocation that is directed in the negative \( y \) direction:

\[ F_y = -\sigma_{xx} \ b \quad (7.12) \]

This result is intuitively acceptable; if the plane of atoms of the edge dislocation is regarded as a crack in a brittle solid, a tensile stress perpendicular to the crack causes it to propagate; a compressive stress perpendicular to the crack causes it to close up, or to move in the direction opposite to the propagation direction. The force given by Eq (7.12) results in climb of the dislocation only if a sufficiently large concentration of

\[ + \text{In an irradiation field, the edge dislocation can climb without applied stress by absorbing the copious quantities of point defects available.} \]
mobile point defects exists in the bulk of the solid. In the absence of a radiation field, high temperatures are required to generate the requisite point defect population.

No diagram for the response of screw dislocation to a normal stress is shown in Fig. 7.15. The dislocation responds to normal stresses by coiling up into a helical shape. However, there is no net motion of the dislocation, so this phenomenon is not of interest in explaining the mechanical properties of solids.

### 7.9 Interaction Forces Between Dislocations

The proximity of two dislocations results in forces between them. Depending on their relative positions and in the case of an edge dislocation, on its orientation, the interaction force may be positive or negative. A positive interaction force means that the two dislocations attract each other; negative interaction corresponds to repulsion. Knowledge of these interaction forces is useful in understanding mechanical properties of metals, including the onset of plasticity (yielding) and the rate at which one dislocation surmounts the repulsion of another (creep).

The forces exerted on dislocation lines by applied stresses was analyzed in Sect. 7.8. The stresses in Eqs (7.10) – (7.12) were said to be “applied”, meaning generated by an external load on the body. However, the stresses in these equations can originate from any source, in particular, the presence of a nearby dislocation. Figure 7.16 shows the simplest of the combinations of the relative positions of the two pairs of parallel dislocations, one pair screw of opposite sign and the other pair edge of the same sign.

![Fig. 7.16 Interactions between two screw dislocations and two edge dislocations](Image)

**Parallel Screw Dislocations**

No slip planes are shown for the screw dislocations because of the many slip directions available to this type of dislocation (see end of Sect. 7.3). However, interaction forces in the x and y directions shown in Fig. 7.16 are readily calculated.

Substituting the second of Eqs (7.5a) into (7.11a) yields:
Substituting the first of Eqs (7.5a) into (7.11b) yields:

\[ F_y = \left( \sigma_{xz} \right)_1 b = -\frac{Gb^2}{2\pi r} \sin \theta \]  

(7.13b)

where the subscript 1 indicates that stress component is generated by dislocation 1 at the location of dislocation 2. The interaction force in the line joining the two dislocations is obtained by resolving \( F_x \) and \( F_y \) in the \( r \) direction:

\[
\text{component of } F_x \text{ along } r = F_x \cos \theta = \left( \frac{gb^2}{2\pi r} \right) \cos^2 \theta 
\]

\[
\text{component of } F_y \text{ along } r = -F_y \cos \left( \frac{\pi}{2} - \theta \right) = -F_y \sin \theta = \left( \frac{gb^2}{2\pi r} \right) \sin^2 \theta 
\]

The sum of these two components is the radial interaction force:

\[ F_r = \left( \frac{gb^2}{2\pi r} \right) \]  

(7.14)

The interaction force between two parallel screw dislocations is along their line of connection. \( F_r \) is attractive if the two dislocations are of opposite sign and repulsive if they are of the same sign.

The force on dislocation 2 due to the presence of dislocation 1 is equal and opposite to the force exerted by 1 due to 2.

**Parallel Edge Dislocations**

The diagram for the edge dislocation problem in Fig. 7.16 shows two parallel slip planes. The edge dislocations are constrained to move on these planes, in contrast to screw dislocations, which have a number of directions to move by slip.

The \( x \) force on dislocation 2 due to the presence of 1 is obtained by substituting Eq (7.7a) into (7.10):
or, in more compact form:

\[ F_x = \Gamma \frac{b}{y} \frac{f_x(\theta)}{y} \quad (7.15) \]

where \( y = r \sin \theta \) is the separation of the slip planes, \( \Gamma \) is given by Eq (7.8), and

\[ f_x(\theta) = \sin \theta \cos \theta (\cos^2 \theta - \sin^2 \theta) = \frac{1}{4} \sin(4\theta) \quad (7.16) \]

For the \( y \) force exerted on dislocation 2 by dislocation 1, the first of Eqs (7.6a) is used in Eq (7.12), giving:

\[ F_y = \Gamma \frac{b}{y} \frac{\sin^2 \theta (1 + 2 \cos^2 \theta)}{y} = \Gamma \frac{b}{y} f_y(\theta) \quad (7.17) \]

The angular functions \( f_x \) and \( f_y \) are plotted in Fig. 7.17.

![Fig. 7.17 Angular dependencies of the forces between two parallel edge dislocations of the same sign](image)

Both angular functions are zero when \( \theta = 0 \) which, for a fixed slip-plane separation \( y \), means the two dislocations are infinitely far apart. As dislocation 2 approaches dislocation 1, the \( f_x \) function is positive. This means that \( F_x \) is positive, or in the geometry of Fig. 7.16, the two dislocations repel each other. The maximum repulsion on the slip planes occurs at \( \theta = \pi/8 \), where \( f_x = \frac{1}{4} \). Thereafter, the repulsive force decreases until the angular region between \( \pi/4 \) and \( \pi/2 \) (when the two dislocations are aligned along the y axis), where the interaction is attraction.

The \( f_y \) function is positive for all angles, meaning that dislocation 2 tends to be pushed upward, away from dislocation 1. This is intuitively correct; according to the first of Eqs (7.6a), the entire upper hemicylinder around the edge dislocation \( (0 \leq \theta \leq \pi) \) is in compression. The compressive normal stress tends to “squeeze out” the half plane of
atoms constituting the edge dislocation, which is equivalent to an upward (y) force on
the dislocation. The normal force \( F_y \) is a driving force for climb of the dislocation, but
whether this occurs depends on the mobility of the vacancies in the lattice, which in turn
is highly temperature-sensitive.

The above analysis applied to parallel edge dislocations of the same sign. If the
dislocation 1 in Fig. 7.16 were turned upside down (i.e., the half-plane coming in from
the bottom instead of from the top), its Burgers vector would change sign. As a result,
the curves in Fig 7.17 would also be flipped; the long-range \( F_x \) force would be attractive
but repulsion would set in for angles greater than 45\(^{\circ}\).

As in the case of the interaction of two screw dislocations, the x and y forces on
dislocation 1 arising from the stress field of dislocation 2 are equal in magnitude but
opposite in direction to the forces on 2 from 1.

The relative positions of the dislocations in Fig. 7.16 (both parallel) represent the
simplest cases of dislocation interaction. Calculation of the interaction forces for other
orientations is more complicated. The more complex situations include: parallel edge
and screw; two parallel edge with perpendicular Burgers vectors; and perpendicular
dislocation lines.

7.10 Equilibrium Dislocations

Dislocations are mobile because of the force acting upon them by the applied
shear stress. The mobile dislocation line: 1) is in the z direction; 2) lies in a slip plane
perpendicular to the y direction; 3) moves in the x direction. The x force on the mobile
dislocation is given by Eq (7.10) for an edge or Eq (7.11a) for a screw. For the present
purposes, these forces are rewritten as:

\[
F_{\text{app}} = \sigma_S b \tag{7.18}
\]

The mobile dislocation encounters an immovable obstacle and responds by either:
i) stopping and assuming a shape or position the creates a back force that
just counterbalances the forward force \( F_{\text{app}} \) (this section);
ii) breaking through or moving past the obstacle (next two sections).

Several examples of the force equilibrium that stops the mobile dislocation are given
below.

Parallel Screw Dislocations

Suppose the dislocations S2 on the left in Fig. 7.16 were constrained to move on
its horizontal slip plane driven by the applied force \( F_{\text{app}} \). Cross slip is neglected.
Suppose in addition that the sign of S1 in Fig. 7.16 were changed so that S1 and S2
now have the same sign. Then, instead of attraction, the two dislocations repel each
other. S1 is assumed to be immobile and constitutes the obstacle to the motion of S2.
By changing the sign of $S_1$, its Burgers vector also changes sign, which results in a sign change of $F_x$ given by Eq (7.13a). This means that the direction of the $F_x$ arrow in Fig. 7.16 is flipped by $180^\circ$. $S_2$ moves to the right in the figure with the forward force $F_{\text{app}}$ and the back force $F_x$ from $S_1$ simultaneously acting on it. $S_2$ stops when the opposing forces just balance, or when $F_{\text{app}} = F_x$. Using Eqs (7.13a) and (7.18), this force equality becomes:

$$\sigma_S b = \frac{G b^2}{2\pi r} \cos \theta$$

If the distance between the slip planes of $S_1$ and $S_2$ is a fixed, specified value $y$, the quantity of interest is the $x$ value of the separation of the two dislocations, denoted by $x_{\text{eq}}$. To obtain this, $r$ and $\theta$ in the above equation need to be converted to Cartesian coordinates. Replacing $r$ in the above equation by $y/\sin \theta$ and using the trigonometric identity $\sin(2\theta) = 2\sin \theta \cos \theta$ results in:

$$\sigma_S = \frac{Gb}{4\pi y} \sin(2\theta_{\text{eq}})$$  \hspace{1cm} (7.19)

**Example:** If a shear stress equal to 1% of the shear modulus is applied to the mobile screw dislocation and the separation of the parallel slip planes of $S_1$ and $S_2$ is five Burgers vectors, what is $x_{\text{eq}}$? Using $\sigma_S/G = 0.01$ and $y/b = 5$ in Eq (7.19) and solving for the angle gives $\sin(2\theta_{\text{eq}}) = 0.63$ or $\theta_{\text{eq}} = 19.5^\circ$. From the geometry of Fig. 7.16, $x_{\text{eq}} = y/\tan \theta_{\text{eq}} = 14b$. What would happen if the applied shear stress were doubled?

**Bowed Dislocation**

An edge dislocation propelled by an applied shear stress $\sigma_S$ encounters a pair of point obstacles in its slip plane. If the portion of the dislocation line between the two pinning points is to attain an equilibrium shape, it must generate a back force to balance $F_{\text{app}}$. It does so by bowing out, as shown in Fig. 7.18. The extent of bowing, as measured by the radius of curvature $\Re$, is obtained from a force balance in the $x$ (upward) direction. The applied force acts normal to the entire length of the curved segment, which is of mixed edge/screw character. The three dimensional analog is the gas pressure in a balloon. When resolved in the $x$ direction, the total upward force on the bowed dislocation is $F_{\text{app}}L = \sigma_S bL$, where $L$ is the distance between pinning points (shown as black dots in Fig. 7.18).
Fig. 7.18 Force balance on a bowed dislocation

This upward force is balanced by the line tension of the dislocation, which is the same as its elastic strain energy per unit length, $Gb^2$ (see Eq (7.9) et seq.). The line tension acts tangent to the bowed dislocation at the pinning points. The force resolved in the downward direction, shown as $2F_{\text{bow}}$ in Fig. 7.18, is $2Gb^2 \cos \theta$. However, $\cos \theta = (L/2)/\Re$, so the downward force is $Gb^2L/\Re$.

Equating the upward and downward forces derived in the preceding two paragraphs leads to the following equation for the radius of curvature at equilibrium:

$$\Re = \frac{Gb}{\sigma_S} \quad (7.20)$$

The radius of curvature of the bowed dislocation is independent of the spacing of the obstacles that pin it. The consequences of an applied shear stress that reduces the radius of curvature to $L/2$ and bows the dislocation into a semicircle is considered below.

Dislocation Loop

The third example of a dislocation acted upon by opposing forces is the loop shown in Fig. 7.5. The existence of the loop requires a shear stress parallel to the Burgers vectors of the loop acting in the glide plane of the loop. Moreover, the direction of the shear stress must be that shown acting on the square loop in the upper left hand diagram in Fig. 7.5. Such a shear stress produces radially outward forces on both the positive and negative pure edge portions. Because the shear stress acts parallel to the screw portions of the loop (at the ends of a vertical diameter in Fig. 7.5), the force here is also radially outward (see Fig. 7.15). In fact, the force due to the shear stress is radially outward over the entire periphery of the loop.

If one cuts a segment of the loop, the force balance on this curved segment would be identical to that derived above for the pinned dislocation; the radius of the loop is given by Eq (7.20).

This result, however, is spurious. If the shear stress is removed, the loop will collapse and disappear, thereby reducing the strain energy of the crystal. Yet Eq (7.20) indicates that as $\sigma_S \rightarrow 0$, $\Re$ becomes very large. At the opposite extreme, as the shear...
stress becomes large, the loop should expand because the force on its periphery is radially outward. Yet Eq (7.20) predicts that as $\sigma_s \to \infty$, the loop should shrink.

The only conclusion that can be drawn from these contradictions is that the loop is not an equilibrium object. Rather, it is formed by an applied shear stress and, provided that the shear stress is maintained, will continue to expand until it encounters an obstacle or exits the body.

### 6.11 Dislocation Multiplication

**Why a dislocation multiplication mechanism is needed**

Consider the following simple model of shear deformation: a shear stress acting on one pair of opposing faces of a parallelepiped-shaped solid would induce motion only on a fraction of the total dislocation density. Assuming for simplicity that all dislocations are of the edge type, a shear stress $\sigma_{yx}$ can exert a force only on dislocations lying in the z direction. Assuming the simplified model of Fig. 7.13, these constitute 1/3 of the total. In addition if the shear stress is represented by only edge dislocations oriented as $\left\langle \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right\rangle$ and $\left\langle \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right\rangle$ would experience a force; those with $\left\langle \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right\rangle$ and $\left\langle \begin{array}{c} 0 \\ -1 \\ 0 \end{array} \right\rangle$ orientations are not affected by the stress. This reduces the fraction of potentially mobile dislocations by another factor of ½. Of the 1/6 of the total dislocation content properly oriented to respond to the applied shear stress, some will be pinned and immobilized. For this argument, assume that 50% are mobile. In all, ~ 1/12 of the total dislocation density responds to an applied shear stress. The objective is to calculate the maximum shear strain if all of these dislocations leave the solid and cause macroscopic deformation.

A sketch of the movable dislocations is shown in Fig. 7.19. The diagram on the left shows the unstrained solid, with dimensions X and Y is the x and y directions, respectively, and unit depth in the z direction. Shown are 4 slip planes each containing 4 properly-oriented mobile dislocations. The sequence that follows shows the strain of the block as dislocations are removed from one slip plane at a time. Note that the symbol for the edge dislocation indicates a complete incomplete plane of atoms extending perpendicularly from the slip plane to the surface of the body. The lower right hand diagram depicts the deformed state of the solid after all mobile dislocations have been swept by the applied shear stress. Note that the extra partial-plane of atoms represent by the vertical segments of the dislocation symbol extend completely to the top or bottom surface of the block. Thus, when the positive edge dislocations in the upper layer are all swept to the right, only the upper layer is displaced to the right (second panel); when the negative dislocations in the second layer from the top are acted upon by the shear applied force, the three lowest layers are displaced to the right but the second layer does not move (3rd panel); movement of the positive dislocations to the right carry with them all three uppermost layers, giving the deformed structure shown in the 4th panel. Finally, sweeping the remaining negative dislocations move the bottom block to the left. What remains of the dislocation-cleansed body is the shape deformed in shear shown in the last panel.
To generalize the above picture, assume \( n \) dislocations per unit length on each slip plane. Then each of the latter contain \( nX \) dislocations and the shear displacement of the layer(s) when all are swept out of the crystal is \( nXb \), where \( b \) is the Burgers vector. If \( L \) is the vertical distance between slip planes with dislocations of the same sign, \( Y/L \) planes have moved to the right. The shear displacement of the entire block is:

\[
u = (nXb)(Y/L)
\]

In a block of volume \((X)(Y)(1)\), the total length of dislocations capable of movement is \( XY\rho/12 \), where \( \rho \) is the total dislocation density and the factor of 12 accounts for the fraction that can glide by the applied shear stress (see above). The same total length of moveable dislocations is \((nX)(2Y/L)\), the factor of 2 arising because the length \( L \) is the distance between every other slip plane (see Fig. 7.19). Equating these two measures:

\[
\frac{n}{L} = \frac{\rho}{24}
\]

Eliminating \( n/L \) from the above two equations and dividing by \( Y \) to give the shear strain gives:

\[
\varepsilon = \frac{u}{Y} = \frac{1}{24\rho} bX
\]

Using the following numerical values:

\( \rho = 10^8 \text{ cm}^{-2} \) (typical of annealed metals, and probably consistent with the assumption used earlier of 50% mobile dislocations)
\( b = 3 \times 10^{-8} \text{ cm} \) (on the order of the interatomic spacing)
\( X = 1 \text{ cm} \) (characteristic dimension of mechanical test specimens)
The maximum shear strain obtained from the above equation is \( \sim 12\% \).

The necessity of a means of generating dislocations inside a stressed solid undergoing plastic deformation arises from the following observations: if deformation occurred solely by movement of pre-existing mobile dislocations, deformation would cease when all of these leave the solid body. In addition, the deformed body should have a lower dislocation density than the original specimen. Neither of these expectations has been verified by experiment: shear strains well in excess of the 12% calculated above are commonly observed (think of bending a paper clip); rather than decrease the dislocation density by strain, higher density of dislocations are observed in strained specimens.

**The Frank-Read Source**

Numerous mechanisms for generating dislocations within a solid are discussed by Hull (Ref. 6, Chap.8). Of these, the one proposed in 1950 independently and simultaneously by F. C. Frank and W. T. Read is the most important. The origin of this mechanism is the consequence of bowing the pinned dislocation segment shown in Fig. 7.18 into a semicircle, or when the radius of curvature is one half of the separation of the pinning points. The stress at which this occurs is termed the critical stress or the Frank-Read stress. It is obtained from Eq (7.20) by setting \( \Re = L/2 \):

\[
\sigma_{FR} = \frac{2Gb}{L}
\]  

Equation (7.21)

Figure 7.20 shows the sequence of shapes adopted by a pinned edge dislocation as the stress is increased up to and beyond the instability limit of Eq (7.21).

1. In the absence of a shear stress, the dislocation is a straight line between the two pinning points. This linear shape is the smallest length of the segment, and therefore the lowest energy configuration.
2. For a shear stress less than the critical value, the dislocation assumes the shape of the circular arc of Fig. 7.18.
3. When the stress is exactly equal to \( \sigma_{FR} \), the semicircular bowed dislocation is on the brink of instability.

4. If the stress is even slightly larger than \( \sigma_{FR} \), the dislocation passes rapidly through a sequence of shapes, all of which are unstable for the same reasons outlined for the circular dislocation loop in Sect 7.10. The loop swirls around the pinning points until the opposite lobes touch. The Burgers vector shown in stage 1 of Fig. 7.20 is the same for all parts of the loop, and at contact, is parallel to the dislocation. This means that the two contact points must be pure screw. At this point only, annihilation occurs because the contact is between two screw dislocations of opposite sense (assuming that the original pinned dislocation is pure edge).

The products of the annihilation process are shown in the last drawing of stage 4 in Fig. 7.20. A circular loop freed from the pinning points becomes ever larger because the stress that initiated the process (\( > \sigma_{FR} \)) exceeds that necessary to maintain and grow a loop. The other segment of dislocation resulting from the annihilation process is still attached to the pinning points and rapidly returns to the stage 1 shape.

As long as the stress is maintained \( > \sigma_{FR} \), the 4-stage sequence of Fig. 7.20 is rapidly repeated. The result is continuous production of loops that expand at speeds determined by obstacles that hinder their motion.

### 6.12 Impediments to Dislocation Motion

Hindrance of dislocation movement is divided into two classes, source hardening and friction hardening. The former is caused by impurity atoms, precipitate particles, or other microstructural defects that are attached to the dislocation in its unstressed state. These entities “lock” dislocations such that initiation of motion or operation of Frank-Reed sources requires a stress greater than that given by Eq (7.21). Friction hardening refers to the effect of obstacles of various types that impede the motion of mobile dislocations. The obstacles include other dislocations, foreign bodies ranging in size from impurity atoms to macroscopic precipitate particles and, in the case of metals irradiated by fast neutrons or fast ions, clusters of point defects. Friction hardening is further subdivided into long-range and short-range effects. Long-range friction hardening implies that the mobile dislocations do not directly contact the obstacle but feel its retarding force at a distance. In short-range hardening, the mobile dislocations encounter obstacles lying in their slip planes.

**Long-Range Friction Hardening**

The conventional picture of this type of impediment to dislocation motion is the interaction of two parallel edge dislocations on different slip planes, as depicted in the

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*The term hardening refers to the increase in yield stress (the stress required to initiate dislocation movement) and to the continual increase in stress required to maintain dislocation motion as plastic strain increases.*

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right hand diagram of Fig. 7.17. A critical value of the applied shear stress is necessary to force mobile dislocation 2 past immobile dislocation 1. From Eqs (7.15) and (7.16), the retarding force exerted on 2 by 1 is:

$$F_x = \frac{\Gamma b \sin(\theta)}{4y} \tag{7.22}$$

where \(\Gamma\) is the collection of elastic constants given by Eq (7.8). The maximum back force occurs at \(\theta = \pi/8\), or:

$$(F_x)_{\text{max}} = \frac{\Gamma b}{4y}$$

The forward force due to the applied shear stress is given by Eq (7.10), which in the present notation is:

$$(F_x)_{\text{app}} = \sigma_S b$$

When the applied shear stress is such that \((F_x)_{\text{app}}\) just exceeds \((F_x)_{\text{max}}\), dislocation 1 slips past dislocation 2 and goes on its way. The critical value of \(\sigma_S\) is:

$$\left(\sigma_S^\text{friction}\right)_{\text{crit}} = \frac{\Gamma}{4y} = \frac{Gb}{8\pi(1-\nu)y} \tag{7.23}$$

where Eq (7.8) has been used.

The separation of the slip planes \((y)\) is related to the dislocation density. From the discussion in Sect. 7.4, the separation of parallel dislocations in the unstressed solid should be of the order of the unit cell radius \(R\) given by Eq (7.4).

**Dislocation Climb**

Even if the applied shear stress is less than the critical value given by Eq (7.22), another mechanism is available for the mobile dislocation to pass the immobile one and continue on its way. By increasing the separation of the slip planes \((y)\) in Fig. 7.16) the critical shear stress is reduced until it eventually equals the applied shear stress. The process by which the mobile dislocation moves perpendicular to its slip plane is climb (see end of Sect. 7.3, and Fig. 7.11 in particular). With reference to Fig. 7.16, the normal stress \(\sigma_{xx}\) on dislocation 2 due to the presence of dislocation 1 is compressive (Eq (7.6a)). The effect of this stress is to reduce the vacancy concentration at the core of dislocation 2 to a value below that in the bulk solid. This concentration difference generates a flux of vacancies to the dislocation, which responds by climbing upward, as shown in Fig. 7.11.

The key to establishing the reduction of the vacancy concentration at the dislocation core is the pressure effect on the equilibrium vacancy concentration. This effect is analyzed in Sect. 3.2 and leads to Eq (3.4). In order to utilize this equation as a boundary condition on the vacancy diffusion equation, two modifications are needed.
First, Fick’s laws of diffusion are cast in terms of the volumetric concentration of the diffusing species. Equation (3.4) is converted to these units by dividing both sides by the atomic volume $\Omega$. Second, the pressure in Eq (3.4) is replaced by the compressive stress, which is the absolute value of $\sigma_{xx}$ in Eq (7.6a). With these modifications, the equilibrium vacancy concentration at the dislocation core is:

$$c_v(r_d) = c_v^{eq} \exp\left(-\frac{|\sigma_{xx}|\Omega}{kT}\right)$$  \hspace{1cm} (7.24)

where $r_d$ is the radius of the dislocation core, $k$ is Boltzmann’s constant, and:

$$c_v^{eq} = \exp\left(\frac{s_v}{k}\right)\exp\left(-\frac{\varepsilon_v}{kT}\right)\frac{1}{\Omega}$$ \hspace{1cm} (7.25)

is the equilibrium concentration in the stress-free solid.

The geometry in which vacancy diffusion towards the dislocation ($J_V$) induced by the above concentrations is shown in Fig. 7.21. The radius of the unit cell $R$ is given by Eq (7.4) in terms of the dislocation density.

The diffusion equation in the cylindrical annulus $r_d \leq r \leq R$ is:

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial c_v}{\partial r}\right) = 0$$

The boundary conditions are given by Eq (7.43) at the dislocation core and by Eq (7.25) at the periphery of the unit cell. The solution is:
\[
\frac{c^e_V - c_V}{c^e_V - c_V(r_d)} = \frac{\ln(R/r)}{\ln(r/r_d)}
\]

The flux of vacancies per unit length of dislocation is:

\[
J_V = 2\pi r_d D_V \left( \frac{\partial c_V}{\partial r} \right)_{r_d} = \frac{2\pi}{\ln(R/r_d)} D_V \left( c^e_V - c_V(r_d) \right)
\]

The first term on the left hand side depends only on the geometry of the unit cell, and is written as:

\[
Z_V = \frac{2\pi}{\ln(R/r_d)} \quad \text{(7.26)}
\]

so that \(J_V\) becomes

\[
J_V = Z_V D_V \left( c^e_V - c_V(r_d) \right) \quad \text{(7.27)}
\]

For typical values \(R \sim 10^{-5}\) cm and \(r_d \sim 3\times10^{-8}\), \(Z_V \sim 1\), a value that will be used throughout for numerical work.

\(c_V(r_d)\) is expressed by Eq (7.24) in which the exponential term is represented by its one-term Taylor series expansion. Eq (7.27) reduces to:

\[
J_V = Z_V D_V c^e_V \frac{\sigma_{xx}}{kT} = Z_V D \frac{\sigma_{xx}}{kT} \quad \text{(7.28)}
\]

where the self diffusion coefficient \((D)\) has been substituted according to Eq (4.32b).

The connection between the vacancy flux and the climb velocity can be deduced with the aid of Fig. 7.11. Let \(\delta t\) be the time required to remove one row of atoms from the bottom of the dislocation’s half plane. The empty volume created (per unit length of dislocation) is \(v_c \delta t \ b\), where \(v_c\) is the climb velocity and \(b\), the Burgers vector, is the approximate width of the half plane of atoms. The volume supplied by the vacancy flux during this time is \(J_V \Omega \delta t\). From this volume balance, \(v_c = J_V \Omega / b\). Substituting Eq (7.28) for \(J_V\) gives the climb velocity:

\[
v_c = \frac{Z_V \Omega D \left| \sigma_{xx} \right|}{bkT} \quad \text{(7.29)}
\]

**Example:** Two parallel edge dislocations on parallel slip planes initially 5 Burgers vectors apart are present in a specimen of iron. The applied shear stress pushing mobile dislocation 2 against immobile dislocation 1 in Fig. 7.16 is one half of the critical value given by Eq (7.23).
Considering two temperatures, 500 K and 1000 K, how long will it take for dislocation 2 to climb until the critical stress is reduced to the actual stress?

Substituting \( \frac{dy}{dt} \) for \( v_c \) and Eq (7.6a) for \( \sigma_{xx} \) (in which \( r = y / \sin \theta \)) into Eq (7.29) yields:

\[
\frac{dy}{dt} = \frac{Z \Omega D \Gamma}{b k T} \sin^2 \theta \left( 1 + 2 \cos^2 \theta \right) y
\]  \hspace{1cm} (7.30)

\( \Gamma \) contains the elastic constants of the medium and is given by Eq (7.8).

In order to integrate the above equation, \( \theta \) is expressed in terms of \( y \) by equating the back force on dislocation 2 due to dislocation 1 given by Eq (7.22) to the forward force \( \sigma_S b \) exerted by the applied shear stress. \( \sigma_S \) is equal to one-half of the initial critical shear stress given by Eq (7.23), or \( \sigma_S = \Gamma/8y_o \), where \( y_o \) is the initial separation of the slip planes. This yields:

\[
\theta = \frac{1}{4} \sin^{-1} \left( \frac{1}{2} \frac{y}{y_o} \right) = \frac{1}{4} \sin^{-1} \left( \frac{1}{2} Y \right)
\]  \hspace{1cm} (7.31)

where \( Y = y/y_o \). Equation (7.30) is made dimensionless using \( y = y_o Y \) and \( t = t^* \tau \), where the characteristic time for climb is given by:

\[
t^* = \frac{bkTy_o^2}{Z \Omega D \Gamma}
\]  \hspace{1cm} (7.32)

For dislocation 2 to overcome the back force of dislocation 1, it must climb to double the separation of the slip planes, or to \( Y = 2 \). This separation of the slip planes requires a dimensionless time given by:

\[
\tau_f = \int_1^2 \frac{YdY}{\sin^2 \theta \left( 1 + 2 \cos^2 \theta \right)}
\]

Using Eq (7.31) to express \( \theta \) in terms of \( Y \), the integral is \( \tau_f = 18.3 \). The time for dislocation 2 to escape dislocation 1 is \( t_f = 18.3 t^* \). In order to evaluate \( t^* \) from Eq (7.32), the following properties of iron are used:

- \( G = 73 \) Gpa (the room temperature value)
- \( b = 0.3 \) nm
- \( \nu = 0.33 \)

These values yield \( \Gamma = 5.2 \) N/m (using Eq (7.8)).

- \( Z_V = 1 \)
- \( y_o = 5b = 1.5 \) nm
- \( \Omega = 1.1 \times 10^{-29} \) m\(^3\)/atom
- \( k = 1.38 \times 10^{-23} \) J/K
- \( D = 1.9 \times 10^{-4} \exp(-28,800/T) \) m\(^2\)/s

At 1000 K, Eq (7.32) gives a characteristic climb time of \( t^* = 2.7 \) ms, or the time for the dislocation to climb to the point that the blocking dislocation is circumvented is 50 ms. At this temperature, climb is so rapid that friction hardening is negligible. At 500 K, on the other hand,
the decrease in the self diffusion coefficient is so pronounced that the comparable escape time is 2 ½ millennia. The only way that a blocked mobile dislocation can pass by an immobile one is to increase the applied shear stress until it exceeds the critical value.

**Jogs**

So far, only the interaction forces between parallel dislocations have been considered. However, even the simplified representation of dislocations in Fig. 7.13 shows that moving dislocations are sure to encounter other dislocations at an angle other that 0°. When this occurs, the two dislocations physically touch each other. Depending on the magnitude of the stress, the two dislocations become immobilized, or if the stress is sufficiently high, one dislocation can cut through the other and both can continue on their way, albeit with more difficulty than before intersection.

There are a large number of combinations of interacting dislocations. Each combination: i) requires a different stress for the crossed dislocations to disengage; ii) leaves one or both disfigured by a kink in the initially straight or continuously curved line; iii) upon disengagement from each other one or both require a larger stress to continue moving than prior to the intersection; iv) the angle of approach of the two dislocations can be anywhere between 0 and 90°. The remnants of the intersection are affected by all of the above features.

In order to provide a rudimentary understanding of the dislocation intersection phenomenon, six combinations grouped into two sets are shown in Fig. 7.22.

**Fig. 7.22** Intersection of three vertical dislocations by a moving edge dislocation (left) and by a moving screw dislocation (right)

The interacting dislocations are at right angles to each other. In the two sets of interactions shown in Fig. 7.22, the three vertical dislocations, which are the same in both sets, are assumed to be immobile. In the set on the left, a horizontal edge

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† Sect. 6.7.9 without applied stress; Sect 6.7.10 with applied stress for screw dislocations; earlier in this section with applied stress for edge dislocations

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dislocation has cut through the three vertical ones. In the right hand set, the moving dislocation is a pure screw.

The intersection of moving edge dislocation \( b_a \) with vertical edge dislocation \( b_1 \) produces a jog in the latter, but not in the former. The length of the jog in \( b_1 \) is of the order of an atomic spacing\(^*\). It is treated as a small segment of a dislocation that maintains the Burgers vector of \( b_1 \). Since the jogged section is perpendicular to \( b_1 \), it must be of edge character (see Fig. 7.2 and p.4). Moreover, because the Burgers vector \( b_1 \) lies in the slip plane of the segment, the jog does not interfere with the glide motion of the vertical edge dislocation that contains it.

In the next case, the Burgers vectors \( b_2 \) and \( b_e \) are parallel. The jogs that appear on both dislocations are parallel to the Burgers vectors, and consequently are short segments of screw dislocations. The shear stress responsible for moving \( b_2 \) and \( b_e \) are parallel to the jogged segments, which move along the main dislocations in a manner similar to the motion of the wave in a garden hose when one end is given a sharp shake. Neither jog impedes the motion of the main dislocation.

When the moving edge dislocation \( b_e \) cuts through the vertical screw dislocation \( b_3 \), the jogged sections are perpendicular to the Burgers vectors of the original dislocations. Hence, the segments are of edge character. An edge dislocation can slip only in a plane containing both the line and its Burgers vector, which is true for the jogs in both \( b_3 \) and \( b_e \). However, for slip, the jog must move in a direction perpendicular to itself. This is true of the jog in \( b_e \), which therefore does not hinder motion of the main dislocation. The extra half plane (or more precisely, half row) edge jog in \( b_3 \) lies in the plane perpendicular to the main screw dislocation. Depending on the orientation of the vertical shear stress component (i.e., \( \sigma_{zx} \) or \( \sigma_{zy} \)), the main screw dislocation is capable of glide in a direction either perpendicular to \( b_3 \) or parallel to it. In the former case, the edge-like jog would need to move parallel to itself, which it normally cannot do. In the latter case, movement of the jog in \( b_3 \) would require climb (because the short “half-plane” of the edge jog is perpendicular to \( b_3 \)). Such movement would require a higher stress than that for glide, and in addition, would require absorption or emission of point defects.

The consequences of the dislocation intersections in the three cases analyzed above illustrate the following general rules:

1. Jogs in pure edge dislocations do not impede glide motion of the main dislocation. However, the slip plane of the jogged section may not be as energetically favored as that of the main dislocation.

2. Jogs in pure screw dislocations hinder movement of the parent dislocation, which requires a higher applied stress to drag along the recalcitrant jog.

The shear stress required to move a screw dislocation decorated with a number of edge jogs is obtained by considering the \( b_3 \) screw dislocation in Fig. 7.22. It is gliding

\(^*\) It may appear odd that a jog one atomic diameter in length has the properties of a line defect. In fact it does not (Ref. 6, p. 131). However, enough of the properties of a true line defect are at least partially retained by the nanometer-length line that treating it as a dislocation segment is useful.
(with difficulty) in the y direction under an applied $\sigma_{xz}$ shear stress. The edge jogs glide freely up and down the main screw dislocation. Some jogs are positive and others are negative, depending on whether the screw dislocation is moving in the +y or –y direction. Jogs of opposite sign annihilate each other until only one kind remains. Since edge dislocations of the same sign repel one another, the jogs assume equidistant positions along the screw dislocation.

At low shear stress the mobile segments of the screw dislocation between the jogs will be pinned by the jogs, and can only bow out in the manner shown in Fig. 7.18. If the applied shear stress is sufficiently large, the array of jogs moves by climb along with the main screw dislocation. At temperatures low enough so that the point defects are immobile, this process leaves a trail of point defects in the wake of each climbing jog. Figure 7.23 shows a moving screw dislocation with climbing vacancy-type jogs. If the applied shear stress tended to move the screw dislocation in the –y direction, a string of self interstitials would be left behind.

**Fig. 7.23  Mechanism of movement of a jogged screw dislocation**

The stress required to initiate the process shown in Fig. 7.23 is obtained by equating the energy to create a vacancy to the work done by the interjog segment of the screw dislocation as it moves one Burgers vector in its glide direction.

The energy required to create a vacancy beneath a jog is assumed to be the same as that for a normal lattice vacancy (Sect. 3.2) even though the surroundings of the vacancy are quite different in the two cases. A normal vacancy resides in an otherwise unperturbed lattice site, whereas the jog vacancy sits in the highly disrupted core of an edge dislocation. In addition, the energy of formation is replaced by:

$$\varepsilon_{v,jog} = 0.2 G b^3$$

The right hand side of this equation is the energy added to that of the dislocation by the presence of the jog; $G b^2$ is the energy per unit length of a dislocation (Eq 7.9), and if the jog is regarded as a dislocation of length b, its energy is $G b^3$. The factor of 0.2 in the above equation arises from the much reduced range of the stress field due to the jog.
compared to that around an ordinary dislocation. However, there is no physical reason why the energy of the jog can be taken to be the energy to form a vacancy. The above evaluation of the formation energy of a jog vacancy is given without physical justification in Ref. 8, p. 105 and Ref. 9, p. 597.

Nonetheless, the approximation is remarkably accurate; for copper, $G = 46$ GPa and $b = 0.26$ nm, for which the above equation gives $\varepsilon_{v,\text{jog}} = 90$ kJ/mole. The normal vacancy formation energy for copper is $\varepsilon_v = 100$ kJ/mole

Creation of a vacancy beneath the jog is accompanied by glide of the screw dislocation a distance $b$, the Burgers vector. On a per-jog basis, the work required is the force per unit length of dislocation given by Eq (7.11a), in which the shear stress is $\left(\sigma^\text{crit}_S\right)_{\text{jog}}$, times $L$, the interjog spacing (Fig. 7.23). The work performed by the stress is

$$W = Lb^2 \left(\sigma^\text{crit}_S\right)_{\text{jog}}$$

Equating the work to the energy expended in producing the vacancy gives the critical stress as:

$$\left(\sigma^\text{crit}_S\right)_{\text{jog}} \cong 0.2 \frac{Gb}{L}$$

(7.33)

Obstacles

Commercial metals contain obstacles to dislocation motion ranging in size from impurity atoms to large irradiation-produced voids. These obstacles are introduced into the solid by a variety of means, including: i) expressly during fabrication - intermetallic precipitates and oxide particles are in this class; ii) by lack of complete purification during fabrication - commercial aluminum is stronger than high-purity aluminum; iii) during operation – the voids, bubbles and defect clusters produced in most metals by irradiation by high-energy neutrons or ions are the most notable of this class of obstacles.

Irrespective of how they were introduced into the metal, obstacles have several features in common. First, they can be approximated by spheres or points for the purpose of analysis. Second, they impede or stop the motion of dislocations. Third, as a consequence of the second, their effect is to harden the metal and at the same time render it more brittle.

Obstacles range in size from individual impurity atoms, small clusters of point defects 1 – 2 nm in size found in metals irradiated by fast neutrons at low temperature, and at the high end, large, widely-spaced precipitate particles. These obstacles have in common a stress field around them that either attracts or repels the dislocation. In the latter case, the applied stress must be large enough to overcome the repulsive force; in the former case, the applied stress must be able to tear the dislocation away from its attractive trap.
Two mechanisms are available to dislocations to overcome an obstacle. 1) if the obstacle is small enough, the dislocation can cut through it; 2) the dislocation can bow around an array of large obstacles. The cutting mechanism is not treated here; brief discussions of this mechanism can be found in Sect. 18.5 of Ref. 3 and in Sect. 10.6(a) of Ref. 5.

The stress required to push a dislocation through an array of impenetrable spherical obstacles depends on their center-to-center spacing in the slip plane, $L$. This in turn is a function of the obstacle’s radius $R$ and its number density in the solid, $N$. These two quantities are related to $L$ with the aid of the sketch on the left of Fig. 7.24.

![Sketch of spherical obstacles and slip plane](image)

**Fig. 7.24** The intersections of volume-distributed spherical objects with a plane

A sphere whose center lies within the square volume $2R \times 1 \times 1$ will touch the plane. The number of spheres within this volume is $2RN$, which is also the number of object intersections with the unit plane.

The right hand sketch of Fig. 7.24 shows unit cells into which the random positions of the circular intersections of the obstacle and the slip plane are arranged for the purpose of calculation. The unit cell is a square of side $L$ with an object’s intersection in its center. The area of this unit cell, $L^2$, divided into the unit area of the plane is also the number of intersections of the objects with unit plane. Equating these two expressions and solving for $L$ yields:

$$L = (2RN)^{-1/2}$$ (7.34)

The average radius of the intersections shown as shaded circles in Fig. 7.24 is $\pi R/4$.

Figure 7.25 shows how a straight dislocation pushed to the right by the applied shear stress interacts with the array of obstacle intersections on its slip plane. Not only the intersections of the volume-distributed obstacles placed on a square grid, but their radii are all accorded the average value of $\pi R/4$. The dislocation interacts with the obstacles at their peripheries, whose separation distance is less than the center-to-center distance of Eq (7.34) by the average diameter:

$$l = L - \pi R/2 = (2RN)^{-1/2} - \pi R/2$$ (7.35)
The sequence of events by which the dislocation passes through the obstacle array on its slip plane can be broken into 4 stages. The straight dislocation approaches the obstacle array (1) and upon encountering the obstacles, bows out between them (2). If the applied stress is too small to bow the line into a semicircle, the dislocation line is stuck in this configuration with the sections bowed to a radius of curvature given by Eq (7.20). At an applied stress sufficiently large to bow the dislocation between obstacles into a semicircle (3), the shape becomes unstable and expands in the same manner as the Frank-Read source shown in Fig. 7.20. When the portions of adjacent expanding bows touch, the dislocation line is reformed and proceeds on its way, leaving behind a circular dislocation loop of the type shown in Fig. 7.5 around each obstacle (4). The stress required to reach the instability of stage 3 is given by Eq (7.20) with $\gamma_R = l/2$:

$$\phi_{S\text{, obstacle}}^{\text{crit}} = \frac{2Gb}{l} \quad (7.36)$$

where $l$ is given by Eqs (7.35) and (7.34).

As successive dislocation lines pass through the obstacle array, the residual loops accumulate around the obstacles and effectively increase their size. Because of this effective growth in the size of the obstacles, the separation of the obstacles decreases below the initial value $l$, and the according to Eq (7.35), the stress to pass the barrier increases. This effect contributes to the macroscopic phenomenon of work hardening.

**Locking and Unlocking**

The addition of impurity atoms, whether they be substitutional or interstitial, harden metals by hindering the start of edge dislocation motion (they have no effect on screw dislocations). The impurities tend to accumulate around edge dislocations because at this location the system’s energy is reduced. The solid above the slip plane (i.e., in the section containing the extra half-plane of atoms) is in compression while the solid below the slip plane is in tension. A substitutional impurity atom that is larger than the host atoms generates a compressive stress field in the immediate vicinity. Consequently,
has a tendency to accumulate on the tensile side of the slip plane, because the compressive stress is relieved by the dislocation’s tensile stress and vice versa. For the same reason, substitutional impurity atoms smaller than the host atoms tend to migrate to the compressive side of the slip plane. Interstitial impurity atoms are small compared to the host atoms (e.g., carbon, nitrogen, boron) yet they always create a compressive stress field around them because they are not in a regular lattice site. Consequently, interstitial impurities tend to collect just below the end of the edge dislocation’s half plane (i.e., in the open space in the middle diagram of Fig. 7.2).

In order for edge dislocations to accumulate impurity atoms, a combination of high temperature and/or long times is needed. This can occur during fabrication as the molten metal cools and solidifies, the principal example being carbon in steel. Alternatively, high temperatures during operation of the component or very long times at low temperatures (called aging) can produce the same result.

Analysis of this phenomenon is based on interstitial impurities, although the same general results apply to substitutional alloying atoms. The first step is to determine the interaction energy between the interstitial atom and the edge dislocation. The interstitial atom is regarded as an oversized rigid sphere inserted into a hole of radius $R_o$ in the solid. $R_o$ is roughly equal to the cube root of the volume of an interstitial site in the host crystal structure. Insertion of an interstitial atom into this hole increases the radius of the hole to $(1 + \varepsilon)R_o$, $\varepsilon$ being << 1 to an extent depending on the radius of the interstitial atom and the size of the interstitial sites in the host lattice. The volume change of the solid in the immediate vicinity of the interstitial is:

$$\Delta V = \frac{4}{3} \pi [(1 + \varepsilon)^3 R_o^3 - R_o^3] \approx 4\pi R_o^3 \varepsilon$$

Expansion of the medium in the vicinity of an edge dislocation performs work against the hydrostatic component of the dislocation’s stress field. From Eqs (7.6) this is:

$$\sigma_h = \frac{1}{3} (\sigma_r + \sigma_{\theta\theta} + \sigma_{zz}) = \frac{Gb}{3\pi (1 - \nu)} \frac{\sin \theta}{r}$$

The work done in expanding against this stress is the interaction energy, $E_i = \sigma_h \Delta V$. Multiplying the above two equations and assuming $\nu = 1/3$ gives:

$$E_i = \frac{8}{3} Gb \varepsilon R_o^3 \frac{\sin \theta}{r}$$

The interstitial atoms are attracted to the atom half plane at the underside of the slip plane because $\sin \theta < 0$ at this location and so $E_i$ is negative. (the geometry defining $\theta$ and $r$ is that shown in Fig. 7.16, with the interstitial replacing dislocation #2)

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# The exact value of the numerical coefficient in Eq (6.7.37) is 4 (Ref. 6, p. 172 et seq).
Because of the reduction of energy with decreasing r, interstitials will tend to congregate around the underside of the edge dislocation. Not all interstitials in the vicinity of the dislocation can occupy the minimum energy sites, which according to Eq (7.37), is at \( \sin \theta = -1 \) and \( r \) is at its minimum value. The latter, designated \( r_0 \), is the distance separating the interstitial located just below the half plane of the dislocation and the terminating atoms of the half plane (i.e., the open space in the middle sketch of Fig. 7.2). First, the minimum energy location is not an unsaturable sink; there is only one interstitial site for every terminating atom of the dislocation half plane. Second, if \( E_i \) is of the order of thermal energy (~kT), vibrations of the host atoms in the lattice (phonons) establish an equilibrium Boltzmann distribution of interstitials around the dislocation:

\[
x(r) = x(\text{bulk}) \exp(-E_i/kT)
\]

(7.38)

\( x(r) \) is the fraction of interstitial sites at distance \( r \) from the dislocation that are occupied by an interstitial atom. \( x(\text{bulk}) \) is the interstitial site fraction far from the dislocation line.

The lion’s share of the interstitials are located beneath the dislocation’s half plane since here \( E_i < 0 \). However, some interstitials remain above the slip plane where \( E_i > 0 \). The overall increase in the interstitial population around the edge dislocation is termed a dislocation cloud.

Eq (7.38) fails when \( x(r) \) exceeds unity at any \( r \). In this case, the interstitial sites just below the half plane are fully occupied. The tensile stress field normally associated with the solid below the slip plane of an edge dislocation now becomes greatly reduced because of the interstitials in the positions just below the half plane. The interaction energy with this interstitial-saturated edge dislocations and interstitials in the lattice is greatly reduced, and the impurity cloud tends to drift away. What remains is a dislocation “locked” by the row of underlying interstitials attached to it. The applied shear stress required to break the dislocation is called the unlocking stress. Calculation of this stress is given in Ref. 5, p. 216, and will be summarized here. Figure 7.26

![Diagram of interstitials and dislocation](image)

Fig 7.26  Row of interstitials filling all sites below the half plane of an edge dislocation. End-on view (left); the unlocking process (right)
interaction energy between the dislocation line and a single interstitial atom given by Eq (7.37) can be written in compact form as:

\[ E_i = A \frac{\sin \theta}{r} \]  

(7.37a)

where A is the collection of constants in Eq (7.37). The angle \( \alpha \) in the triangle in Fig. 7.26 is equal to \(-\theta\), so that \( \sin \theta = -\sin \alpha = -r_o/r \) and Eq (7.37a) becomes:

\[ E_i = -A \frac{r_o}{r^2} = -\frac{Ar_o}{x^2 + r_o^2} \]  

(7.37b)

The force between the dislocation line and the single impurity interstitial atom intersected by the triangle in Fig. 7.26 is the derivative of Eq (7.37b). Since interstitial atoms in the row are separated by approximately one Burgers vector, there are \( 1/b \) interstitials per unit length of the row. The force between the dislocation located a distance \( x \) from its locked position and the interstitials in unit length of the row is:

\[ F(x) = -\frac{1}{b} \frac{dE_i}{dx} = -\frac{2Ar_o}{b} \frac{x}{x^2 + r_o^2} \]

The force is negative, meaning that the row of interstitial atoms restrains the movement of the dislocation. Counter-intuitively, the force is not a maximum when \( x = 0 \). The above function of \( x \) has a maximum at \( x = r_o/\sqrt{3} \); since \( r_o \) is of the order of a Burgers vector, the maximum force occurs at a very small \( x \) displacement from the dislocation's locked position. The maximum restraining force is:

\[ F_{\text{max}} = -\frac{3\sqrt{3}A}{8br_o^2}, \quad \text{in \ N/m} \]

\( F_{\text{max}} \) is the maximum force between the entire dislocation line and a unit length of the row of interstitials. By symmetry, it also represents the maximum retarding force on a unit length of dislocation exerted by the entire row of interstitial atoms.

For the dislocation to become unlocked, the opposing applied force (given by Eq (7.18)) must equal \( F_{\text{max}} \), or the unlocking shear stress is:

\[ (\sigma_{S,S})_{\text{unlock}} = \frac{3\sqrt{3}A}{8b^2r_o^2} = \sqrt{3} \varepsilon \frac{R_o^3}{br_o^2} G \]  

(7.38)
Summary

The four mechanisms that impede the motion of dislocations analyzed in this section are summarized in Table 7.4, which also gives best-estimate numerical values for the critical stresses (relative to the shear modulus).

Table 7.4 Numerical comparison of the mechanisms hindering dislocation motion

In formulas, \( b \) = Burgers vector = 0.3 nm; \( R \) = radius of dislocation unit cell = \( \rho^{-1/2} \)

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>( \frac{\sigma^\text{crit}}{G} ) (formula)</th>
<th>Equation</th>
<th>( \frac{\sigma^\text{crit}}{G} ) (value)</th>
<th>Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction</td>
<td>( \frac{b}{R} \frac{1}{8\pi(1-\nu)} )</td>
<td>7.23</td>
<td>2\times10^{-5}</td>
<td>( \rho = 10^8 \text{ cm}^{-2} )</td>
</tr>
<tr>
<td>Jogs</td>
<td>( \frac{0.2b}{L} )</td>
<td>7.33</td>
<td>6\times10^{-4}</td>
<td>( L = 2R ) = separation of jogs on screw dislocation – Fig.7.23; ( \rho = 10^{10} \text{ cm}^{-2} )</td>
</tr>
<tr>
<td>Obstacles</td>
<td>( \frac{2b}{l} )</td>
<td>7.36</td>
<td>6\times10^{-4}</td>
<td>( l = 1 \mu m ) = spacing of obstacles on slip plane</td>
</tr>
<tr>
<td>Locking</td>
<td>( \sqrt{3} \varepsilon \frac{R_o^3}{br_o^2} )</td>
<td>7.38</td>
<td>4\times10^{-3}</td>
<td>( \varepsilon = 0.02 ) = fractional radial expansion of interstitial site when occupied; ( r_o = b ) = separation of interstitial row and bottom of half plane of the edge dislocation – Fig. 7.26; ( R_o^3 = \Omega/4 = 1.2\times10^{-23} \text{ cm}^3 ) ( R_o ) = interstitial site radius</td>
</tr>
</tbody>
</table>

The critical stresses due to friction and jogs depend only on the presence of other dislocations at a reasonably high density. These critical stresses are in satisfactory accord with the critical resolved shear stress measured on single crystals (Sect. 7.3). The critical stresses from the obstacle and locking mechanisms are very sensitive to the concentration of impurities in the crystal.

The first three mechanisms are active for moving dislocations. It is possible that a mobile dislocation is subject to all three at once. In this case, the critical shear stress to maintain motion would be the sum of the critical stresses for the individual mechanisms.

The critical stress for unlocking a dislocation acts only to initiate motion of a dislocation. Once freed from the row of impurity atoms, the stress to keep the dislocation moving may be lower than the unlocking stress. This is observed in engineering stress-strain tests and is called a yield drop.
Problems

7.1 A single crystal bar of a bcc metal is cut into a uniaxial tension specimen with the (100) planes perpendicular to the direction of the applied force F. The cross sectional area of the specimen is A. Slip occurs on the (110) planes in a [111] direction. The critical resolved shear stress of the metal is $\sigma_C$. At what value of $F/A$ does slip occur? Note that the shear stress on the (110) plane is the resolved force parallel to this plane divided by the area of the plane.

Diagram on original

7.2

Draw the end-on view analogous to that of Fig. 6.6 for the $(a_0/2)[111](110)$ edge dislocation in the bcc crystal structure. Note that the bcc structure can be viewed as a stacking of (111) planes. What is the magnitude of the Burgers vector of this dislocation? A diagram of the (110) plane in the bcc lattice is shown below for your convenience.
7.3 Derive the force per unit length between parallel edge dislocations with perpendicular Burgers vectors. Note: the Burgers vectors in the diagram on p. 12 are parallel. For this problem, rotate E₂ by 90° clockwise.

7.4

(a) The common dislocation on the (111) plane of the fcc lattice has the designation of a₀/2 [110]. This dislocation is b₁ in the sketch. Another dislocation, called a Shockley partial, b₂ is also shown. What is its designation?

(b) A dislocation, called the Frank partial (b₃), is formed by the periphery of a disk-shaped layer of interstitials. This disk, or loop, is inserted between (111) planes in the fcc structure. The periphery of the disk is an edge dislocation. The separation of (111) planes in the fcc structure is a / \sqrt{3}. What is the designation of this dislocation?

7.5 Consider slip in a single crystal as shown in Fig. 6.3.

(a) For a given value of \( \phi \), at what value of \( \lambda \) is the resolved shear stress \( \sigma_S \) a maximum? What is \( (\sigma_S)_{\text{max}} \) for this condition?

(b) At what value of \( \phi \) is the result of part (a) an absolute maximum, \( (\sigma_S)_{\text{max}} \)? What is the ratio of \( (\sigma_S)_{\text{max}} \) to the applied stress F/A?
7.6
Consider a unit cube of a crystal (1 cm on a side) having a dislocation density \( \rho \) cm\(^{-2}\). Assume all dislocations are in one direction and parallel to each other. If on application of a shear stress in the right direction, all of these dislocations emerge at the surface of the cube, what is the shear strain? The magnitude of the Burgers vector is \( b \).

If \( \rho = 10^6 \) cm\(^{-2}\) and \( b = 3 \times 10^{-8} \) cm, what is the shear strain? Does this result explain the observed behavior of ductile materials?

7.7
The critical resolved shear stress of high-purity single-crystal copper is 0.5 MPa. A specimen of this metal is loaded in tension in the [110] direction. At what applied tensile stress will (111)[101] slip occur?

7.8
Consider two parallel edge dislocations of the same sign on parallel glide planes that are five Burgers vectors apart (diagram on p. 12). The lower dislocation is immobile but the other reacts to an applied shear stress equal to 1% of the shear modulus acting perpendicular to the mobile dislocation. How close (radial distance in units of the Burgers vector) are the two dislocations at equilibrium?

7.9.
Calculate the magnitude (in nm) of the burgers vector for \( \alpha \)-Fe (bcc Fe) and for hcp Zr. From this result and using the shear modulus given in the notes calculate an approximate value for the dislocation self-energy per unit line (J/m) in these metals.
References


