STRAIN CHARACTERISTICS NEAR THE FLAME ATTACHMENT POINT IN A SWIRLING FLOW

Qingguo Zhang, Santosh J. Shanbhogue, Shreekrishna, Tim Lieuwen, and Jacqueline O’Connor

School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA

Swirling flows are widely used in industrial burners and gas turbine combustors for flame stabilization. In many cases, the flame is stabilized in the separating shear layer near the centerbody or abrupt expansion, where the high-speed nozzle flow transitions into the larger combustor. Several prior studies have shown that the flame position becomes increasingly unsteady as it approaches blowoff, due to local extinction and reattachment of the flame at one or both of these locations. This is apparently due to the local strain rate irregularly oscillating about the extinction strain rate values near the attachment point. In order to characterize these flame strain characteristics, particle image velocimetry measurements were obtained of several hydrogen/methane mixtures in this attachment region. The fluid mechanic straining of the flame in this region is dominated by two gradients in velocity — that due to the strong shear near the centerbody and to the bulk flow deceleration as it expands from the smaller diameter nozzle into the combustor. These two velocity gradients cause positive and negative stretching of the flame sheet, respectively. The shearing velocity gradient is an order of magnitude larger than the flow deceleration term but, due to the fact that the flame is essentially parallel to the shear layer, actually has a secondary influence relative to the flow deceleration. As a result, the dominant flame straining term near the attachment point is compressive – a somewhat counter-intuitive result, given that the flame is stabilized in the positively straining shear layer.

Keywords: Extinction; Flame attachment; Flame strain; Swirl flow

I. INTRODUCTION

The objective of this paper is to consider the factors influencing flame position and stabilization location in swirling flows with a centerbody, such as shown in Figure 1. Such a geometry is very typical for low NOx combustor configurations utilizing lean, premixed technology (Lefebvre, 1999; Vandervort, 2001) and in fundamental studies of swirling flows (Schefer et al., 2002; Yang and Huang, 2004). The flow field of this geometry, in a time-averaged sense, consists of four main regions (Gupta et al., 1984; Lu et al., 1999; Wicksall et al., 2004), see Figure 1: (a) the outer recirculation zone (ORZ), a toroidal recirculating regime generated by the rapid...
expansion of the nozzle into the combustor; (b) the inner recirculation zone (IRZ; also referred to here as the vortex breakdown bubble), due to vortex breakdown accompanying the swirling flow and the bluff body wake; (c) the high-velocity, annular fluid jet that divides these regions; and (d) two annular shear layers that divide the ORZ and annular jet (denoted here as the outer shear layer [OSL]) and the IRZ and the annular jet (inner shear layer [ISL]).

Consider Figure 2, which illustrates several flame configurations that have been observed for this geometry. Note that there are three basic flame holding locations: (a) OSL (see configuration b and d), (b) ISL (configurations c and d), and (3) vortex breakdown bubble (VBB; configurations a and b).

The location and spatial distribution of the flame in a combustion chamber is a fundamental problem that has important ramifications on combustor operability, durability, and emissions. It can be seen that flame stabilization location has strong influences on flame shape that in turn influences heat loadings to combustor hardware (e.g., centerbody, walls, dome plate). For example, the heat transfer to the

Figure 1 Illustration of the key fluid mechanical features in the geometry of interest.

Figure 2 Illustration of basic flame configurations possible for geometry of interest.
centerbody is fundamentally different in configurations c and d than in a and b. This, in turn, has implications on centerbody design and life. Similarly, the degree of flame spreading to combustor walls will vary between, for example, configurations b and c.

Next, flame location has important influence on combustion instability boundaries (Lieuwen et al., 2008). It is known that combustor stability limits are controlled by the time delay between when a fuel/air ratio disturbance or vortex is created and when it reaches the flame. This time delay will certainly vary between, for example, configurations a and d. This also illustrates that discontinuous changes in combustor stability behavior may occur when the flame abruptly bifurcates from one stabilization location to another.

Next, stabilization locations influence the blowoff limits of the system. In reality, shifts in flame location can be thought of as a sequence of local blowoff events (e.g., a flame nominally looking like configuration d will bifurcate to configuration b due to local blowoff of the flame from the centerbody shear layer).

How the flame is stabilized plays a crucial role in which physical processes control its dynamics. For example, the configuration shown in Figure 2d is clearly affected by the dynamics of both shear layers, while that in Figure 2b may be less affected by the centerbody shear layer. Similarly, the dynamics of the central portion of the flame is strongly influenced by vortex breakdown bubble dynamics in Figures 2a and 2b, while 2c and 2d are presumably less affected. Furthermore, the time-averaged stabilization location can vary with perturbation amplitude during combustion dynamics, implying that one set of fluid mechanic processes are important at low amplitudes, and another at higher amplitudes. The interactions of all these fluid mechanic instabilities are clearly shown in the OH-PLIF images in Figure 3, reproduced from Thumuluru and Lieuwen (2009).

The specific focus of this paper is on the processes influencing flame stabilization in the ISL, Figures 2c and 2d, a common mode of stabilization in practice. Because of the centerbody, there is a low-velocity region in the separating shear layer where the flame can stabilize. However, it is known that shear introduces

![Figure 3](image-url)  
**Figure 3** PLIF images showing (a) flame interacting with VBB, (b) flame interacting with ORZ, and (c) flame interacting with ISL and OSL.
aerodynamic straining on the flame (Law and Sung, 2000), which alters the local temperature and burning rate (Law, 2006). If the shear rate and consequent flame strain rate is too large, the flame will locally extinguish and either blow out of the combustor completely or stabilize at another location, such as transitioning from stabilization configuration d to b or from configuration c to a. As such, characterization of the local strain rate magnitudes of the flame in the attachment point regions is needed in order to understand these factors influencing flame location. In particular, predicting flame stabilization requires a further understanding of the local strain characteristics of the flow field, as well as the maximum strain that the flame can withstand.

The significance of aerodynamic straining of the flame sheet in the shear layer near the attachment point was apparently first discussed by Karlovitz et al. (1953). They noted that holes appeared in the side of the flame as flow velocity increased, apparently due to local extinction. Similar observations of such holes in flames near blowoff are detailed in a review by Shanbogue et al. (2009) or Khosla and Smith (2007). A discussion of flame stabilization in shear layers can be found in Law (2006). The next section further discusses the strain characteristics of shear layer stabilized flames.

II. BACKGROUND

There is extensive literature available that has quantified flame strain statistics for turbulent flames—see the work of Im and Chen (2002), Chen and Im (2000), Driscoll et al. (1994), Echekki and Chen (1996), Donbar et al. (2001), and Veynante and Vervisch (2002). Turbulent flames can be locally quenched in regions of high turbulence (Driscoll, 2008; Poinset et al., 1991)—the Klimov-Williams criterion and the conditions under which this occurs is still a subject of much discussion (Bedat and Cheng, 1995). It is useful to distinguish between the time-averaged flame strain rate and the instantaneous fluctuations about this value (which can be quite large). Most fundamental measurements of extinction strain rates have been performed in steady-state experiments, often utilizing opposed flow burners that allow a controllable flame strain rate (Holley et al., 2006; Kee et al., 1988; Law, 2006). The response of flames to unsteady stretch has also been extensively analyzed, and it is known that the flame can withstand instantaneous strain rates that can substantially exceed the quasisteady extinction strain rate if they occur over a short enough time scale (Im et al., 1995; Luff et al., 2004). Thus, the general flame-quenching problem requires understanding both the magnitudes and time scales associated with the unsteady flame strain rate. Moreover, for a significant class of physically interesting problems, the time-averaged flame strain rate is relatively small—thus, the turbulent flame-quenching problem by its very nature requires understanding the unsteady problem.

For the problem of interest to this paper—flame stabilization in shear layers—there is good reason to focus first on the steady-state problem before considering the fully unsteady problem. The reason for this is that the time-averaged flame and flow strain rates are so large by themselves. For example, the fluid mechanic straining associated with flow shear in many combustion applications is very large—for example, assuming a boundary layer thickness on the order of millimeters, and a...
mean flow velocity of tens of meters, leads to flow shearing rates of well over 10,000 \(1/s\). These are values that easily approach or exceed typical extinction strain rates of hydrocarbon/air flames. This observation is a key motivator for the work described in this paper. In particular, a focused characterization of strain rate in the vicinity of the high shear attachment point of the flame does not appear to have been previously performed. The objective of this study is to obtain such measurements in order to determine the key processes influencing flame stabilization near the centerbody attachment point. As will be shown next, scaling the mean flame strain rate is much more complex than it may initially appear. Indeed, the very fundamental question of whether the local time-averaged flame strain rate is positive or negative (much less its magnitude) is itself a difficult question, with an answer that is very sensitive to details of the flow and flame speed.

It is useful to consider the expression for hydrodynamic flame strain under certain simplifying assumptions in order to illustrate some key points. Consider Figure 4, which illustrates a flame anchored upon a centerbody. The transverse and axial coordinates are given by \((x, z)\) and velocity by \((u, v)\), respectively. The flame’s location is given by \(x = X_f(z)\).

The hydrodynamic flame stretch may be expressed in terms of the velocity gradients as (Poinsot and Veynante, 2001):

\[
\kappa_s = -n_n n_j \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_i}
\]  

(1)

Assuming incompressible flow upstream of the flame sheet (i.e., \(\partial u_i/\partial x_i = 0\)) and a 2-D flow,

\[
\kappa_s = -n_x \frac{\partial u}{\partial x} - n_z \frac{\partial v}{\partial z} - n_z n_x \frac{\partial u}{\partial x} - n_z n_x \frac{\partial v}{\partial x}
\]  

(2)

Figure 4 Flow and flame coordinate system for centerbody stabilized 2-D flame.
Equation (2) can be written as

\[ \kappa_s = (n_x^2 - n_z^2) \frac{\partial v}{\partial z} - n_x n_z \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) \] (3)

From Eq. (3), we can explicitly identify the normal (symmetric) and shear (antisymmetric) strain terms as

\[ \kappa_{\text{normal}} = (n_x^2 - n_z^2) \frac{\partial v}{\partial z} \] (4)

\[ \kappa_{\text{shear}} = -n_x n_z \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) \] (5)

The manner in which these normal and shear flow strain terms translate into flame strain is illustrated in Figure 5.

The left image illustrates a material line just upstream of the flame. The right image shows the deformation of this material line due to flow strain an instant later. The increase–decrease in length of this material line leads to flame strain, as detailed in Law (2006). These images were drawn assuming a high velocity axial flow, so that the flame is nearly parallel to the flow and \( n_x = 1 - \gamma^2 \) and \( n_z \approx -\gamma \), where \( \gamma \) is the angle of the flame with respect to the vertical, see Figure 4. The top set of images shows the effect of normal strain, assuming that the axial flow is decelerating (as it would be in the configurations shown in Figures 1 and 3)—this image shows that normal strain leads to negative stretching or compression of the flame. The bottom set of images show the effect of shear strain (assumed to be dominated by shearing in the separating boundary layer)—this image shows that shear strain leads to positive stretching of the flame. Thus, the flame can be either positively or negatively stretched (with a resulting increase or decrease in local burning rate), depending on which term dominates.

**Figure 5** Illustration of the manner in which (a) flow deceleration and (b) flow shearing causes compressive and extensional flame straining. Solid line = material line, dash-dotted line = flame front element; Rectangle = material volume.
To address this further, assume that $\partial u/\partial z \ll \partial v/\partial x$ and that $\gamma \ll 1$, then rewrite Eq. (3), neglecting terms of $O(\gamma^2)$ or higher:

$$\kappa_s \approx \frac{\partial v}{\partial z} + \gamma \frac{\partial v}{\partial x}$$  \hspace{1cm} (6)

In the shear layer, we can expect both velocity gradient terms in this equation to be nonzero, and for the shear gradient to be much larger than the axial flow deceleration (i.e., $\partial v/\partial x \gg \partial v/\partial z$). However, the much larger shearing term is multiplied by the small flame angle, $\gamma$. Thus, it can be seen that neither term obviously dominates, indicating that the near-field flame strain could be either positive or negative. Moreover, $\kappa_s$ is a sum of two numbers of opposite sign, but potentially similar magnitudes. This implies that relatively small changes in flame angle, axial flow deceleration, or shear layer characteristics could fundamentally alter the value of the local strain from positive to negative values, or vice versa. The objective of the rest of this paper is to provide some insight into these issues.

III. INSTRUMENTATION AND FACILITY

Measurements were obtained in a lean, premixed swirl combustor that was duplicated from an experimental rig at Sandia National Laboratories (Williams et al., 2007). The combustor is schematically shown in Figure 6a.

The facility consists of a swirler/nozzle, combustor, and exhaust sections. Premixed gas, consisting of $\text{H}_2/\text{CH}_4$ mixtures and air flows through the swirler/nozzle section. The nozzle is an annular tube with inner diameter of 28 mm. The center body has an outer diameter of 20 mm. The overall flow area remains constant at

![Figure 6](downloaded_by:itlevenen, Tim At: 16:21 5 April 2011)

**Figure 6** Schematic of the (a) combustor facility and PIV measurement window and (b) time-averaged flow field.
3.0 cm² inside the nozzle. Tests were performed with a six-vane, 45° swirler, which is located in the annulus between the centerbody and nozzle wall. The theoretical swirl number, S = 0.85, is calculated from the relation (Lilley, 1977)

\[ S = \frac{2}{3} \left( \frac{1 - (d_h/d)^3}{1 - (d_h/d)^2} \right) \tan \theta \]

(7)

where \( d_h \) and \( d \) are the diameters of centerbody and swirler, respectively, and \( \theta \) is the swirler vane angle. The fuel is injected 150 cm upstream of the combustor to achieve a premixed condition. The combustor consists of a 305 mm long quartz tube, with a 115 and 120 mm inner and outer diameter, respectively. It rests in a circular groove in a base plate. The exhaust nozzle has a 152 mm contraction section with an area ratio 0.44, and a 102 mm long, 51 mm inner diameter chimney section.

The air and fuel flow rates are measured with calibrated flowmeter and mass flow controllers, respectively. The uncertainty in equivalence ratio is less than 0.03 and in \( \text{H}_2/\text{CH}_4 \) ratio less than 4%. The air is choked before the mixing section, and the premixed air/fuel is choked again inside the inlet tube of the combustor (not shown) upstream of the swirler to minimize the impact of perturbations in the combustor impacting the fuel/air mixing process. Data were obtained for four different \( \text{CH}_4/\text{H}_2 \) flames performed at room temperature, atmosphere pressure, and constant nozzle exit velocity of \( U_{\text{nozzle}} = 33 \pm 1 \text{ m/s} \).

The velocity field in the combustor was measured using particle image velocimetry (PIV; Raffel et al. 1998). The system consists of a dual head Nd:YAG laser, a high-resolution CCD camera, a mechanical shutter, and a centralized timing generator. In addition a cyclone seeder was used to supply anhydrous aluminum oxide (\( \text{Al}_2\text{O}_3 \)) with an average particle size of 3 μm. Each laser head delivered a 5 mm, 120 mJ/pulse beam at a wavelength of 532 nm. The light sheet is generated with a convex spherical \( (f = 1 \text{ m}) \) and a convex cylindrical lens \( (f = 25.4 \text{ mm}) \) which provides a 1 mm thick sheet at the center of the combustor. The CCD camera captured the images of the illuminated particles at a resolution of 1600 × 1200 pixels (corresponding to 52 mm × 39 mm) in frame straddling mode. The duration between laser shots was set at 5 μs. The camera was also fitted with a 532 nm laser line filter with a FWHM of 3 nm to restrict background light from the flame. The images were processed using the DaVis software package, provided by LaVision Inc. This software uses an adaptive algorithm to obtain the velocity field. The grid size was 32 × 32 pixels with a 50% overlap. The output spatial resolution is 0.50 mm.

IV. DATA REDUCTION AND ERROR ANALYSIS

The key quantity of interest to this paper is the flame stretch rate, which has two contributing terms, hydrodynamic strain rate and curvature (Poinsot and Veynante, 2001):

\[ \kappa = \nabla \cdot \bar{u} + S_d (\nabla \cdot \bar{n}) = (\delta_{ij} - n_i n_j) \frac{\partial u_i}{\partial x_j} + S_d \frac{\partial n_i}{\partial x_j} = \kappa_s + \kappa_c \]

(8)
where $\vec{n}$ is the normal vector of the flame, pointing at the cold reactants, and $S_d$ is the local displacement flame speed.

Attention here is focused on the hydrodynamic strain rate just upstream of the flame where gas expansion is negligible and, thus, the flow is essentially incompressible (the flow Mach number at the nozzle exit is $\sim 0.1$). Utilizing the same assumptions as in the prior section, $\kappa_s$ can be written in a cylindrical coordinate system as

$$
\kappa_s = -n_r^2 \frac{\partial u}{\partial r} - n_z^2 \frac{\partial v}{\partial z} - n_r n_z \frac{\partial u}{\partial z} - n_n \frac{\partial v}{\partial r} - n_z \frac{\partial w}{\partial r} - n_n \frac{\partial u}{\partial r} - n_r \frac{\partial w}{\partial z}
$$

Terms 1–4

$$
\frac{n_r^2 \partial u}{\partial r} - n_z \frac{\partial v}{\partial z} + \frac{n_r n_z \partial u}{\partial z} + \frac{n_n \partial v}{\partial r} - \frac{n_z \partial w}{\partial r} - \frac{n_n \partial u}{\partial r} - \frac{n_r \partial w}{\partial z}
$$

Terms 5–9

Note the change in coordinate system from that utilized in Figure 4 for the discussion in the Background section. Definitions of coordinate system in Eq. (9) are indicated in Figure 6b, where $u$, $v$, and $w$ indicate the velocities in the $r$, $z$, and $\theta$ directions, respectively. Terms 1–4 in Eq. (9) are resolved by the PIV measurements and terms 5–9 are unresolved, being out of plane. The implications of this is discussed in the subsequent error analysis.

A second $(t, n)$ coordinate system was also defined that is parallel $(t)$ and orthogonal $(n)$ to the streamline at the nozzle centerline of the average velocity field, which is indicated by the solid black line in Figure 6b. This line was determined from the locus of maximum average axial velocity points at each axial location. The corresponding velocities are given by $u_t$ and $u_n$ and flame normal by $n_t$ and $n_n$.

Mie scattering from the PIV seed particles was used to visualize the thermal boundary between product and reactants. Simultaneous PLIF and PIV measurements have shown that this procedure is appropriate (Pfadler et al., 2007; Pfadler et al., 2008). Each pixel is 0.03 mm, the spatial resolution of the PIV camera. A $10 \times 10$ pixel (0.3 x 0.3 mm) Gaussian filter was used to smooth the raw particle images. The flame sheet or thermal boundary, $X_f(z)$, is then extracted from the gradient of particle densities.

The local flame normal is needed to evaluate Eq. (9). Both instantaneous and time-averaged flame positions were determined. The time-averaged flame position, $\bar{X}_f(z)$, was determined from the instantaneous flame locations at each axial location:

$$
\bar{X}_f(z) = \sum_{i=1}^{K} X_{f,i}(z)
$$

where $K$ denotes the number of PIV shots used for the averaging. The flame surface was determined by fitting a least squares third-order polynomial curve through 11 points (5 points each side, ±0.15 mm) along the flame. The flame normal then was calculated based on the fitted flame surface. The resultant uncertainty in the angular estimate is ±3°, based on analysis of the variance of these 11 points about the curve.

Two different flame strain calculation approaches were performed at each axial location, $z$: the time-averaged conditioned, $\bar{\kappa}_s(z)$, and nonconditioned, $\kappa_s(z)$, strain
rates. To illustrate the conditional averaging approach, consider the estimated time average of the first term in Eq. (9):

$$\bar{\kappa}_s(z) = -\sum_{i=1}^{K} n_i^2(X_f,i(z), z) \frac{\partial u(X_f,i(z), z)}{\partial r} + \cdots$$  \hspace{1cm} (11)

Thus, determining $\bar{\kappa}_s(z)$ requires manually determining the conditional, unburned velocity just prior to the flame from each PIV image and multiplying it by the instantaneous flame normal, at each axial location. Note that because the flame is moving, the spatial location at which the velocity is evaluated is different in each image, $i$. The nonconditioned mean strain rate, $\bar{\kappa}_s(z)$, can be calculated using the time-averaged flame normal and the time-averaged velocity gradient evaluated just upstream of the time-averaged flame position:

$$\bar{\kappa}_s(z) = -\bar{n}_r^2(X_f(z), z) \sum_{i=1}^{K} \frac{\partial u(X_f(z), z)}{\partial r} + \cdots$$  \hspace{1cm} (12)

Thus, the average velocity is not a conditional velocity. These terms can be related to analogous ones that arise in combustion simulations using Reynolds-type decompositions of the flow field into time-averaged and fluctuating terms, where modeling closure of $\bar{\kappa}_s$ is a key issue (Poinsot and Veynante, 2001). For example, as shown in Veynante and Vervisch (2002), $\bar{\kappa}_s$ can be decomposed into a sum of two terms, $\bar{\kappa}_s = A_T + a_T$, where $A_T$ and $a_T$ denote the contributions of the time-averaged flow and turbulent fluctuations, respectively, to the mean strain rate. Their notation and that used in this paper are then related by $A_T = \bar{\kappa}_s$ and $a_T = \bar{\kappa}_s - \kappa_s$. We show comparisons of $\bar{\kappa}_s(z)$ and $\bar{\kappa}_s(z)$ in the Results section.

Determining the velocity gradients required to evaluate Eq. (9) requires getting as close as possible to the flame sheet in the strongly shearing flow, without allowing any bins to overlap the flame sheet itself, which would induce bias errors in the measurement. For this reason, derivatives were estimated using single-side differencing, as opposed to central differencing, which has larger uncertainties but allows one to get 16 pixels closer to the flame. In general the velocity gradients were taken 0.5–1.0 mm away from the flame sheet in the reactant side. This method introduces systematic errors into estimation of the derivatives in the radial direction, such as $\partial u/\partial r$ (note that the shear layer is approximately 1–2 mm thick). The errors are much smaller for derivatives in the flow direction, such as $\partial u/\partial z$. Typical flame thicknesses are 0.1–0.2 mm, much smaller than this distance.

There are a number of potential systematic and random error sources in PIV measurements, as detailed in various sources (Raffel et al. 1998; Sergei et al., 2003). Error due to out-of-plane motion is important to account for in swirling flow. The laser sheet thickness, the duration between laser shots, and the seed density have been carefully optimized to minimize this error. Assuming a $10$ m/s tangential velocity, the distance a particle travels between two successive laser shots is $50 \mu m$, $5\%$ of the laser sheet thickness. Errors associated with identifying the cross-correlation peak are less than $2\%$ (Wicksall et al., 2005). Contributions from thermophoretic velocity bias is negligible in this high-speed flow, where the average nozzle exit velocity is $\sim 30$ m/s. Errors in instantaneous velocity measurements due to eddies
smaller than the PIV spatial resolution (0.5 mm) are difficult to estimate; note, however that the interrogation window size is less than 10% of the 7 mm longitudinal integral length scale, measured at the jet centerline. Moreover, since the majority of the analysis in this paper is based on time-averaged velocity fields, this error source is not important. Based on the previous analysis and the results presented in Wicksall et al. (2005) and Ji et al. (2002), time-averaged velocity and velocity derivative uncertainties in this study are estimated to be on the order of 5 and 20%, respectively.

Consider next the uncertainty in the overall strain rate, Eq. (9), which is a sum of several terms, each of which is a product of flame normals and velocity derivatives. Note that the flame is nearly vertical, implying that \( n_r \sim 1 \) and \( n_z \sim 0 \). With regard to derivatives, the largest velocity gradient is the shearing term, \( \partial v/\partial r \). The overall uncertainty in flame strain rate is largely driven by two factors—first, term 4 in Eq. (9), \(-n_r n_z (\partial v/\partial r)\), which is the product of the largest velocity gradient and the smallest normal component. Uncertainty in \( n_z \) is the single largest contributor to strain estimate uncertainties. Second, this term is positive, while the first term in Eq. (9) is negative. As is shown, these two terms are of similar magnitude, which implies that uncertainties in their individual values are magnified substantially in their difference. Because the uncertainties in several quantities are not small, particularly in the normal vectors, linearized error propagation approaches are not appropriate. As such, a Monte Carlo technique was utilized, where the angle and local velocity were assumed to have a normal distribution with a mean equal to the measured value and standard deviation equal to the uncertainties quoted earlier. Then, \( 10^5 \) realizations of Eq. (9) were generated, resulting in a distribution of strain rate that was also close to Gaussian in shape. The uncertainties quoted in the results equal one standard deviation from this distribution (thus, an \( \sim 68\% \) confidence interval); typical values are 1100 1/s.

Finally, consider the errors in the strain rate estimate due to out of plane motion. Recall that only terms 1–4 in Eq. (9) are resolved by the PIV measurements and terms 5–9 are unresolved, being out of plane. The resulting errors for the time-averaged conditioned, \( \bar{\kappa}_c(z) \), and nonconditioned, \( \bar{\kappa}_n(z) \), are quite different. For the nonconditioned strain, \( \bar{\kappa}_n(z) \), these out-of-plane terms are essentially zero, because of the time-averaged axisymmetry of the flame (i.e., \( \bar{n}_\theta = 0 \)). This is not true for the conditioned calculation. These unresolved terms are small relative to the resolved, in plane terms in the near field of the attachment point because the flame is stabilized in the shear layer of the cylindrical centerbody. As such, the azimuthal normal component, \( n_\theta \), must be close to zero (only if the flame is lifting off in an unsteady manner, which it is not, could this term be nonzero). Slices of the in-plane flame position indicate significant levels of flame wrinkling, starting between 5–10 mm downstream. It certainly seems reasonable that this is accompanied by corresponding wrinkling in the azimuthal direction. This suggests that calculations of \( \bar{\kappa}_c(z) \) ignore potentially significant unresolved, out of plane effects at axial locations beyond about 5–10 mm downstream.

**V. RESULTS AND DISCUSSION**

Results were obtained using four different CH\(_4\)/H\(_2\) flames. These 0/100, 20/80, 50/50, and 75/25% H\(_2\)/CH\(_4\) flames blow off at stoichiometries of approximately
\( \phi = 0.51, 0.44, 0.32, \) and \( 0.23, \) respectively. However, the near-attachment point region starts to become unsteady and exhibit local extinction events, manifested as holes in the flame sheet, at equivalence ratios of approximately \( 0.62, 0.56, 0.43, \) and \( 0.33. \) The data shown here were taken at stoichiometries close to, but sufficiently removed from these points, at \( \phi = 0.65, 0.59, 0.46, \) and \( 0.36, \) respectively. The calculated strain sensitivities of these experimental blends are plotted in Figure 7, showing that they all have comparable extinction strain rates of about \( 1,000 \text{ s}^{-1} \) for this simulated opposed flow configuration. Note that, in general, \( \text{H}_2 \) flames are significantly more stretch sensitive than \( \text{CH}_4 \) flames (Law, 2006) due to their lower molecular weight and higher diffusivity. However, this figure shows that the stretched flame speeds and stretch sensitivities, \( dS_L/d\kappa, \) are quite comparable for all of these fuels in the \( \kappa > 400 \text{ s}^{-1} \) range.

Figure 8 shows four typical instantaneous flow field and flame position snapshots for different mixtures. In addition, the 2D vorticity component, \( \omega = \frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}, \) and instantaneous flame front is shown. The inner and outer flame sheets are stabilized in the shear layers of the centerbody and rapid expansion, respectively. This study focuses on the inner flame sheet, which rides along the periphery of the annular jet and is located in the shear layer. Because of the very high nozzle velocity, the flame is oriented very nearly vertical in the immediate vicinity of the nozzle exit and bends outward farther downstream, as it follows the trajectory of the annular jet.

In many instances, the vorticity is concentrated into discrete blobs, apparently associated with the quasiperiodic rollup of the shear layer due to the Kelvin-Helmholtz instability. In some cases (e.g., Figure 8c), the entire flame and shear layer is undulating, possibly a manifestation of the instability of the annular jet column.

![Figure 7](image.png)  
**Figure 7** Calculated strain sensitivities of experimental mixtures (1) 0/100 \( \text{H}_2/\text{CH}_4, \) \( \phi = 0.65, \) 2) 20/80 \( \text{H}_2/\text{CH}_4, \) \( \phi = 0.59, \) 3) 50/50 \( \text{H}_2/\text{CH}_4, \) \( \phi = 0.46 \) and 4) 75/25 \( \text{H}_2/\text{CH}_4, \) \( \phi = 0.36 \) \( \text{H}_2/\text{CH}_4 \) blends) using a symmetric, premixed, opposed flow configuration. Chemical kinetic mechanism is GRI 3.0 with a nozzle separation distance of 2.0 cm.
Figure 9a shows the time-averaged flow velocity gradients at a location just upstream of the time-averaged flame front for the 100% CH\textsubscript{4} case. Flow velocity gradients were normalized by the ratio of averaged nozzle velocity and annular gap, which are 33 m/s and 4 mm under this configuration. Because of the rapid expansion and divergence of the jet due to its swirl, there is an axial deceleration of the high-speed jet, $\partial v/\partial z << 0$. Correspondingly, there is a rapid increase in radial velocity of the jet in the radial direction, $\partial u/\partial r >> 0$. Because the flow is nearly incompressible and the time-averaged flow is axisymmetric, the sum of these two terms has a small magnitude relative to the other terms (i.e., $\partial u/\partial r + \partial v/\partial z \approx -(u/r)$). Indeed, evaluation of this sum provides an independent assessment of the errors in calculation of time-averaged derivatives. This can be quantified by considering the ratio $(u/r)/(\partial u/\partial r)$ or $(u/r)/(\partial v/\partial z)$, which has average values of around 20% and equals our estimated uncertainty in velocity gradients.

The largest flow velocity gradient term, $\partial v/\partial r$, is associated with the strong shearing associated with the separating boundary layer. This term is larger than $\partial v/\partial z$ by a factor of two. Finally, the variation in radial velocity in axial direction, $\partial u/\partial z$, has the lowest magnitude, being smaller than the axial flow deceleration terms by a factor of 2–4.

Figure 8 Instantaneous vorticity field and flame location (solid and dashed lines denote inner and outer flame branches) for (a) CH\textsubscript{4}, $\phi = 0.65$, (b) 20/80% H\textsubscript{2}/CH\textsubscript{4}, $\phi = 0.59$, and (c) 50/50% H\textsubscript{2}/CH\textsubscript{4}, $\phi = 0.46$, 75/25% H\textsubscript{2}/CH\textsubscript{4}, $\phi = 0.36$; $U_{\text{nozzle}} = 33$ m/s.
Because the flame and flow bend outward, it is difficult to distinguish between downstream changes in the flow field and simple changes in orientation of the annular jet. For this reason, Figure 9b plots the velocity derivatives in the rotated $n$-$t$ system, see Figure 6b. $\partial u_t/\partial n$, reflecting the strong shearing in the flow, still has the largest derivative value. $\partial u_t/\partial t$, representing streamwise deceleration, is much smaller than the shear term but has a large magnitude immediately at the nozzle exit. It should be emphasized that the values of these velocity derivatives are very sensitive to the coordinate system, because the flame is almost parallel to this rotated coordinate system.

Having considered the trends in velocity derivatives, we next consider the flame orientation characteristics, $n_r$, $n_z$, $n_n$, and $n_t$ (Figure 10). These orientation characteristics play an important role in determining the overall flame strain rate, and which term in Eq. (6) is the dominant contributor to this strain rate. Figure 10 shows the values of $n_z$ and $n_t$ along the flame front. In general, the angle between flame front
and dump plane decreases as the flame bends outward in the downstream direction. Although the flame angle almost changes 20° from 2 to 20 mm axial location, \( n_r \) (not shown) only changes around 10%. However, the magnitude of \( n_z \) changes greatly. Under the rotated coordinate system, which is attached to the jet flow, \( n_r \) (not shown) almost equals unity along the whole flame, while \( n_t \) has a much smaller magnitude.

Figure 11 is the key result of this study. It plots the four contributing flame strain terms, plus their sum (the total estimated hydrodynamic flame strain rate) along the flame for the methane flame. For reference, note that a dimensionless strain rate of 0.1 corresponds to an absolute value of 825 \( \text{1/s} \) for these conditions. In Figure 11a, start with the strain rate characteristics near the attachment point, \( z < 10 \text{ mm} \). In this region, the two dominant contributors to the flame strain flame remain the same as for the velocity gradients—the axial deceleration term, \( \partial v/\partial z \), and the axial shearing term, \( \partial v/\partial r \). However, because the flame is oriented nearly parallel to the flow (\( n_r \approx 0.95 \) and \( n_z \approx 0.15 \)), the relative significance of these terms on the strain rate is inverted. As such, the largest straining term comes from the \( -n_r^2(\partial u/\partial r) \approx n_r^2(\partial v/\partial z) \) term, a negative, or compression, strain. The other symmetric strain rate term, \( -n_z^2(\partial v/\partial z) \), is negligible due to the low value of \( n_z \). The large axial velocity shear term, \( \partial v/\partial r \), is actually of secondary importance in this region because of the low magnitude of \( n_z \) in this region, \( -n_0 n_4(\partial v/\partial r) \). This term is a positive, extensional straining term. Thus, we see that the flame strain characteristics are dominated by two factors—one due to flow deceleration and the other due to flow shear as discussed in the introduction of this paper.

However, distinguishing the role of shear and flow deceleration becomes more difficult farther downstream because of the rotation of the flame in the downstream direction. As such, it is also useful to consider the strain results in the rotated coordinate system (Figure 11b). Since this new coordinate system follows the high speed jet centerline, deceleration and shear terms can be better distinguished. The sum of
these terms, the total strain rate, is unaffected. However, this graph provides a different view of the key contributing terms. It shows that flow deceleration is the key contributor to flame strain over the entire viewing window. In general, the flame front bends with the high speed jet, so that the flame normal is almost coincident with the \( n \) axis. Thus, \( n_n \approx 1.0 \) and \( n_t \approx 0.01 \). The magnitude of the shearing term \(-n_n n_t (\partial u_t / \partial n)\) is, consequently, small. Physically, the flame front stays near the edge of the high-speed jet at the exit of premixer, so flow deceleration strongly compresses the flame surface. Moving downstream, the flame front is still located inside the shear layer, but spreads into a region where the flow is accelerating and, thus, is positively stretched. Due to the flame front and high-speed jet orientations, this result suggests that the shear terms are never important in determining flame stretch. Rather, annular jet flow deceleration, then acceleration (at the flame location), dominates the hydrodynamic flame strain at all measured axial locations. However, some caution should be exercised in drawing this conclusion. The relative roles of what is interpreted as shear and flow deceleration are strongly dependent on the coordinate system—for example, a very small rotation in coordinate that would cause a change in \( n_t \) from 0.01 to 0.03, for instance, has a large effect on the magnitude of the shear term. Moreover, it can be seen that the magnitude of the shear term in Figure 11b is

Figure 11 Axial variation of \( \kappa_c(z) \) and its contributing terms in (a) cylindrical and (b) rotated coordinate systems (CH\(_4\) flame, \( \phi = 0.65 \); \( U_{\text{nozzle}} = 33 \text{ m/s} \)). Error bars only shown for total hydrodynamic strain rate.
actually negative, due to the relative inclinations of the flame to the rotated coordinate system, very likely not a good interpretation of the effects of shear in this region, which provides positive strain, as shown in Figure 5. To summarize, it seems clear that the very near field of the jet ($z < 10\,\text{mm}$) is dominated by flow deceleration—however, it is less clear whether the $10 < z < 20\,\text{mm}$ region should be interpreted as shear or deceleration dominated.

A comparison of $\bar{\kappa}_s(z)$ and $\tilde{\kappa}_s(z)$ is shown in Figure 12, based on conditional averaging of the strain rate just upstream of the flame from 40 PIV shots. As discussed in the Background section, the difference between these two quantities, $\alpha_T$, denotes the contribution from resolved turbulent fluctuations to the time-averaged strain rate. Note also that, in reality, $\kbar_s(z)$ has an additional component due to out of plane motion, estimated to become significant for axial locations beyond about 5–10 mm, see the Error Analysis discussion. It can be seen that $\kbar_s(z)$ and $\tilde{\kappa}_s(z)$ have similar trends, but that there are appreciable quantitative differences, reflecting the role of turbulent fluctuations in causing time-averaged flame strain. For example, at axial locations of 2.7 mm and 19.6 mm normalized $\tilde{\kappa}_s(z)$ values are $-0.22$ and $0.23$, while $\kbar_s(z)$ values are $-0.13$ and $0.42\,1/\text{s}$, respectively. Note that the error bars for the latter averaging procedure are estimates based on the same reasoning as that derived from the unconditioned results, and are probably underestimates.

In order to illustrate the sensitivity of these strain profiles to fuel (or, more fundamentally, the turbulent flame speed), the flame strain rates for different hydrogen mixtures are plotted in Figure 13. It shows that the flame strain varies considerably with the mixture composition. The 100% CH$_4$ and 80%/20% CH$_4$/H$_2$ flame strain characteristics are similar, as they are compressed near the exit and positively stretched farther downstream. However, the 25/75% flame shows a different trend. This is apparently due to the higher turbulent flame speed of the high H$_2$ mixture, as manifested in the angle of the flame with respect to the flow, see Figure 13b. This figure shows that $n_o$ is smaller than the low hydrogen flames near the exit of nozzle (0.97 vs. 0.99), which means that the flame bends outward more (or toward the center of the annular jet). This figure is one example showing the sensitivity of the attachment point strain characteristics to flame properties. Further discussion of
these sensitivities to other geometric parameters and operating conditions is included in the Concluding Remarks section.

VI. CONCLUDING REMARKS

This study has characterized the flame strain characteristics of a swirling flow near its attachment point. The fluid mechanic straining of the flame in this region is dominated by two gradients in velocity—that due to the strong shear near the center-body and due to the bulk flow deceleration as it expands from the smaller diameter nozzle into the combustor. These two velocity gradients cause positive and negative stretching of the flame sheet, respectively. The shearing velocity gradient is larger than the flow deceleration term but, due to the fact that the flame is essentially parallel to the shear layer, actually has a secondary influence relative to the flow deceleration. As a result, the dominant flame straining term near the attachment point for this swirling flow is apparently compressive—a somewhat counterintuitive result.
given that the flame is stabilized in the positively straining shear layer. This, in turn, indicates that the local burning rate is enhanced for lean propane/air mixtures and decreased for lean methane/air or hydrogen/air mixtures.

The sign of the flame strain rate, its spatial profile, and its magnitude are sensitive to geometry, operating conditions, and fuel. For example, the previous conclusions could be altered as the area ratio of the combustor to nozzle exit is changed, because this flow expansion sets the large-scale mean flow deceleration. For instance, smaller area ratios could result in lower flow deceleration, leading to a more important influence of the shearing term on the flame strain. In addition, these effects will also be dependent on swirl number, as highly swirling flow leads to radial flow divergence at the nozzle exit, leading to large flow deceleration. Lower swirl numbers could also lead to more importance of the shearing term. The length of the center-body prior to the combustion section would also influence the boundary layer development, influencing its thickness at the separation point and the associated shearing strain value. Next, operating conditions will be very important. For example, the flame angle, $\gamma$, is a function of the flow velocity. Changes in flame angle, in turn, influence the magnitude of the shearing term on the flame strain rate, as shown by Eq. (6). In addition, pressure and preheat temperature will influence the Reynolds number, and therefore the boundary layer thickness, as well as the flame speed. Finally, as shown by Figure 13, fuel composition influences the laminar and turbulent flame speed, and therefore the flame angle, $\gamma$. As such, it is likely that other combustors have quite different flame strain characteristics. For example, flow past a two dimensional bluff body will likely have a much smaller flow acceleration/deceleration term, suggesting that the shear term could be dominant, causing the near-field strain rate to be positive. As such, further consideration of this problem in other geometries and operating conditions is needed.

REFERENCES


