

### 32.3 Taguchi's Robust Design Method

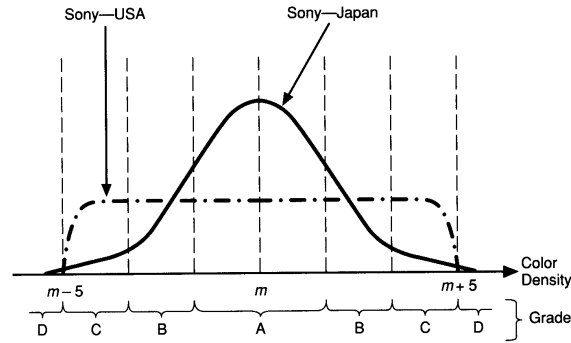
Since 1960, Taguchi methods have been used for improving the quality of Japanese products with great success. During the 1980's, many companies finally realized that the old methods for ensuring quality were not competitive with the Japanese methods. The old methods for quality assurance relied heavily upon inspecting products as they rolled off the production line and rejecting those products that did not fall within a certain acceptance range. However, Taguchi was quick to point out that no amount of inspection can improve a product; quality must be designed into a product from the start. It is only recently that companies in the United States and Europe began adopting Taguchi's robust design approaches in an effort to improve product quality and design robustness.

What is robust design? *Robust design is an "engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost"* (Phadke, 1989). The idea behind robust design is to improve the quality of a product by minimizing the effects of variation without eliminating the causes (since they are too difficult or too expensive to control). His method is an off-line quality control method that is instituted at both the product and process design stage to improve product manufacturability and reliability by making products insensitive to environmental conditions and component variations. The end result is a *robust design, a design that has minimum sensitivity to variations in uncontrollable factors.*

Dr. Genichi Taguchi bases his method on conventional statistical tools together with some guidelines for laying out design experiments and analyzing the results of these experiments. Taguchi's approach to quality control applies to the entire process of developing and manufacturing a product—from the initial concept, through design and engineering, to manufacturing and production. Taguchi methods are used to specify dimension and feature detail and normally follow DFM activities. In the next section we discuss Taguchi's concept of a quality loss function. This is then followed by a detailed description of Taguchi's approach to parameter design.

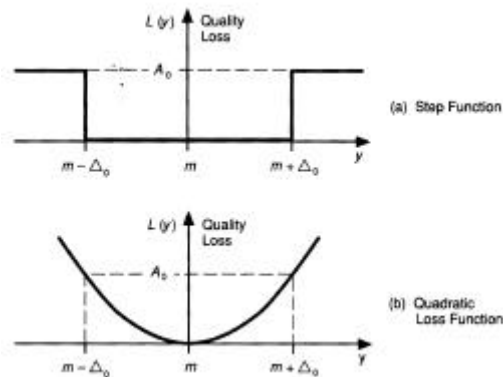
#### 32.3.1 Taguchi's Quality Loss Function

Consider the popular story of Sony and two of their television production facilities, one in the USA, the other in Japan. The color density of the televisions manufactured by Sony-USA were uniformly distributed and fell within the tolerance limits,  $m \pm 5$  (where  $m$  is the target for color density), while the televisions from Sony-Japan followed a normal distribution, more televisions were on target but about 0.3% fell outside of the tolerance limits. The color density distributions are illustrated in Figure 1. The differences in customer perceptions of quality resulted from Sony-USA paying attention only to meeting the tolerances whereas in Sony-Japan, the focus was on meeting the target and minimizing the variance around that target. If we assign a grade to each television based on its color density as done in Figure 12, then we see that Sony-Japan produced many more Grade A sets and fewer Grade C sets in comparison to Sony-USA. Overall, a much larger portion of Sony-Japan's televisions receive higher grades than those made by Sony-USA; hence, the customer's preferred the televisions sets produced by Sony-Japan over those produced by Sony-USA.



**Figure 1 Color Density Distribution of Television Sets (Phadke, 1989)**

To measure quality, Taguchi defines a Quality Loss Function. The quality loss function is a continuous function that is defined in terms of the deviation of a design parameter from an ideal or target value, see Figure 2b. Taguchi's view on the nature of the quality loss function represents a fundamental paradigm shift (particularly in the U.S.) in the way in which manufacturers consider whether or not a product is good. The traditional approach employed by U.S. manufacturers (as evidenced by Sony-USA) has been to use a "step function" that ensures that performance fell within the upper and lower specification limits as shown in Figure 2a.



**Figure 2 Quality Loss Function (Phadke, 1989)**

Taguchi's loss function can be expressed in terms of the quadratic relationship:

$$L = k (y - m)^2 \quad [32.1]$$

where  $y$  is the critical performance parameter value,  $L$  is the loss associated with a particular parameter  $y$ ,  $m$  is the nominal value of the parameter specification,  $k$  is a constant that depends on the cost at the specification limits (can be determined conservatively by dividing the cost of scrap in \$, by the square of the lower or higher tolerance values). This function penalizes the deviation of a parameter from the specification value that contributes to deteriorating the performance of the product, resulting in a loss to the customer. The key here is that a product engineer has a good understanding of what the nominal size of the specification is. The usual lower and upper limits for the tolerance of a given design parameter are changed to a continuous function that presents any parameter value other than the nominal as a loss. The loss function given in Eq.32.1 is referred to as "nominal is best," but there are also expressions for cases when higher or lower values of parameters are better (Phadke, 1989).

If a large number of parts are considered, say  $N$ , the average loss per part is equal to the summation of the losses given by Eq. 32.1 for each part, divided by the total  $N$ . The average quality loss results from deviation around the average value of  $y$  from the target and the mean square deviation of  $y$  around its own mean. The average quality loss can be expressed as:

$$L = k[S^2 + (\mu - m)^2] \quad [32.2]$$

where  $\mu$  is the average value of  $y$  for the set of parts, and  $S^2$  is the variance around the average.

To minimize loss, the traditional approach is to monitor the process variables during production and adjust the process to reduce manufacturing imperfections so that response parameters fall within the specified tolerances. This method adds cost to the manufacturing process does not improve the quality of the product. Using Taguchi's approach the average response has to be adjusted, and the variance must be reduced in order to minimize loss. Reducing the variation is accomplished by product and process engineers who use off-line quality control techniques; adjustments to the average response are realized by process and production engineers during production using on-line quality control techniques. Within the Taguchi philosophy both quality improvement methods are considered; however, building quality into the product during the design stage (i.e., off-line) is the ultimate goal.

To achieve desirable product quality by design, Taguchi suggests a three-stage process: **system design**, **parameter design**, and **tolerance design**. *System design is the conceptualization and synthesis of a product or process to be used.* The system design stage is where new ideas, concepts and knowledge in the areas of science and technology are utilized by the design team to determine the right combination of materials, parts, processes and design factors that will satisfy functional and economical specifications. To achieve an increase in quality at this level requires innovation, and therefore improvements are not always made. In parameter design the system variables are experimentally analyzed to determine how the product or process reacts to uncontrollable "noise" in the system; parameter design is the main thrust of Taguchi's approach. *Parameter design is related to finding the appropriate design factor levels to make the system less sensitive to variations in uncontrollable noise factors, i.e., to make the system robust.* In this way the product performs better, reducing the loss to the customer.

The final step in Taguchi's robust design approach is tolerance design; *tolerance design occurs when the tolerances for the products or process are established to minimize the sum of the manufacturing and lifetime costs of the product or process.* In the tolerance design stage, tolerances of factors that have the largest influence on variation are adjusted only if after the parameter design stage, the target values of quality have not yet been achieved. Most engineers tend to associate quality with better tolerances, but tightening the tolerances increases the cost of the product or process because it requires better materials, components, or machinery to achieve the tighter tolerances as we discussed in earlier chapters. Taguchi's parameter design approach allows for improving the quality without requiring better materials or parts and makes it possible to improve quality and decrease (or at least maintain the same) cost. Parameter design is discussed in detail in the next section.

### 32.3 Taguchi's Parameter Design Approach

In parameter design, there are two types of factors that affect a product's functional characteristic: **control factors** and **noise factors**. *Control factors are those factors which can easily be controlled* such as material choice, cycle time, or mold temperature in an injection molding process. *Noise factors are factors that are difficult or impossible or too expensive to control.* There are three types of noise factors: outer noise, inner noise, and between product

noise. Examples of each type of noise factor and controllable factors in product and process design are listed in Table 1. Noise factors are primarily response for causing a product’s performance to deviate from its target value. Hence, parameter design seeks to identify settings of the control factors which make the product insensitive to variations in the noise factors, i.e., make the product more robust, without actually eliminating the causes of variation.

	<b>Product Design</b>	<b>Process Design</b>
<b>Outer Noise</b>	Consumer’s usage conditions Low temperature High temperature Temperature change Shock Vibration Humidity	Ambient Temperature Humidity Seasons Incoming material variation Operators Voltage change Batch to batch variation
<b>Inner Noise</b>	Deterioration of parts Deterioration of material Oxidation (rust)	Machinery aging Tool wear Deterioration
<b>Between Product Noise</b>	Piece to piece variation where they are supposed to be the same, e.g., Young’s modulus shear modulus allowable stress	Process to process variation where they are supposed to be the same, e.g., variations in feed rate
<b>Controllable Factors</b>	All design parameters, e.g., • dimensions • material selection	All process design parameters All process setting parameters

**Table 1 Examples of Noise and Control Factors (adapted from Byrne and Taguchi, 1987)**

Design of experiments techniques, specifically Orthogonal Arrays (OAs), are employed in Taguchi’s approach to systematically vary and test the different levels of each of the control factors. Commonly used OAs include the L<sub>4</sub>, L<sub>9</sub>, L<sub>12</sub>, L<sub>18</sub>, and L<sub>27</sub>, several of which are listed in Table 2. A complete listing of OAs can be found in text such as (Phadke, 1989). The columns in the OA indicate the factor and its corresponding levels, and each row in the OA constitutes an experimental run which is performed at the given factor settings. For instance, in Fig. 2 experimental run #3 has Factor 1 at Level 2, Factor 2 at Level 1, and Factor 3 at Level 2. It is up to the experimental designer to establish the appropriate factor levels for each control factor; typically either 2 or 3 levels are chosen for each factor. Selecting the number of levels and quantities properly constitutes the bulk of the effort in planning robust design experiments.

Factors	
Run	A B C
1	1 1 1
2	1 2 2
3	2 1 2
4	2 2 1

(a) L<sub>4</sub> (2<sup>3</sup>) array

Factors						
Run	A B C D E F G					
1	1 1 1 1 1 1 1					
2	1 1 1 2 2 2 2					
3	1 2 2 1 1 2 2					
4	1 2 2 2 2 1 1					
5	2 1 2 1 2 1 2					
6	2 1 2 2 1 2 1					
7	2 2 1 1 2 2 1					
8	2 2 1 2 1 1 2					

(b) L<sub>8</sub> (2<sup>7</sup>) array

Factors			
Run	A B C D		
1	1 1 1 1		
2	1 2 2 2		
3	1 3 3 3		
4	2 1 2 3		
5	2 2 3 1		
6	2 3 1 2		
7	3 1 3 2		
8	3 2 1 3		
9	3 3 2 1		

(c) L<sub>9</sub> (3<sup>4</sup>) array

**Table 2 Some Commonly Used Orthogonal Arrays**

To implement robust design, Taguchi advocates the use of an “inner array” and “outer array” approach. The “inner array” consists of the OA that contains the control factor settings; the “outer array” consists of the OA that contains the noise factors and their settings which are under investigation. The combination of the “inner array” and “outer array” constitutes what is called the “product array” or “complete parameter design layout.” Examples of each of these arrays are given in the case study in the next section. The product array is used to systematically test various combinations of the control factor settings over all combinations of noise factors after which the mean response and standard deviation may be approximated for each run using the following equations.

- Mean response: 
$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad [32.3]$$

- Standard deviation: 
$$S = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}} \quad [32.4]$$

The preferred parameter settings are then determined through analysis of the “signal-to-noise” (SN) ratio where factor levels that maximize the appropriate SN ratio are optimal. There are three standard types of SN ratios depending on the desired performance response (Phadke, 1989):

- Smaller the better (for making the system response as small as possible):

$$SN_S = -10 \log \left( \frac{1}{n} \sum_{i=1}^n y_i^2 \right) \quad [32.5]$$

- Nominal the best (for reducing variability around a target):

$$SN_T = 10 \log \left( \frac{\bar{y}^2}{S^2} \right) \quad [32.6]$$

- Larger the better (for making the system response as large as possible):

$$SN_L = -10 \log \left( \frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right) \quad [32.7]$$

These SN ratios are derived from the quadratic loss function and are expressed in a decibel scale.

Once all of the SN ratios have been computed for each run of an experiment, Taguchi advocates a graphical approach to analyze the data. In the graphical approach, the SN ratios and average responses are plotted for each factor against each of its levels. The graphs are then examined to “pick the winner,” i.e., pick the factor level which (1) best maximize SN and (2) bring the mean on target (or maximize or minimize the mean, as the case may be). Using this information, the control factors can also be grouped as follows.

1. Factors that affect both the variation and the average performance of the product.
2. Factors that affect the variation only.
3. Factors that affect the average only.
4. Factors that do not affect either the variance or the average.

Factors in the first and second groups can be utilized to reduce the variations in the system, making it more robust. Factors in the third group are then used to adjust the average to the target value. Lastly, factors in the fourth group are set to the most economical level. Finally,

confirmation tests should be run at the “optimal” product settings to verify that the predicted performance is actually realized. A demonstration of Taguchi’s approach to parameter design serves as our case study in the next section.

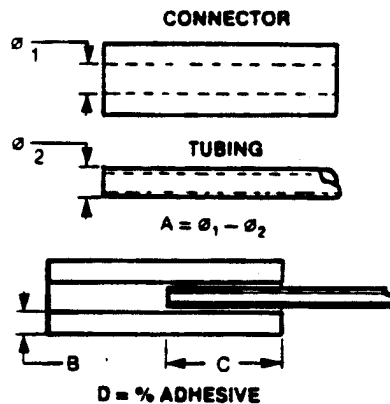
### 32.4 Case Study: Parameter Design of an Elastometric Connector

The following case study is taken from “The Taguchi Approach to Parameter Design,” by D. M. Byrne and S. Taguchi, *Quality Progress*, Dec. 1987, pp. 19-26. The case uses Taguchi’s parameter design approach to integrate product and process design decisions for elastometric connector used in an automotive engine application.

#### 32.4.1 The Problem

The experiment that is being conducted seeks to determine a method to assemble an elastometric connector to a nylon tube while delivering the requisite pull-off performance suitable for an automotive engineering application. The primary design objective is to maximize the pull-off force while secondary considerations are made to minimize assembly effort and reduce the cost of the connector and assembly.

Four control factors and three noise factors have been identified for the connector and tube assembly. The control factors consist of the (A) interference, (B) connector wall thickness, (C) insertion depth, and (D) percent adhesive in connector pre-dip; a sketch of the control factors for the connector and tube is given in Figure 3.



**Figure 3 Control Factors for Connector and Nylon Tube Experiment (Byrne and Taguchi, 1987)**

The noise factors in the experiment are (E) conditioning time, (F) conditioning temperature, and (G) conditioning relative humidity. Each control factor is to be tested at three levels while each noise factor is tested at two levels. The factors and levels of concern for the experiment are listed in Table 3. In terms of product and process design, Factors A and B represent product design parameters while Factors C-G represent process design parameters. However, during routine operation the noise factors (E-G) are uncontrollable and are thus taken as “noise” which can adversely affect product performance. Fortunately, these noise factors can be controlled for the purposes of this experiment. In this regard, Taguchi’s parameter design approach can be used to help make product and process design decisions to improve the robustness of a system.

<b>Controllable Factors</b>	<b>Levels</b>		
A. Interference	Low	Medium	High
B. Wall thickness	Thin	Medium	Thick
C. Insertion depth	Shallow	Medium	Deep
D. Percent adhesive	Low	Medium	High

<b>Noise Factors</b>	<b>Levels</b>	
E. Conditioning time	24 h	120 h
F. Conditioning temperature	72°F	150°F
G. Conditioning relative humidity	25%	75%

**Table 3 Factors and Levels for Connector and Tube**

### 32.4.2 The Experiment

Following Taguchi's method, two experimental designs are selected to vary (i) the control factors and (ii) the noise factors. An  $L_9$  orthogonal array is selected for the controllable factors while an  $L_8$  orthogonal array is chosen for the noise factors, see Table 4. The ones, twos, and threes in the  $L_9$  array in Table 4a correspond to the low, medium, and high levels identified for each control factor and listed in Table 3. Similarly, the ones and twos in the  $L_8$  in Table 4b corresponding the low and high levels for each of the noise factors. Note that only the columns labeled E, F, and G in Table 4b are actually used in the experiment. Since there are only three noise variables, the remaining columns in the  $L_8$  array are used to estimate the interactions between certain noise factors (e.g., ExF represents the interaction between conditioning time, E, and temperature, F). Finally, the last column in the  $L_8$  array is used to estimate the variance in the experiment.

Run	Factor			
	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Run	Factor						
	E	F	ExF	G	ExG	FxG	e
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

(a)  $L_9$  Orthogonal Array  
for the Control Factors

(b)  $L_8$  Orthogonal Array  
for the Noise Factors

**Table 4 Designs for the Control and Noise Factors**

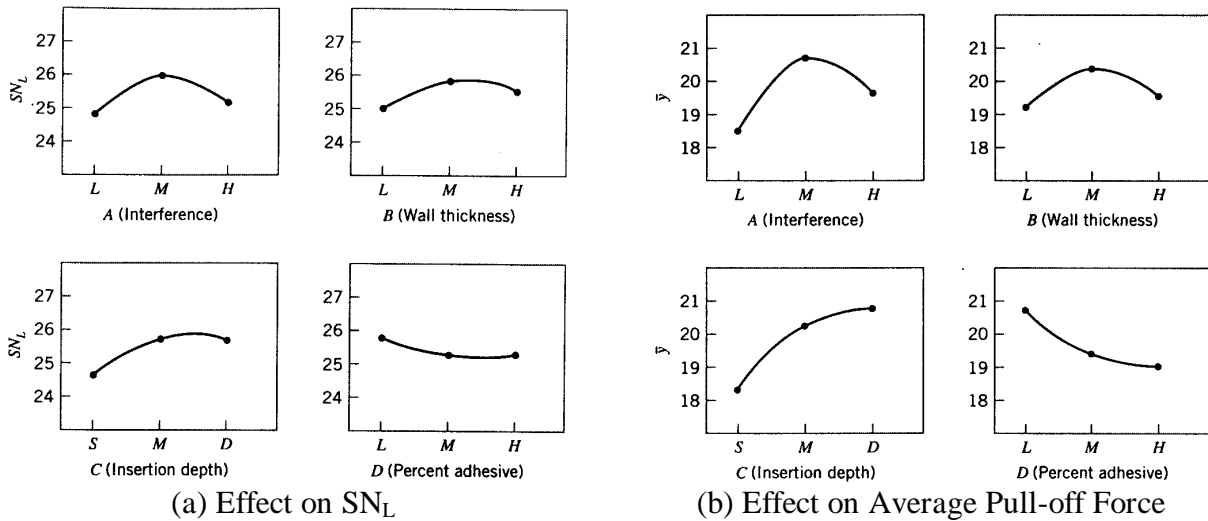
The total set of experiments that are performed is obtained by combining the  $L_9$  array of control factors (the outer array) with the  $L_8$  array of noise factors (the inner array). The total number of experiments is the product of the number of runs of each array, i.e.,  $9 \times 8$  or 72 experiments. For each experiment, the pull-off force is measured using the specified settings for each control factor level and noise factor level. The average pull-off force for each combination of the control factors A-D. Since the objective in the experiment is to maximize the pull-off force, the signal-to-noise ratio for "Larger is Better" is also computed for each set of runs. These results are summarized in Table 5 and discussed in the next section.

Run	Inner Array (L <sub>9</sub> )				Outer Array (L <sub>8</sub> )								Responses				
	A	B	C	D	E	F	G	1	1	1	1	2	2	2	2	$\bar{y}$	SN <sub>L</sub>
1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	17.525	24.025
2	1	2	2	2	1	1	2	2	1	1	2	1	1	2	2	19.475	25.522
3	1	3	3	3	1	2	1	2	1	2	1	2	2	1	2	19.025	25.335
4	2	1	2	3	2	1	2	3	1	2	3	1	2	2	2	20.125	25.904
5	2	2	3	1	2	2	3	1	2	3	1	2	2	2	2	22.825	26.908
6	2	3	1	2	2	3	1	2	3	1	2	2	2	2	2	19.225	25.326
7	3	1	3	2	3	1	2	3	2	3	1	2	2	2	2	19.850	25.711
8	3	2	1	3	3	2	3	1	2	3	2	3	1	2	2	18.838	24.852
9	3	3	2	1	3	3	2	1	3	2	3	1	2	3	2	21.200	26.152

**Table 5 Pull-Off Force for Connector and Tube Parameter Design Experiment**

### 32.4.2 Data Analysis

In this experiment, Taguchi’s graphical approach is used to plot the “marginal means” of each level of each factor and “pick the winner” to determine the best setting for each control factor. The average pull-off force and SN ratio for each level of each of the control factors are plotted in Figure 4. These values are computed by averaging the mean pull-off force or SN<sub>L</sub> for each factor for each level. For example, the average pull-off force for the shallow setting (Level 1) of the insertion depth (Factor C) is obtained by averaging Runs 1, 6, and 8 in Table 5, i.e., (17.525 + 19.225 + 18.838)/3 = 18.4. The same procedure is employed to compute the average SN<sub>L</sub> for each level of each factor and the remaining pull-off force averages.



**Figure 4 Control Factor Effects**

Figure 4 reveals that of the product design factors, interference (A) and wall thickness (B), the interference has a larger impact on SN<sub>L</sub> and the average pull-off force. The medium level for A (A<sub>medium</sub>) is clearly the best choice for maximizing SN<sub>L</sub> and the average pull-off force. As for the wall thickness, B, levels B<sub>medium</sub> and B<sub>high</sub> are slightly better than B<sub>low</sub>; however, B<sub>medium</sub> is preferred to B<sub>high</sub> in order to maximize the average pull-off force, see Figure 4b.



For the process design parameters, insertion depth (C) and percent adhesive (D), the insertion depth has a much larger impact on  $SN_L$  and the average pull-off force than does the percent adhesive. From Figure 4a,  $C_{\text{medium}}$  and  $C_{\text{deep}}$  yield nearly equal  $SN_L$ , but  $C_{\text{deep}}$  should be selected to maximize the average pull-off force. What is the implication of two levels of  $SN_L$  being nearly equal? The answer is that the average pull-off force increases when  $C_{\text{deep}}$  is chosen, and  $SN_L$  increases beyond its value when  $C_{\text{medium}}$  is chosen. However, the variance in the pull-off force also increases when  $C_{\text{deep}}$  is chosen and  $SN_L$  decreases. The end result is that  $C_{\text{medium}}$  and  $C_{\text{deep}}$  yield nearly equal  $SN_L$ . Notice that the  $SN_L$  values are nearly the same for  $D_{\text{medium}}$  and  $D_{\text{high}}$  as well. It appears that level  $D_{\text{low}}$  should be selected for percent adhesive since it yields a slightly higher  $SN_L$  and average pull-off force compared to  $D_{\text{medium}}$  and  $D_{\text{high}}$ .

Hence, the best settings to maximize  $SN_L$  are  $A_{\text{medium}}$ ,  $C_{\text{deep}}$ ,  $B_{\text{medium}}$ , and  $D_{\text{low}}$  based on the experimental results for maximizing pull-off force. In the actual study, further analysis of the data revealed that (1) the variance in the experiment was not constant and depended on the specific levels of each control factor, and (2) there were several interactions between some of the control factors and noise factors (Byrne and Taguchi, 1987). A summary of the results is given in the next section for the full experiment.

### 32.4.3 Summary of Results

While it was easy to “pick the winner” from the effects plots in Figure 4, these factor settings are only best for maximizing the pull-off force. In the complete set of experimental results which also includes assembly effort and cost ratings, the choice of factor levels is not as easy since many objectives are conflicting. A summary of the results is given in Table 6 where insignificant effects are noted with a dash. Based on the full set of data, the best levels for each factor are:  $A_{\text{medium}}$ ,  $B_{\text{low}}$ ,  $C_{\text{medium}}$ , and  $D_{\text{low}}$ . Only two of these choices are the same as those obtained by examining Figure 4 since the most desirable settings for reducing cost rating and assembly effort are not the same as those required to maximize pull-off force. The final choice of factor level requires a trade-off between the multiple competing objectives which is often the case in product and process design.

Factor	Level	Assembly Effort	Pull-Off Force	Cost Rating	Performance Rating		Selected Level
					Assembly	Pull-off	
Interference (A)	1 Low	8.1	18.7	Least	Best	Worst	
	2 Medium	8.3	20.7	–	–	Best	X
	3 High	8.7	19.8	Most	Worst	–	
Wall thickness (B)	1 Thin	–	–	Least	–	–	X
	2 Medium	–	–	–	–	–	
	3 Thick	–	–	Most	–	–	
Insertion depth (C)	1 Shallow	7.7	18.4	Least	Best	Worst	
	2 Medium	8.3	20.3	–	–	–	X
	3 Deep	9.1	20.6	Most	Worst	Best	
Percent adhesive (D)	1 Low	8.3	20.5	Least	Best	Best	X
	2 Medium	8.4	19.5	–	Worst	–	
	3 High	8.4	19.2	Most	Worst	Worst	

**Table 6 Summary of Parameter Design Results for Elastometric Connector**

This chapter is an excerpt from Wysk, R. A., Niebel, B. W., Cohen, P. H., and Simpson, T. W. Manufacturing Processes: Integrated Product and Process Design, McGraw Hill, New York, 2000.