Further Characterization of the Disturbance Field in a Transversely Excited Swirl-Stabilized Flame

This paper describes an analysis of the unsteady flow field in swirl flames subjected to transverse acoustic waves. This work is motivated by transverse instabilities in annular gas turbine combustors, which are a continuing challenge for both power generation and aircraft applications. The unsteady flow field that disturbs the flame consists not only of the incident transverse acoustic wave, but also longitudinal acoustic fluctuations and vortical fluctuations associated with underlying hydrodynamic instabilities of the base flow. We show that the acoustic and vortical velocity fluctuations are of comparable magnitude. The superposition of these waves leads to strong interference patterns in the velocity field, a result of the significantly different wave propagation speeds and axial phase dependencies of these two disturbance sources. Vortical fluctuations originate from the convectively unstable shear layers and absolutely unstable swirling jet. We argue that the unsteady shear layer induced fluctuations are the most dynamically significant, as they are the primary source of flame fluctuations. We also suggest that vortical structures associated with vortex breakdown play an important role in controlling the time-averaged features of the central flow and flame spreading angle, but do not play an important role in disturbing the flame at low disturbance amplitudes. This result has important implications not only for our understanding of the velocity disturbance field in the flame region, but also for capturing important physics in future modeling efforts. [DOI: 10.1115/1.4004186]

Introduction

Combustion dynamics, a coupling between resonant combustor acoustics and flame heat release fluctuations, has been an issue with propulsion and power generation technologies since the middle of the twentieth century [1]. Initially explained by Rayleigh [2], this coupling can lead to high-cycle fatigue and engine damage, reduced operability windows, and increased emissions. For gas turbines, these instabilities have become more pronounced as engines have been optimized for low NOx emissions [3]. The main emissions abatement strategy, lean combustion, has lead to a rise in the severity of instabilities and the more frequent appearance of transverse instabilities in these engines.

Transverse instabilities are a common instability mode in rockets [4–6], augmenters [7–9], and annular combustors [10,11], but have only recently become a significant issue for can-annular gas turbine systems [12]. These instabilities are characterized by acoustic pressure and velocity perturbations that oscillate normal to the direction of flow. Traditionally, longitudinal instabilities have been the dominant mode in can-annular engines and significant work has been done to understand the coupling mechanisms present in these instabilities [13–15]. More recently, work has been initiated to shed light on some of the flame response characteristics and coupling mechanisms for transversely forced flames [16–22].

Understanding the underlying mechanism for instability is a critical step in predicting the conditions under which instabilities do and do not appear. Two dominant coupling mechanisms in gas turbines are equivalence ratio [23–25] and velocity [26–29] coupling, where other mechanisms, such as pressure coupling [1], are believed to play a negligible role. In this work, we focus on velocity coupled disturbances, by which we mean the flame response to acoustic and vortical velocity perturbations.

Several studies have provided detailed characterizations of the way in which velocity disturbances lead to heat release oscillations. First, acoustic velocity perturbations at the flame attachment point excite flame wrinkles that propagate the entire length of the flame at a speed approximately equal to that of the mean flow [30]. Vortical velocity disturbances originate at the nozzle (e.g., the rollup of the separating shear layer) and distort the flame as they propagate axially at the vortex convection speed [31]. Acoustic disturbances also excite wrinkles as they propagate axially and/or transversely at the sound speed. Finally, several researchers have pointed to a swirl fluctuation mechanism that causes the flame angle to fluctuate with the swirl number [32–35]. Put together, the flame is being simultaneously wrinkled by several sources, each with its own phase and convection speeds.

As discussed in O’Connor et al. [22], the velocity disturbance field in a transversely forced flame is significantly more complex than in the longitudinal problem. The incident transverse acoustic perturbation disturbs the flame in an intricately nonaxisymmetric manner. In addition, the acoustic pressure fluctuation over the nozzle leads to longitudinal acoustic fluctuation in the nozzle region, as shown in simulations by Staffelbach et al. [17], and in experiments by O’Connor et al. [22]. A similar phenomenon occurs in transverse rocket instabilities and is referred to as “injector coupling” [36–39]. Additionally, vortex roll-up at the base of the flame leads to further velocity disturbance sources. These disturbance mechanisms and their pathways are outlined in Fig. 1.

Each of these processes can be notionally described with a transfer function, as is shown in Fig. 1. For example, the coupling between acoustic fluctuations at the nozzle and the resulting vortex rollup is characterized using an acoustic to hydrodynamic velocity transfer function. Additionally, the transverse to longitudinal acoustic coupling process provides an important connection back to the previous flame transfer function work performed for longitudinally excited flames [23,29,32,34,40].
In general, the flame response is due to a superposition of the effects shown in Fig. 1. This superposition manifests in the flame response both locally and globally [27,32,34,35,41–45]. The experimentally observed oscillation in the flame transfer function with frequency is attributed to constructive and destructive interference between disturbances at different parts of the flame. Similarly, the spatial distribution of the flame heat release response is strongly influenced by interference effects. Shanbhogue et al. [46] show that the response amplitude of the flame, as measured by the flame displacement from the centerline axis, first rises and then falls as a function of downstream distance. Recent studies from Emerson et al. [47] show nodes in the response of a vitiated bluff-body stabilized flame at certain axial locations.

Interference phenomena cannot only be seen in the flame response, but also in the velocity disturbance field. Acoustic and vortical disturbances travel downstream and constructively and destructively interfere. This interference effect is particularly striking when the two disturbances are of similar amplitudes. If the amplitudes of the two waves are roughly equal, the nodes in the interference pattern are approximately zero, resulting in a zero velocity fluctuation in certain spatial locations. This effect was briefly discussed in O’Connor et al. [22] and is more thoroughly investigated in this work.

We next consider the unsteady flow field characteristics in swirling flow in more detail. Swirl flows exhibit a variety of behaviors depending on combustor geometry, swirl number, Reynolds number, and many other parameters. Indeed, it is currently not possible to draw general conclusions about flow structure from specific papers, as results are highly configuration specific. Moreover, it appears that a general flow classification for the type of unsteady flow structures, their mechanism of occurrence, and how they manifest themselves, does not exist yet for high Reynolds number swirling flows. Finally, it is known that heat release has important effects upon both the time-averaged and unsteady flow structure, so that nonreacting swirl flow results with vortex breakdown, several different sources of instability coexist. Vortex breakdown is a manifestation of an absolute instability of the swirling jet. In other words, the vortex breakdown region, characterized above as a flow feature controlling the “base” state in the central part of the flow field, is itself intrinsically unsteady. In addition, the shear layers, in both the azimuthal and streamwise directions, are convectively unstable and rollup and pair into larger concentrated regions of vorticity. This shear layer rollup is shown in Fig. 2(b).

We postulate that the distinction between the absolutely unstable (AI) vortex breakdown bubble and convectively unstable (CI) shear layers is key to understanding the response of the system to low to intermediate amplitude acoustic excitation. The absolutely unstable breakdown bubble exhibits intrinsic dynamics that are relatively independent of the excitation at low amplitudes [49,50]. Therefore, the basic dynamics of this flow remains unchanged in the presence of low amplitude acoustic forcing. Only in the presence of large amplitude dynamics that alter the vortex breakdown bubble [51], and cause frequency locking of bubble dynamics, are the interactions between the excitation and vortex breakdown bubble dynamically significant, although this effect too may be limited to a change in the time-averaged “base” state.
The convectively unstable shear layers are amplifiers that respond to the external forcing. Moreover, since the flame lies in the shear layer, as shown in Fig. 2(b), the instability characteristics of the shear layers dominate the flame response. Although the “base” state of the flow changes with heat release, the essential features of the unstable shear layers do not. This implies that key dynamical features of the flow that excite the flame remain qualitatively similar between nonreacting and reacting situations. This point should not be pressed to far, as heat release certainly has important influences on the shear layer development. For example, the dilatation effect of heat release acts as a vorticity sink in the inner shear layer where the flame is stabilized. Nonetheless, it is an important simplification for interpretation of results in this complex flow field.

A hypothesis put forward here is that the key unsteady flow features responsible for disturbing the flame are essentially the same between reacting and nonreacting flow configurations. In the rest of this paper, we further investigate the velocity disturbance field characteristics in a transversely forced annular swirling jet, both for nonreacting and reacting flows. First, we discuss the experimental configuration and diagnostic systems used. Next, an overview of the important disturbance field characteristics is outlined. This is followed by a more detailed look at the convection velocity, mode shape, and relative amplitude of the disturbances in the flow field. These parameters are used to develop a two wave fit that describes the interference phenomenon observed in the fluctuating velocity field.

Experimental Setup

In this section we overview the experimental facility and diagnostic systems used in this study. For more experimental facility details, see O’Connor et al. [22]. The combustor mimics an annular combustor configuration and was designed to support a strong transverse acoustic mode. A swirler nozzle is situated at the center of the chamber with an outer diameter of 31.75 mm, inner diameter of 21.84 mm, and swirl number of 0.85. The fuel is natural gas and the equivalence ratio is 0.9. Six acoustic drivers, three on each side, provide the acoustic excitation for the system.

The acoustic drivers on either side of the combustor can be controlled independently. By changing the phase between the signals driving each side of the combustor, different wave patterns, both standing and traveling waves, can be created. When the drivers are forced in-phase, a pressure antinode and velocity node are created at the center of the experiment. When the drivers are forced out-of-phase, a pressure node and velocity antinode are created at the center. The difference between these acoustic driving conditions can be seen in the data by looking at the transverse velocity along the centerline. In the case of out-of-phase forcing, the centerline velocity is high amplitude and sinusoidal, where in the in-phase forcing case, the signal is random and low amplitude. The nonzero value of the transverse velocity observed for the in-phase case is likely due to imbalances in excitation amplitudes in the left and right speakers and random motion in the vortex breakdown region.

Particle image velocimetry is used to measure the velocity field in this experiment. A LaVision Flowmaster Planar Time Resolved system allows for two-dimensional velocity measurements at 10 kHz. The laser is a Litron Lasers, Ltd. LDY303He Nd:YLF laser with a wavelength of 527 nm and a 5 mJ/pulse pulse energy at the 10 kHz repetition rate used for these experiments. The Photron HighSpeed Star 6 camera has a 640 x 448 pixel viewing window and a resolution of 0.18 mm per pixel at a frame rate of 10 kHz. Velocity field calculations were performed using DaVis 7.2 software from LaVision. 500 velocity fields were obtained at each test condition, using 30 ms between laser pulses. 32 x 32 pixel interrogation windows with 50% overlap were used for the velocity calculation.

All results are presented in nondimensional form. The velocities are normalized by the bulk approach flow velocity, $U_0 = \frac{m}{\rho S}$ (where $\rho$ and $S$ denote approach flow density and annulus area), the spatial coordinates by the nozzle diameter, $D$, time by the forcing frequency, $f_0$, and the vorticity by the bulk velocity divided by the annular gap width, $U_0/(r_2 - r_1)$.

Results

This section consists of three parts: explanation of the time-averaged flow properties, an overview of the disturbance flow field topology, and a discussion of the fluctuating disturbance field characteristics and the interference behavior found within.

The time-averaged axial velocity and out-of-plane vorticity are shown in Fig. 3, with the nonreacting flow on top and reacting on bottom. The time-averaged flow fields of the nonreacting and reacting cases are different in several ways. First, the vortex breakdown bubble changes in size and shape, growing wider at the dump plane in the reacting case. Second, the jet spreading angle is higher in the reacting case, presumably because of the expansion of the vortex breakdown bubble. Third, the average shear layer locations, as shown by the vorticity plots, also spread as a result of the two aforementioned effects. Note that this flow contains two distinct shear layers, one emanating from the inner edge and one from the outer edge of the annulus. The flame configuration is nominally in a V-shape flame, stabilized in the inner shear layer.

Flow Field Topology

At a given mean flow velocity, the vorticity field is influenced by the acoustic forcing configuration and the presence of a flame. First, the effect of acoustic forcing is considered. The transverse acoustic velocity causes significant side-to-side flow field and flame motion. Additionally, the transverse acoustics create pressure disturbances that, as described earlier, lead to axial velocity fluctuations in the flame.
fluctuations at the nozzle. The phasing of these axial fluctuations on either side of the nozzle determines the symmetry of the disturbance field, similar to the observations of Rodriguez and co-workers for a rocket injector [39]. Figure 4 shows a notional sketch of a cross-section of the flow field.

The figure shows three main features of the flow field in both the reacting and nonreacting flow. First, the center of the flow is dominated by the vortex breakdown flowfield. On either side of the bubble is the annular jet column. The inner shear layer, the region of shear between the jet column and the vortex breakdown region, and the outer shear layer, the region between the jet column and the ambient fluid, contain coherent vortices. These structures are formed as a result of fluctuating axial velocity at the nozzle, and the phase of the fluctuations on one edge with respect to the other edges determines the symmetry of the vorticity field.

Figure 4(a) shows the case of an asymmetric flow field, caused by out-of-phase acoustic forcing. Here, a helical disturbance is created within each shear layer, resulting in a staggered vortex pattern in the plane formed by the laser sheet. Figure 4(b) shows an example of an axisymmetric flow field, where vortex rings propagate downstream from the inner and outer edges of the annulus. As they convect downstream, these structures deform and locally bend the jet column. In addition to the aforementioned processes, the transverse acoustic motion periodically shifts the flow field from side to side, shifting the angle of the jet column and the trajectories of the coherent structures.

The plots in Fig. 5 show the filtered velocity and vorticity field, obtained by plotting the sum of the vorticity fluctuation at the forcing frequency and the mean vorticity. This calculation, similar to the analysis described in Ref. [22], involves taking the FFT of the instantaneous velocity and vorticity at each point and harmonically reconstructing the signal at the forcing frequency. This is effectively a filtering, or phase locking, process that captures only the motions at the forcing frequency, eliminating turbulent noise. This process also spatially smears the instantaneous vorticity, due to cycle-cycle phase jitter in axial location of the vortical structures. Figure 5, like the notional pictures in Fig. 4, shows the out-of-phase and in-phase velocity and vorticity fields for one phase of the acoustic cycle.

In the out-of-phase forcing case, a pressure node is present along the centerline of the flow. The pressure fluctuations on either side of the node are out of phase, creating out-of-phase axial velocity fluctuations on either side of the nozzle. This asymmetry in the axial velocity fluctuations leads to asymmetry in the vorticity field, which is evident in Fig. 5(a). For example, in the outer shear layer, the structure closest to the dump plane is on the right-hand side at x/D = 0.2. The next shear layer structure is on the left-hand side at x/D = 0.5. The final structure of significant strength in the outer shear layer is again on the right-hand side at x/D = 0.9. This vortex structure suggests a helical vortex pattern...
in both the inner and outer shear layers, similar to LES simulations [52–54].

In the in-phase forcing case, the formation of structures in each shear layer is nominally axisymmetric, as shown in Fig. 5(b). For example, the inner shear layer shows two sets of structures, one located at $x/D = 0.3$ and one at $x/D = 0.9$. The outer shear layer structures are also axisymmetric, but not aligned with the structures in the inner shear layer. Similar trends for both the in-phase and out-of-phase forcing are observed in the nonreacting cases as well.

The behavior of the vorticity field in these filtered velocity and vorticity images is very similar to that of the instantaneous fields, shown in Fig. 6. The harmonic reconstruction acts as a phase-locking process that smears the effect of phase jitter and neglects the motions at other frequencies. Despite that, the major coherent features of the flow are similar. The images shown in Fig. 6 correspond to the phases depicted in Fig. 5.

The local behavior and downstream evolution of the shear layer structures appear qualitatively similar between the nonreacting and reacting cases, further emphasizing the point that heat release does not affect the basic mechanisms responsible for the appearance of unsteady flow structures in the shear layers that distort the flame. The major difference between the two is difference in dissipation rate of the vorticity, as seen in Fig. 7. In the reacting case, significant vorticity is seen until $x/D = 1$, while it has decayed by $x/D = 0.4$ in the nonreacting case.

Convection velocities of the vortex structures were estimated from the axial phase dependence of the vortical disturbances shown in Fig. 8. These phases were calculated from the phase of the FFT at the forcing frequency along the inner and outer shear layers, as shown in Fig. 3. The shear layer locations were estimated from the time-averaged vorticity magnitude maxima.
Linear fits of the phase data were calculated as a function of downstream distance. The slope of this line was taken from the fit and used to calculate the convection velocity using the formula

$$u_{c,v} = \frac{2\pi f_0}{(d\phi/dx)}.$$  \hspace{1cm} (1)

The convection speeds of the vorticity along the six lines of travel were calculated and averaged, leading to a value of 13 and 10 m/s for the nonreacting and reacting cases, respectively. Each convection speed falls within the range of convection speeds measured in nonswirling flows, which are

$$\frac{1}{2}C_{24} \leq U_0 < \frac{1}{2}C_{14}$$

The uncertainty in phase is 6 ± 15 deg. These phase differences indicate the shape and symmetry of the disturbance field on either side of the nozzle. For example, the phases of the in- and out-of-phase reacting cases are 130 deg and −50 deg, respectively. This can be seen by looking at snapshots of the vorticity field at any point in time, shown in Fig. 5.

### Fluctuating Disturbance Field Characteristics

This section focuses on the unsteady disturbance field characteristics, as opposed to the total instantaneous/filtered field structure detailed earlier. There are fundamental differences between the appearance of the unsteady flow structures as visualized by their instantaneous and fluctuating values. For example, the location and characteristics of vortical structures can look fundamentally different between the two, and conclusions about topological flow features should probably only be drawn from the total, instantaneous or filtered field characteristics. That said, the unsteady flame response is closely linked to the fluctuating quantities. In this section, we look only at the fluctuating quantities, by subtracting the time-averaged behavior, to more carefully investigate the behavior of the different velocity disturbances in the flow field.

Surface plots of the amplitude of both the velocity and vorticity fluctuations show interesting results. Figures 9, 10, and 11 show the amplitude of the axial velocity, transverse velocity, and vorticity fluctuations at the forcing frequency, respectively, for both nonreacting and reacting flow. One of the most prominent features

<table>
<thead>
<tr>
<th>Phase (± 15 deg)</th>
<th>Nonreacting, out-of-phase</th>
<th>Reacting, out-of-phase</th>
<th>Nonreacting, in-phase</th>
<th>Reacting, in-phase</th>
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<tr>
<td>Phase (deg)</td>
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<td>−50</td>
<td>100</td>
<td>130</td>
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</table>

**Table 1** Average phase difference between vorticity disturbances in the left and right jet centers at several conditions at a bulk velocity of $U_o = 10$ m/s and a forcing frequency of $f_0 = 400$ Hz.
of these plots is the highly nonmonotonic characteristics of the unsteady velocity field, suggesting a cancellation phenomenon. For example, in Fig. 10(b), which shows the transverse velocity fluctuations at the forcing frequency for a reacting case, the fluctuation amplitude peaks at approximately $x/D = 0$, decreases to a minimum value at $x/D = 0.3$, and then increases until a downstream distance of $x/D = 0.6$ where it again peaks and decreases.

While both the axial and transverse velocity fluctuation fields show the interference phenomenon, there is another important factor that differentiates these two types of fluctuations. The transverse velocity fluctuation surface, shown in Fig. 10, has a constant offset across the field. This offset is a result of the acoustic velocity fluctuations, which are of nearly constant amplitude across the field of view for the out-of-phase forcing case shown. Conversely, there is no offset of the axial velocity surfaces, indicating that there are no longitudinal acoustics in the flow field downstream of the immediate nozzle exit. Thus, longitudinal acoustics are important in the immediate vicinity of the nozzle, but these fluctuations do not contribute in any significant way farther into the combustor.

We believe that the nonmonotonic spatial dependence of velocity amplitude is due to the simultaneous presence of both acoustic and vortical velocity disturbances. This can be shown by constructing a simple representation of the unsteady transverse velocity field, with disturbances propagating at two different axial phase speeds. The input parameters for this fit are the initial amplitude of each wave ($A_1$ and $A_2$), the decay rate ($\lambda$), and convolution speed ($u_{cv}$) of the vortical disturbance, and the phase ($\phi$) between the two disturbance types, see Eq. (2)

$$u_{\text{acoustic}} = A_1 e^{-\lambda t} e^{iu(t + \phi)},$$

$$u_{\text{vortical}} = A_2 e^{-\lambda t} e^{iu(t - x/u_{cv}} e^{x/t}.$$  \hspace{1cm} (2)

The convection velocity of the vortical wave was estimated from the mean of the velocities calculated from Eq. (1). The decay rate was then calculated by fitting an exponential to the decay of time-averaged vorticity as a function of downstream distance, the results of which can be seen in Fig. 12. This was done for both the nonreacting and the reacting case.

The parameters for the acoustic wave were less obvious to extract from experimental data and were used as fit parameters to match the data. The parameters used for a variety of nonreacting and reacting cases are shown in Table 2.

Two example results of both the data and the fit in the nonreacting and reacting cases are shown in Fig. 13. The results of the data and the fit align reasonably well. An important feature of these graphs is the peak spacing in the interference pattern, which is only a function of the difference in acoustic and vortical phase velocities. This is captured quite well in the fit.

An important implication of this fit is that the amplitudes of the acoustic and vortical waves are essentially the same. For example, both the acoustic and vortical fluctuations in the reacting case, shown in Fig. 13(b), are 25% of the bulk flow velocity. This result has important implications on flame response modeling, as models often assume that it is the vortical disturbances that dominate the
flame response [46]. These results show that the acoustic and vortical disturbances have comparable magnitudes, and emphasize the complexity of the disturbance field.

These results can also be used to deduce the amplitude and phase of $F_{TL}F_{TL} + F_{TL}x + F_{TL}$, the transfer function that describes the relationship between the transverse acoustic motion and the vortex rollup at the nozzle, as shown in Fig. 1. The ratio of the amplitudes, $A_1$ and $A_2$, gives the magnitude of the transfer function at this particular forcing frequency, while the phase between the two disturbances, $\phi$, is the phase of the transfer function at the forcing frequency. Note that this results suggests then, that $F_{TL}F_{TL} + F_{TL}x + F_{TL} \approx 1$.

Conclusions

Key conclusions from this work are:

1. Swirling jets contain two important types flow instability: an absolute instability of the swirling flow that sets the base flow state, and the shear layer instability. The fluctuating velocity field, and hence flame response, are dominated by the convectively unstable shear layer and its response to acoustic excitation. The behavior of these structures does not fundamentally change in the presence of the flame.

2. The disturbance field of a transversely excited swirling jet consists of different types of velocity disturbances with relationships described in the disturbance pathway diagram in Fig. 1. The acoustic field fluctuates in both the transverse and axial direction, owing to the coupling between transverse acoustic pressure fluctuations and axial velocity fluctuations at the nozzle. Acoustic fluctuations excite flow instabilities, particularly in the convectively unstable shear layers, which lead to additional sources of flame response.

3. Out-of-phase acoustic forcing cases result in asymmetric vorticity disturbances, presumably associated with a helical structure, while in-phase forcing results in nominally axi-symmetric vortical structures.

4. The resulting disturbance field is dominated by both acoustic and vortical disturbances. These disturbances constructively and destructively interfere to cause interference patterns in the fluctuating velocity field. Using convection velocity and decay rate parameters calculated from the vorticity data, a two wave fit captures the interference pattern in the velocity data for both reacting and nonreacting data.

Nomenclature

$A_1 =$ amplitude of acoustic wave (fit)
$A_2 =$ amplitude of vortical wave (fit)
$D =$ outer diameter of the swirler nozzle
$S =$ nozzle annular area
$f_0 =$ forcing frequency
$m =$ mass flow
$r =$ radius
$t =$ time
$U_o =$ bulk velocity
$u_{c,v} =$ convection velocity
$u' =$ axial velocity fluctuation
$\hat{u} =$ Fourier transform of axial velocity fluctuations
$v' =$ transverse velocity fluctuation
$\hat{v} =$ Fourier transform of transverse velocity fluctuations
$\alpha =$ vortical decay rate (fit)
$\rho =$ density
$\phi =$ phase of vorticity
$\phi =$ phase difference (fit)
$\omega =$ angular frequency

References


Table 2 Two-wave fit conditions for non-reacting and reacting cases at several acoustic forcing and velocity conditions. The first two columns are the parameters used in Fig. 13.

<table>
<thead>
<tr>
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<th>400 Hz,</th>
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<tr>
<td></td>
<td>10 m/s, nonreacting</td>
<td>10 m/s, reacting</td>
<td>10 m/s, nonreacting</td>
<td>10 m/s, reacting</td>
<td>40 m/s, nonreacting</td>
<td>40 m/s, nonreacting</td>
</tr>
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| $A_1$  | 0.3     | 0.25     | 0.1      | 0.2      | 0.2      | 0.12     |
| $\phi$ | $\pi/6$ | $\pi/4$  | 0        | $\pi/2$  | $\pi/14$ | 7$\pi/16$ |
| $A_2$  | 0.3     | 0.25     | 0.1      | 0.2      | 0.2      | 0.12     |
| $x (1/m)$ | 43       | 27       | 39       | 25       | 20       | 29       |
| $u_{c,v}$ (m/s) | 13       | 10       | 10       | 10       | 18       | 19       |

Fig. 13 Comparison of transverse velocity fluctuation amplitude in left and right jet of the (a) nonreacting and (b) reacting data and the two-wave interaction fit. The bulk velocity was $U_o = 10$ m/s and the forcing frequency was $f_0 = 400$ Hz out-of-phase.