Response of an Annular Burner Nozzle to Transverse Acoustic Excitation

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Transverse combustion instabilities are increasingly problematic in land based and aero-gas turbine engines. A common problem in rockets and augmenters, these transverse modes have a circumferential and/or radial structure and are often associated with high frequency oscillations. Depending on its location in the combustor, a flame experiences different parts of a standing circumferential acoustic wave, or in the case of a spinning waveform, a flame sees different parts of the wave structure over time. The current study investigates the response of an annular, swirling nozzle to two limits of the standing wave transverse excitation structure, associated with a pressure node and antinode at the nozzle centerline.

The resulting disturbance field is significantly different for these two cases. This paper presents results showing how the disturbance field is composed of long wavelength, multi-dimensional acoustic disturbances and shorter wavelength convecting vortical disturbances. The flow vorticity originates in the separating boundary layers of the inner and outer annulus and rolls up into larger structures inside and outside of the annular jet. These structures, in turn, merge downstream in a staggered fashion into a single, larger vortex that convects downstream in the annular jet centerline. This paper quantifies the relative strengths and phasing of these disturbances, and discusses the key features of the disturbance field exciting the flame.

Nomenclature

\begin{align*}
\hat{A} & = \text{Gain of Fourier transform} \\
\hat{D} & = \text{Outer diameter of nozzle} \\
f & = \text{Frequency} \\
G & = \text{Flame position} \\
p & = \text{Pressure} \\
p_o & = \text{Time averaged pressure} \\
\bar{p} & = \text{Spatially averaged instantaneous pressure} \\
r & = \text{Radius from nozzle centerline} \\
S_L & = \text{Laminar flame speed} \\
t & = \text{Time} \\
u & = \text{Axial velocity} \\
\bar{u} & = \text{Spatially averaged instantaneous axial velocity} \\
u_o & = \text{Time averaged axial velocity} \\
\hat{u} & = \text{Harmonically reconstructed axial velocity} \\
\bar{u} & = \text{Total velocity vector} \\
v & = \text{Transverse velocity} \\
\bar{v} & = \text{Spatially averaged instantaneous transverse velocity} \\
v_o & = \text{Time averaged transverse velocity} \\
\hat{v} & = \text{Harmonically reconstructed transverse velocity}
\end{align*}

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The objective of this study is to improve understanding of transverse combustion instabilities in premixed combustors. It is a companion study to work presented in O’Connor et al. [1]. Combustion dynamics is a problem that has plagued numerous rocket and air-breathing engine development programs. The coupling of resonant acoustics in the combustion chamber and heat release fluctuations from the flame [2] can cause structural damage, and decrease operational flexibility and performance. In particular, instabilities have caused major challenges in the development of lean, premixed combustors used in low NOx gas turbines [3].

Transverse instabilities have been discussed extensively in the afterburner [4-7], solid rocket [8, 9], and liquid rocket literature [10-15]. For low NOx gas turbines, most of the work over the last two decades has focused on the longitudinal instability problem [16-18]. Significantly less work has been done on the analysis and characterization of transverse instabilities for gas turbine type applications, characterized by a swirling, premixed flame [19-24]. There are, however, two key application areas where transverse acoustic oscillations are of significant practical interest. The first occurs in annular combustion systems, such as the instabilities in the Solar Mars 100, GE LM6000, Alstom GT24, Siemens VX4.3A, and others [20, 25-27]. Because of the larger length scales involved, these instabilities often occur at similar frequencies (e.g., 100’s of Hertz) as the longitudinal oscillations encountered in can type combustion systems. As these frequencies cannot be simulated without the full annulus, several of these companies report the results of longitudinal instability tests obtained on single nozzle rigs scaled to have similar longitudinal acoustic frequencies as the observed transverse instabilities [25, 28, 29]. This type of test presumes that the key physical processes occurring in a transversely excited flame can be simulated in a longitudinally excited one, a hypothesis that will be discussed extensively in this paper. The second application where transverse instabilities are of interest is the higher frequency transverse oscillations encountered in can combustion systems. These instabilities occur at relatively high frequencies, in the 1-5 kHz range, and scale with the combustor can diameter. While relatively little treatment of these high frequency oscillations can be found in the technical literature, there is significant discussion of them in the gas turbine operator/user community; e.g., see Combined Cycle Journal [30] or Sobieski and Sewell’s chapter in Combustion Instabilities in Gas Turbine Engines [26].

In both types of combustion systems, annular systems and can systems, nozzles are distributed around the annulus or cylinder. As such, in both the case of standing azimuthal waves and traveling azimuthal waves, each nozzle experiences a different disturbance field than its neighbors. This point is illustrated in Figure 1, which shows the pressure fluctuation of the first azimuthal mode in an annular combustor from work done by Sensiau et al. [31].

\[ x = \text{Axial coordinate} \]
\[ \gamma = \text{Ratio of specific heats} \]
\[ \phi = \text{Phase of Fourier transform} \]
\[ \omega = \text{Vorticity} \]
\[ \omega_0 = \text{Time averaged vorticity} \]
\[ \hat{\omega} = \text{Harmonically reconstructed vorticity} \]

**Figure 1.** Pressure fluctuation of the first azimuthal mode in an annular combustor, flow left to right. Pressure and velocity nodes shown with arrows. Reproduced from Sensiau et al. [31].

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A nozzle situated near the velocity node and another near the pressure node experience significantly different disturbance fields, and possibly different flame excitation physics. In the case of a standing wave, the flame response between each nozzle would vary in space, but in the case of the spinning wave, the flame response would not only vary around the annulus or can, but also vary in time at each nozzle. Additionally, the disturbance field characteristics vary greatly as frequency of the acoustic excitation changes. At low acoustic frequencies, corresponding to long wavelengths, the transverse disturbance does not vary significantly across the flame region. When the flame is at a velocity node, the transverse acoustic velocity fluctuation at the flame is low, and when the flame is at a pressure node, corresponding to a velocity anti-node, the velocity fluctuation is large in the flame zone. At a higher frequency, though, the fluctuation amplitude of the wave may vary spatially over the flame region, and so the disturbance field can be significantly different.

Transverse acoustic oscillations in combustors introduce a variety of new physics and processes relative to longitudinal modes. The focus of the next two sections is to elucidate these differences. The key problem we focus on here is the manner in which the flame responds to oscillations in flow velocity. This is referred to as the “velocity coupled” response mechanism [32-36], to be distinguished from the also important “fuel/air ratio coupling” [37, 38] mechanism, or the probably negligible “pressure coupling” mechanism [39]. Flow oscillations lead to flame wrinkling that, in turn, causes oscillations in flame surface area and rate of heat release. In the discussion below, we break this problem into two components, (1) nature of the velocity field that is exciting the flame, and (2) flame response to this flow excitation.

A. Velocity Field Disturbing the Flame

While acoustic oscillations can directly excite the flame, they also excite organized flow instabilities associated with shear layers, wakes, or the vortex breakdown bubble [40-42]. Taken together, there are a number of potential sources of flow disturbances that can lead to heat release oscillations. Two of these sources are directly acoustic in origin - transverse acoustic motions associated with the natural frequencies/mode shapes of the combustion system, and longitudinal oscillations due to reflection from the flame [19, 20] and the oscillating pressure drop across the nozzle. An alternative way of thinking about the latter case is that transversely propagating acoustic wave propagate into the nozzle and enter the nozzle passages, leading to longitudinal mass flow oscillations at the burner exit. Recent simulations by Staffelbach [19] suggest that it is these longitudinal mass flow oscillations that most significantly control the flame response to the imposed transverse disturbance.

Most studies suggest that it is vortical flow oscillations that most significantly control the flame’s heat release response [43-45]. In other words, acoustic oscillations excite flow instabilities that, in turn, excite the flame. As such, the receptivity of these flow instabilities to both transverse and longitudinal acoustic oscillations must be considered. For example, consider the convectively unstable shear layers, which form tightly concentrated regions of vorticity through the action of vortex rollup and pairing [46]. The transverse acoustic oscillation will disturb them in both normal and parallel directions, see Figure 2. Normally incident transverse acoustic oscillations will push them from side to side in a flapping manner, while parallel incidence will graze across them as shown in Figure 2.

![Figure 2. Transverse excitation interacts both normal to the flame and grazing the flame.](image_url)

In contrast, longitudinal oscillations will lead to an axially oscillating core flow velocity that will, in essence, excite an oscillation in shear layer strength [47, 48]. Other literature on the response of wakes and swirling flows to longitudinal and transverse excitation can be found in [41, 49, 50]. These potential sources of excitation are summarized in the figure below.
As illustrated in the figure, although the initial disturbance is transverse, the disturbance field actually exciting the flame is much more complex. In addition, the fundamental character of the disturbance field can vary along the flame length. For example, a point in the middle of the flame may experience a strong transverse acoustic excitation source locally, as well as the disturbance from a vortex that was excited at the nozzle exit and has convected downstream.

The significant impedance jump across the flame also introduces important differences between the disturbance field resulting from incident longitudinal and transverse disturbances. In the longitudinal excitation case, a nominally axisymmetric burner will see roughly the same amplitude/phase of acoustic excitation at all azimuthal points; e.g., the source of shear layer forcing will be uniform. In contrast, the flame will partially reflect/refract a transversely propagating traveling acoustic wave, resulting in a different excitation field on either side of the flame. In other words, one side of the flame is “shielded” by the impedance jump imposed by the other side. Several studies [51-53] have shown that an acoustic disturbance is considerably changed as it travels across a flame as a result of the change in temperature and density. This point has been dramatically illustrated in recent experiments of Ghosh and Yu for transversely excited H₂/O₂ cryogenic flames, as seen in Figure 4. Note the significantly different degree of flame wrinkling of the two flame branches.

\[
\frac{\partial G}{\partial t} + (\bar{u} \cdot \bar{n}) \nabla G = 0
\]  

Figure 3. Disturbance pathway in transversely excited flame.

Figure 4. OH* Chemiluminescence at various phases of a flame forced at 1150 Hz showing the “shielding” effect. Reproduced from Ghosh et al. [51].

**B. Flame response to Flow Excitation**

In this section, we consider how transverse excitation influences how flames respond to disturbances. Within the flame sheet approximation, the response of the flame to flow excitation is largely controlled by flame kinematics, i.e., the propagation of the flame normal to itself into an oscillatory flow field. This is mathematically described by the so-called G-equation [54]: 

\[
\frac{\partial G}{\partial t} + (\bar{u} \cdot \bar{n}) \nabla G = 0
\]
In this equation, the flame position is described by the parametric equation $G(\bar{x}, t) = \mathbf{0}$ whose local normal vector is given by $\bar{\mathbf{n}}$. Also, $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}(\bar{x}, t)$ and $S_L$ denote the flow field just upstream of the flame and laminar burning velocity, respectively. In the unsteady case, the flame is being continually wrinkled by the unsteady flow field, $\tilde{\mathbf{u}}$. The key thing to note from this equation is that the flame does not distinguish between disturbance sources that are axial or transverse in nature – rather, its response is controlled by the scalar $\mathbf{u} \cdot \bar{\mathbf{n}}$, i.e., the component of the flow velocity in a direction normal to the instantaneous flame front. As such, the response of a two-dimensional flame to a given perturbation field is fundamentally the same, regardless of whether the disturbance field is transverse or axial in nature.

However, additional physics are present for axisymmetric flames, because transverse disturbances will necessarily create a non-axisymmetric disturbance field. In contrast, it can be expected that longitudinal disturbances excite an essentially axisymmetric acoustic and vortical disturbance field, leading to axisymmetric flame wrinkling. The non-axisymmetric excitation of the flame introduces an additional degree of freedom into the flame dynamics, as it leads to azimuthal disturbance propagation along the flame sheet in a helical fashion. Some work on this problem has been reported by Acharya et al. [55].

Having overviewed some basic features of the transverse forcing problem, we conclude this section with an overview of the rest of this paper. Characteristics of the two-dimensional velocity disturbance field in a transversely forced, swirl-stabilized flame are investigated using time resolved particle image velocimetry (PIV) [56]. First, we explain the experimental setup, diagnostics systems, and data analysis techniques used in this study. Next, we review the basic characteristics of this disturbance field using several metrics to capture the different types of disturbances found in the flame region. Following that is a comparison of the disturbance fields that arise from a velocity node along the centerline and a pressure node along the centerline. Next, we discuss the effect of excitation frequency on the disturbance field in the region of a flame with results at both low and high frequencies. Finally, we will draw several conclusions as to the effect of these different disturbances field on the flame behavior.

II. Experimental Facility and Data Analysis

The experimental facility used in this study is a single nozzle rig that was designed to support transverse acoustic modes in the combustion chamber. The inner chamber dimensions are 1.14 m x 0.36 m x 0.08 m, with the 1.14 m dimension is in the transverse direction. This configuration mimics the shape of an azimuthal mode in an “unwrapped” annular combustor for the flame stabilized in at the nozzle in the center. The transverse acoustic field is produced by two sets of drivers on either side of the combustor. Each driver sits at the end of an adjustable tube that allows for acoustic tuning. The drivers on either side of the combustor can be controlled independently. By changing the phase between the signals driving each side of the combustor, different wave patterns, both standing and traveling waves, can be created inside the combustor. As shown in Figure 6, when the drivers are forced in phase, a pressure anti-node and velocity node are created at the center of the experiment in the flame region. When the drivers are forced out of phase, a pressure node and velocity anti-node are set up at the center. The resulting disturbance field from these two forcing configurations can be significantly different. The experimental rig is shown in Figure 5.

![Figure 5. Tunable transverse forcing combustion facility.](image)

Although up to three nozzles can be supported, the current configuration has a single nozzle. The nozzle consists of a swirler with 12 non-aerodynamic blades with a 45° pitch and a 5.1 cm centerbody. The outer diameter...
of the swirler is 3.18 cm and the inner diameter is 2.21 cm, similar in size to several industrial aeroderivative engine nozzles [28]. The nozzle configuration, including the theoretical mode shapes around the nozzle for different acoustic forcing conditions, is shown in Figure 6.

Figure 6. Nozzle configuration and acoustic velocity and pressure perturbations for in phase and out of phase acoustic forcing.

Particle Image Velocimetry (PIV) is used to measure the velocity field and the flame edge. The window at the front of the combustor is a 22.9 cm x 22.9 cm quartz window originating 0.64 cm downstream of the dump plane. The PIV system is a LaVision Flowmaster Planar Time Resolved system that allows for two-dimensional, high-speed velocity measurements. The laser is a Litron Lasers Ltd. LDY303He Nd:YLF laser with a wavelength of 527 nm and a 5 mJ/pulse pulse energy at a 10 kHz repetition rate. The laser is capable of repetition rates up to 10 kHz in each head. The Photron HighSpeed Star 6 camera has a 640x448 pixel resolution with 20x20 micron pixels on the sensor at a frame rate of 10 kHz. The PIV calculation was done using DaVis 7.2 software from LaVision. In the current study, PIV images were taken at a frame rate of 10 kHz with a time between laser shots of 30 microseconds at a bulk approach velocity of 10 m/s and 15 microseconds at a bulk approach velocity of 40 m/s. 500 velocity fields were calculated at each test condition.

Several methods were used to process the data. DaVis 7.2 was used to calculate velocity fields, vorticity fields, and divergence fields from the particle images. Spectra were calculated at each spatial point in the velocity fields, the vorticity fields, and the divergence fields. The amplitude and phase of the Fourier transform, \( \hat{A}(\xi) \) and \( \phi(\xi) \), were calculated at each spatial point and used to harmonically reconstruct the time varying flow field at the forcing frequency. The resulting coherent fluctuation is shown in Equation 2.

\[
\hat{u}(\xi, t) = \text{Re} \left[ \hat{A}(\xi)e^{-i(\omega t + \phi(\xi))} \right] 
\]

Equation 2

Second, spatially averaged, instantaneous velocities were calculated at the nozzle in order to assess bulk flow characteristics. This spatially averaged axial velocity was calculated on the left and right halves of the nozzle, as well as the overall average of the two. The calculation of spatially averaged transverse velocity was calculated along the centerline of the flow, over an axial distance of one nozzle diameter, see Equation 3. These signals are filtered using a 2nd order Butterworth bandpass filter at ±200 Hz around the forcing frequency.

\[
\bar{u}(t) = \frac{2\pi}{\int_0^\xi \mu(x = 0, r, t) rdr} , \bar{v}(t) = \frac{1}{D} \int_0^\xi v(x, r = 0, t) dx 
\]

Equation 3
Finally, the local unsteady pressure was estimated using a linearized energy equation as seen in Equation 4.

\[
\frac{1}{\gamma \rho_o} \frac{\partial p(t)}{\partial t} + \nabla \cdot \vec{u}(\vec{x}, t) = 0
\]  

(4)

The dilatation field is estimated from the two-dimensional velocity field. Note that dilatation in the out of plane direction is not included. However, due to the nature of the disturbance field, this is expected to be small. Because of the large length scale of the dilatation field (e.g., the acoustic wavelength is 0.85 meters at 400 Hz), this calculation is extremely sensitive to noise. As such, indicated pressures are averaged axially over a length of 2 diameters using the formula:

\[
\bar{p}(r, t) = \frac{1}{2D} \int_0^{2D} \bar{p}(x, r, t) dx
\]  

(5)

All results are shown in dimensionless form. The velocities are normalized by the bulk approach flow velocity, \( U_o = \frac{\dot{m}}{\rho A} \) (where \( \rho \) and \( A \) denote approach flow density and annulus area), the spatial coordinates by the nozzle diameter, \( D \), time by the inverse of the forcing frequency, \( 1/f_o \), and the vorticity by the bulk velocity divided by the annular gap width, \( U_o/(r_2-r_1) \).

III. Results and Discussion

The discussion of the experimental results will be broken in to three sections: description of basic flow field characteristics, the effect of in phase vs. out of phase acoustic forcing on the disturbance field, and the effect of frequency on the disturbance field.

A. Flow field and disturbance field characteristics

The time averaged axial velocity and time averaged vorticity fields are shown in Figure 7. From the axial velocity field, the strong annular jet is shown originating from the left, with a vortex breakdown region. The shear layers shown on the right arise from the mixing of the annular jet and the outer media. The inner shear layer is significantly stronger than the outer, presumably because the velocity gradient between the jet and the vortex breakdown region is greater than that between the jet and the quiescent fluid outside the jet.

![Figure 7. Time averaged axial velocity and vorticity fields at \( U_o=40 \) m/s, non-reacting flow. Jet centers are shown in black lines on the left, shear layer centers are shown in black lines on the right.](image)

We next turn to the unsteady flow characteristics. Our analyses to date indicate that the qualitative features of the non-reacting and reacting flow disturbance field are quite similar, so we primarily present non-reacting results here. In addition, we primarily present results from the highest velocity case, \( U_o=40 \) m/s, where the two shear layers remain distinct for a longer axial distance. In order to see the basic features of velocity field and the vorticity field, the harmonically reconstructed velocity and vorticity are plotted in Figure 8.
In this figure, the flow travels from left to right and several key features are evident. First, vortex formation and downstream convection can be clearly seen in the vorticity isocontours. At 0 degrees, two vortices outlined in red form at the nozzle exit, and by 115 degrees through the cycle have traveled to a downstream position of half of one diameter. Considering the velocity in these shear layers, note that the inner and outer shear layers are oscillating out of phase with each other. In other words, when the inner shear layer velocity field is directed outward, the outer shear layer is directed inward, and vice versa. For example, at 0 degrees, the velocity in the outer shear layer at approximately $x/D=0.5$ has a significant negative axial component, while the corresponding inner shear layer has a significant positive axial component. Finally, the velocity fluctuation transitions from a complex two-dimensional field near the nozzle dump plane to dominantly transverse farther downstream.

Alternative views of these features can be seen from the harmonically reconstructed vorticity, axial velocity, and transverse velocity plots below. Figure 9 shows the harmonically reconstructed vorticity through several parts of an
acoustic cycle. Four arrows indicate the initial vorticity disturbances the 0 degree plot of Figure 9. The unsteady formation and downstream convection of vorticity is present. The unsteady vorticity is generated in the two separating boundary layers and rolls up into larger structures in the wakes associated with the inner and outer recirculation zones. While two counter rotating structures are clearly evident at the annular slot exit, see Figure 8 at 0 degrees, they subsequently merge farther downstream to form a single structure that is located in the jet core, see Figure 8 at 115 degrees. Note that this merging process occurs between staggered vorticity whose vorticity direction is the same. Figure 9 also shows this merging and convection phenomenon. For example, the vorticity disturbance created at the outer edge, shown by the two blue troughs at a downstream distance of x/D=0.2 at 0 degrees phase, have convected to approximately x/D=0.5 by 173 degrees through the cycle.

Figure 9. Harmonically reconstructed vorticity field at f_o=800 Hz out of phase acoustic forcing, U_o=40 m/s bulk flow, non-reacting.
The fluctuating vorticity induces fluctuations in the axial velocity and transverse velocity as well. The axial velocity fluctuations are dominant in the shear layers, while the transverse velocity fluctuations are dominant in the jet centers. The axial velocity fluctuations are show in Figure 10. Four arrows indicate the initial axial velocity disturbances in the shear layers in the 0 degrees plot. As noted in the vector plots in Figure 8, the axial velocity fluctuations in adjoining shear layers have opposite signs. Each disturbance convects downstream as with the vortex, so that the disturbances in the inner shear layer at 0 degrees are at a downstream distance of approximately $x/D=0.5$ at 115 degrees through the cycle.

![Harmonically reconstructed axial velocity field at several phases through an acoustic cycle, $f_0=800$ Hz out of phase forcing, $U_o=40$ m/s bulk velocity](image)

Finally, the harmonically reconstructed transverse velocity fluctuations are shown in Figure 11. Here, the fluctuations are dominant in the jet centers, not the shear layers. The disturbances in the jet are in phase and convect...
downstream. For example, the positive fluctuation formed at 115 degrees has traveled $x/D=0.5$ downstream by 230 degrees through the cycle. Additionally, the entire surface moves in bulk throughout the cycle, indicating the bulk transverse motion created by the out of phase acoustic forcing.

Figure 11. Harmonically reconstructed transverse velocity fluctuations at several phases in an acoustic cycle, $f_a=800$ Hz out of phase forcing, $U_o=40$ m/s bulk velocity.

With these basic properties of the time average flow field and the coherent disturbance field, we next consider the effect of acoustic forcing configuration and frequency on the basic characteristics of the disturbance field. The next two sections will cover these issues in detail.

B. Effect of acoustic forcing configuration on disturbance field

As shown in Figure 6, we considered two standing wave acoustic fields in this study, manifested by a pressure anti-node or velocity anti-node at the center of the flow field and flame. A velocity anti-node is caused by out of
phase forcing of the acoustic drivers, and disturbance field results for this configuration are shown above. In general, this configuration creates a periodic fluctuation of the transverse velocity across the entire flow field, as well as vorticity and velocity fluctuations in the shear layers and jets. The vorticity fluctuations originate from the separating boundary layers and then convect downstream in the jet center. The axial velocity fluctuations in adjacent shear layers are of opposite signs and convect downstream. The transverse velocity fluctuations manifest themselves in the jet core and have the same sign on either side of the centerline.

The in-phase forcing configuration creates a pressure anti-node and velocity node along the centerline of the flow and results in a significantly different disturbance field. First, the amplitudes of the coherent fluctuations in vorticity, axial velocity, and transverse velocity in the in phase forcing case are much smaller, having peak to peak values between half and a third as large as those in the out of phase forcing case. The comparisons of one of these quantities are shown in Figure 12.

Figure 12. Comparisons between harmonically reconstructed vorticity at $f_o=400$ Hz out of phase (left) and in phase (right) forcing, $U_o=40$ m/s bulk velocity.

In addition to the reduced amplitude of the coherent fluctuations, the change in forcing configuration has also changed the phases between the fluctuations in adjacent shear layers and jets. The vorticity disturbances in the jet on the left and right sides of the burner are now of opposite sign, as opposed to the symmetric response discussed in the previous section. The axial velocities of the inner and outer shear layers are still out of phase, but now these phase relationships are symmetric with respect to the burner centerline. In addition, the transverse velocity fluctuations in the jets have opposite signs in the case of in phase acoustic forcing, as expected given the centerline velocity node. These relations are summed up in Figure 13.

Figure 13. Instantaneous snapshot of fluctuations in out of phase acoustic forcing (left) and in phase acoustic forcing (right), with an acoustic velocity maximum, acoustic pressure node, asymmetric mass flow fluctuations, asymmetric vorticity fluctuations, asymmetric axial velocity fluctuations, and unidirectional transverse velocity fluctuations.
The phase change in the fluctuations between the two acoustic forcing conditions can be explained by analyzing the effect of the central location of the pressure node in the out of phase case and the velocity node in the in phase forcing case. Take first the transverse velocity fluctuations; in the out of phase forcing configuration, the transverse velocity fluctuations in the jets have the same sign because the transverse acoustic field has the same phase on the left and right sides of the burner centerline. Conversely, the transverse velocity fluctuations are out of phase on either side of the centerline when the acoustic forcing is in phase, where there is velocity node along the centerline. The out of phase transverse acoustic fluctuations dictate the out of phase transverse velocity fluctuations in the jet cores.

Next, consider the vorticity at the jet exit, which is controlled by the action of the fluctuating burner mass flow rate and acoustic pressure field. In the case of out of phase forcing, the pressure fluctuation on either side the nozzle centerline is of opposite sign due to the node in the center. This creates a mass flux variation through either side of the annulus that is also of opposite sign. Conversely, when there is a pressure anti-node at the center, the mass flux of the jet pulses in phase, as also indicated by Staffelbach and coworkers in [19]. These variations in pressure and axial mass flux on either side of the nozzle can be seen in Figure 14, showing the out of phase forcing configuration and the in phase forcing configuration, respectively.

![Figure 14. Average axial velocity fluctuation and pressure fluctuation over either side of the annular nozzle at \( f_o = 400 \) Hz out of phase forcing (left) and in phase forcing (right), \( U_o = 10 \) m/s bulk velocity. The lower velocity is shown for clarity, higher velocity time signals have significantly more noise.]

These results show that the location of the nozzle with respect to the mode shape of the transverse mode has a great effect on the disturbance field created by the transverse forcing. Next, the frequency dependence of the disturbance field will be discussed.

C. Frequency Effects on the Vorticity Field

The size, strength, and pairing locations of the vortical structures convecting down the jet centerlines changes with frequency. This can be seen from Figure 15, where the strength of the vortex is quantified by using the circulation, calculated by integrating the vorticity over a 16.2 mm x 9.2 mm window at several non overlapping axial locations along the jet center. The jet center is defined by the time average axial velocity profile, as seen in Figure 7, between the \( U_o = 0 \) lines on either side of the jet at the base. The circulation has been non-dimensionalized by the axial velocity fluctuation amplitude on the right side of the annulus at \( f_o \) multiplied by the annular gap width.
Figure 15. Non-dimensionalized circulation fluctuation, $\Gamma'/\overline{U}_{right}(f_\omega)(r_2-r_1)$, decay with downstream distance in the right jet at three frequencies, $f_\omega=400$ Hz, $f_\omega=800$ Hz, and $f_\omega=1200$ Hz out of phase forcing for non-reacting, $U_o=40$ m/s bulk flow velocity.

Increasing $f_\omega$ is accompanied by an increase in shedding frequency, an earlier axial pairing location, and a change in size and strength of the paired vortical structures. The movement in vortex merging region is somewhat difficult to infer from this graph, but evident from the movies, which indicate merging locations of $x/D=0.7$ for $f_\omega=400$ Hz, $x/D=0.4$ for $f_\omega=800$ Hz, and $x/D=0.3$ for $f_\omega=1200$ Hz. Note also that the peak circulation increases with frequency. This trend is not fully understood. It could reflect the frequency dependent receptivity of the shear layer rollup process, the burner nozzle acoustics, or some other process.

IV. Conclusions

This work describes the disturbance field of a swirl-stabilized flame that arises from the presence of transverse acoustic forcing. Discussion focuses on the effects of changing the transverse wave structure and frequency of forcing. Three main conclusions can be drawn from this work.

1. Transverse acoustic forcing produces a highly two-dimensional disturbance field. Vorticity fluctuations originate on either side of the nozzle exit, merge and then convect downstream in the jet. The fluctuations in axial velocity are limited mainly to the shear layers, while the fluctuation in transverse velocity is present throughout the entire flow field due to the approximately one dimensional nature of the acoustic excitation, though convecting structures of transverse velocity perturbations are seen in the jet centers.

2. The placement of the nozzle with respect to the transverse acoustic mode shape has a significant impact on the characteristics of the disturbance field. Not only does magnitude of the response change, but also the relative phases between the convecting vorticity and velocity disturbances.

3. The frequency of the acoustic forcing also affects the shape of the disturbance field, particularly when there is a velocity node at the nozzle centerline. The location of the vortex merging region and the rate of decay of the vortices in the jet center change with frequency.

It is critical to have knowledge of the disturbance field in order to understand the response of flames to acoustic excitation, and the work presented here begins to fulfill this need for transversely forced flames. Future work will include tracking of the flame edge and vortex motion with simultaneous chemiluminescence measurements. These measurements will allow for correlation between vortex motion, flame disturbance, and heat release fluctuations.

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