This paper presents a novel testbed for vehicle control experiments: the Illinois Roadway Simulator (IRS). This is a scaled roadway suitable for easily visualizing preliminary vehicle control studies. The concept of the IRS is given along with the details of its design and construction. A review of vehicle dynamics gives the standard Bicycle Model for linear operating conditions. The parameters of the model are then estimated for representative IRS vehicles. The resulting vehicle dynamics are then compared with dynamics of full-scale vehicles for dynamic similitude. The dynamic similitude comparison is the key to gaining confidence in the scaled testbed as an accurate representation of actual vehicles. A series of experimental verifications are used to match the identified vehicle dynamics to the responses predicted by the standard vehicle model with some additional augmentations.

I. INTRODUCTION & MOTIVATION

There are several motivational factors that cause control practitioners to experiment with scaled systems before making suggestions for full-scale systems. One of the key motivators is cost. Scaled systems are usually significantly less expensive than their full scale counterparts. The impact of this is particularly true in cases where resources are limited: e.g., academic institutions or smaller companies. Another motivational factor is safety. Usually, the scaled systems are safer to operate than their full-sized counterparts and are much more forgiving of mistakes. This allows for many novel and radical control approaches to be tried out in an environment that is relatively tolerant of failure. This is particularly important because the new controllers may turn out to be unstable in their initial implementation. Convenience is a third motivation. Usually, scaled systems are easier to work with than full size systems. Flexibility of the experimental apparatus will be the final motivation given here. Since much of the control electronics do not have to be embedded in the system, the controllers can be implemented with common real-time software running on personal computers. This allows for rapid alteration of the control strategy thereby affording an easy comparison between different controllers under consideration.

Much of the non-proprietary, published vehicle control work to date has been limited to simulation because the use of a full-size vehicle to test controllers is often prohibitively expensive as well as dangerous. The focus of the research presented in this work has been to develop a scale version of a vehicle and a roadway for safe and economic testing of controller strategies: the Illinois Roadway Simulator (IRS). Previous investigations using scaled vehicles (Sampei et al 1995, Matsumoto & Tomizuka 1992, Kashroo et al, 1995) have mostly involved moving the vehicles along some fixed surface. This may incar a host of interfacing and sensing issues. The IRS is an experimental testbed consisting of scaled vehicles, running on a simulated road surface, where the vehicles are held fixed with respect to inertial space and the road surface moves relative to the vehicle. The rest of the paper is organized as follows. In Section 2, the physical system design is presented. Section 3 details the system dynamics. This starts with an idealized bicycle model and then gives the model parameters identified for scaled vehicles. Section 4 presents experimental data that is used to verify the approach taken in Section 3. Section 5 then details the dynamic similitude analysis used to verify the validity of using the scaled IRS system. Previous studies by the authors (Brennan et al, 1998) did not formally address the dynamic similitude problem and simply relied on identified I/O data to justify similarity based on eigenvalue locations in the complex plane. Here we use the Buckingham Pi theorem to perform a more formal analysis. It is shown that the IRS system can be made to match most standard vehicles with a high degree of dynamic similitude. A conclusion then summarizes the main points of the paper.

2. THE ILLINOIS ROADWAY SIMULATOR

The IRS's scaled roadway surface consists of a 4 x 8 ft. treadmill capable of top speeds of 15 mph. Scale vehicles are run on the treadmill via multiple wall-mounted transmitter systems operating between 50 and 100 MHz. The remainder of the IRS consists of a driver console, DSP and PC based interface computers, A/D and D/A converters, a significant amount of electronic interface equipment, several separate receiver systems, a vehicle position sensor system, and the vehicles. The vehicle controller hardware loop uses a reference signal that can come either indirectly via the manual driver console or directly from a computer-generated signal. If the signal is from the manual driver console it is first input to a computer via an Analog Devices RTI-815 Analog I/O board sampling at 1 kHz. The computer then outputs analog voltage commands, via an Analog Devices RTI 802 Analog Output board, to the vehicle's transmitter. This voltage signal is then converted to a FM signal. The receiver system on the vehicle transforms the transmitter's FM signals into pulse-width modulated signals, which are then sent to the vehicle actuators. Each actuator has a built-in analog controller that converts the pulse-width-modulated signals into reference commands.

The treadmill road surface regulates the vehicle position with respect to an inertial reference point. The roadway speed is monitored via an optical encoder. To maintain the vehicle on the treadmill, a separate computer uses the vehicle's inertial position as feedback and sends an output voltage signal to the treadmill. The treadmill uses an industrial motor controller that converts the input voltage level to a reference speed, and adjusts the DC drive motor current to match this speed accordingly. Figures 1 and 2 give a representation of the entire system.
As designed, the treadmill does not allow speed reversing. Acceleration of the treadmill is accomplished by a DC motor applying torque to the treadmill belt. Deceleration is accomplished by allowing friction to slow the treadmill down. The deceleration torque contributions due to viscous and sliding friction values actually change with treadmill speed due to the creation of an air bearing between the sliding treadmill surface and the underlying supporting panel.

The feedback loop begins with a position sensor connecting the vehicle with a known inertial reference point. The sensor consists of a 3-bar linkage with encoders at each joint. The joint angles are then used to determine the position and orientation of the vehicle on the treadmill. Figure 3 shows a sensor arm, as well as the angle and length conventions used to determine vehicle position. The corresponding vehicle coordinates are given as:

\[
\begin{align*}
  x &= l_1 \cdot \cos \Theta_1 + l_2 \cdot \cos(\Theta_1 + \Theta_2) \\
  y &= l_1 \cdot \sin \Theta_1 + l_2 \cdot \sin(\Theta_1 + \Theta_2) \\
  \psi &= (\Theta_1 - \Theta_0) + (\Theta_2 - \Theta_0) + (\Theta_3 - \Theta_0)
\end{align*}
\]

where \(l_1, l_2\) are the link lengths, \(\Theta_1, \Theta_2, \Theta_3\) are the joint rotations and \(\Theta_0, \Theta_0, \Theta_0\) are the reference positions calibrated to inertial space. Initial experiments used potentiometers for the joint angle sensors to enhance system robustness. However, after several experimental design iterations, high-resolution encoders are currently being used for their linearity and resistance to wear. Custom mounted brackets were designed so that the encoders themselves saw no side loading.

There are several vehicles in use on the IRS, each with different operating capabilities. They range from a simple 2WD front steer vehicle to a 4WS vehicle with independent drive motors for each wheel shown in Figure 4.

Commercial off-the-shelf (COTS) transmitter systems were used to send signals to the on-board motor and steer servo controllers. This communication system induced a time delay in the control loop. The COTS system was retained in the control loop because of the simplicity involved with interfacing the transmitters which used potentiometers to generate analog control signals. It is possible to directly interface the on-vehicle controllers and this will reduce or eliminate the delay. However, this will greatly increase the system complexity.

All DAQ and control features are handled via Wincon, a windows based control program that runs real-time code generated by Matlab/Simulink’s Real Time Workshop toolbox. Custom drivers were written in C to communicate with the Analog Devices boards. This Wincon interface eliminated lower level C-programming and allowed all functions to be handled with a GUI type of Simulink interface. Additionally, it provided for real-time viewing of data.

### 3. VEHICLE SYSTEM DYNAMICS

The well known Bicycle Model (Genta, 1997) was taken as an initial estimate for the dynamics of the scaled IRS vehicle. The Bicycle Model assumes a constant longitudinal velocity of the vehicle and consists of two dynamic degrees of freedom, lateral velocity and yaw rate. Define:

- \(m\) = mass of the vehicle
- \(I_v\) = vehicle inertia about vertical axis at the c.g.
- \(V\) = vehicle forward velocity
- \(C_{f}, C_{r}\) = front, rear cornering stiffnesses
- \(L_1, L_2\) = distance from front, rear axle to the c.g.
- \(L = L_1 + L_2\)
- \(d_s\) = distance between sensor and c.g. along vehicle axis
- \(Y_s\) = lateral distance measured from reference to sensor
- \(\delta_f, \delta_r\) = front, rear steering angle
- \(\psi\) = Yaw Angle
The state space formulation (Peng & Tomizuka, 1993) is as follows:

\[
\begin{bmatrix}
\frac{\partial Y_m}{\partial t} \\
\frac{\partial Y_m}{\partial \phi_a}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
Y_m \\
\phi_a
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\delta_2
\end{bmatrix}
\]

where

\[
Y_m = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}
\]

In Equation (2), the output equation measures the lateral displacement at a point ahead of the c.g. similar to a AHS configuration (Peng & Tomizuka, 1993). The transfer function from front input steer angle to output lateral displacement is given as:

\[
Y_m(s) = \frac{e^s}{s^2 (s + \beta)}
\]

and the transfer function from front steer angle to yaw rate is:

\[
\dot{\phi_a}(s) = \frac{e^s}{s^2 (s + \beta)}
\]

We can note that the above equation consists of many values that are experimentally measurable, such as vehicle speed, mass, and moment of inertia. If these values are measured and substituted into the transfer function given above, then a reasonable approximation of the vehicle's transfer function should be obtained. Although the measurement of the vehicle mass is trivial, measuring the other values is not intuitively obvious.

The vehicle’s mass was determined simply by weighing it on a standard balance scale. The center of gravity was determined by balancing the vehicle and determining the location where zero net gravitational moment acted. To determine the 2-axis moment of inertia, the vehicle was suspended by a spring whose force is proportional to angle, the governing equation is given as:

\[
\sum M_x = -\beta \dot{\theta} - k \theta
\]

where \(\beta\) is a damping term [N*m/sec/rad], \(I_x\) is the 2-axis moment of inertia [kg-m²], and \(k\) is a spring constant [N*m/rad]. If we take the Laplace transform of the equation and consider a free response situation, we obtain:

\[
\frac{1}{I_x} \dot{s} + \beta s + \lambda \dot{\phi} = 0
\]

If we solve for \(s\), we obtain

\[
s = \frac{\beta}{2I_x} \pm \sqrt{\frac{\beta^2}{4I_x^2} - \lambda}\]

If the system is underdamped, we can measure the exponential decay term, \(-\beta/(2\sqrt{\lambda}) = \lambda\), as well as the spring constant \(k\) and the frequency of the response. From these measurements, we note that

\[
I_x = \frac{k}{\lambda^2 + \omega^2}
\]

The following figure shows a sample of the time response, as well as the exponential fit to determine \(\lambda\).

\[\text{Figure 6: Time Response.}\]

\(\lambda\) is approximately 0.051 rad/sec and the frequency of the system can be identified as 0.965 rad/sec. Using the equation above, the moment of inertia for this particular case is calculated to be 0.0730 [kg*m²].

\[\text{Figure 7: The C₄ testing stand.}\]
To determine the cornering stiffness of the tires, the special test rig shown in Figure 7 was devised. With this test stand it is possible to control both the slip angle and the normal force on each tire. Figure 8 shows the results of testing a particular scale tire at 3 different normal loads. By determining the tangent line at zero slip angle it is possible to determine the actual cornering stiffness for the tire. The cornering stiffness characteristics will change with tire type. Figure 9 below shows the cornering stiffness determined for a low and high Ca tire.

A summary of typical measured parameters is in the following table:

<table>
<thead>
<tr>
<th>Vehicle #1</th>
<th>Vehicle #2</th>
<th>Vehicle #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1.47 kg</td>
<td>4.025 kg</td>
</tr>
<tr>
<td>I&lt;sub&gt;x&lt;/sub&gt;</td>
<td>0.024 kg m&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.119 kg m&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>L&lt;sub&gt;1&lt;/sub&gt;</td>
<td>0.13 m</td>
<td>0.139 m</td>
</tr>
<tr>
<td>L&lt;sub&gt;2&lt;/sub&gt;</td>
<td>0.15 m</td>
<td>0.189 m</td>
</tr>
<tr>
<td>Ca&lt;sub&gt;L&lt;/sub&gt;</td>
<td>1.53 N/deg</td>
<td>0.53 N/deg</td>
</tr>
<tr>
<td>Ca&lt;sub&gt;S&lt;/sub&gt;</td>
<td>1.53 N/deg</td>
<td>0.53 N/deg</td>
</tr>
</tbody>
</table>

Although the system of Eq (2) uses the steer angle as the input, for the IRS system the following dynamics occur between the voltage steer command and the actual steer angle.

\[
\begin{align*}
\text{Volt} & \xrightarrow{e^{-sT}} s^2 + 2\epsilon s + \epsilon^2 \\
& \xrightarrow{\text{2nd order dynamics}} K \\
& \xrightarrow{\text{rate limit}} \delta
\end{align*}
\]

Figure 10: Steer Actuator Dynamics.

There is a communication time delay of approximately 15 msec from the D/A computer signal to the actuator's reference signal. The steer actuator is an electric motor controlled by an analog feedback device. The rate limiter occurs due to the gearing in the motor necessary for sufficient output torque.

Figure 11 shows the time and frequency domain characteristics of two separate steer servos used on IRS vehicles. The slower servo is in the left column. The responses are given at several different operating amplitudes. The approximate bandwidth is 1 Hz for the slower servo and 8 Hz for the faster servo.

4. EXPERIMENTAL VERIFICATION

In this section, we examine the accuracy of the parameters identified in Section 3. Figure 12 shows the frequency response of the entire vehicle from input to yaw rate. Two measured, experimental frequency responses are compared with a transfer function obtained by directly substituting the identified parameters for Vehicle #2 along with the fast steer servo model of Figure 11 and the communication time delay.
The system dynamics used in the model are:

\[ w(s) = \frac{90}{s^2 + 95.4s + 2809} \cdot \frac{s + 5.999}{s^2 + 9.754s + 36.39} \cdot e^{-0.015s} \]

where the yaw rate is in deg/sec and the input is in volts. Not shown in Eq (9) is the rate limit of 340 deg/sec that is included in the steer servo dynamics. As can be seen in the figures, the fits in both the time and frequency domain are very good. The fit could be made even better since the steer servo was identified while the vehicle wasn’t actually running on the IRS and the vehicle’s inertia and mass weren’t measured with the sensor arm attached. By slightly tuning transfer function coefficients the results of Figures 12 and 13 could be improved significantly.

Figure 12: Frequency domain model comparison.

Figure 13 shows the time response of the vehicle for a series of lane change maneuvers. In the figure, the noisy experimental yaw rate is closely matched by the smoother simulated yaw rate.

Figure 13: Time domain model comparison.

Note that the angles such as the steer angle and slip angle are unitless and thus form their own Pi groups. It is clear that the basic unit dimensions are mass, length, and time. Thus, there are 3 primary dimensions in the unit space, abbreviated M, L, and T with 5 parameters in question. If we choose m, V, and L as repeating parameters, we can express the remaining 2 parameters as dimensionless groups, to create 2 additional Pi groups. First, a dimensional equation is formed in powers of the repeating parameters.

Equating the powers, three equations are obtained:

1. mass \( I + a = 0 \)
2. time \( -2 + b = 0 \)
3. length \( 1 + b + c = 0 \)

Solving the equations gives \( a = -1, b = -2, \) and \( c = 1 \). Hence, the first Pi group is \( \frac{c}{mL^2} \). Solving for the second Pi group:

Equating the powers, three equations are obtained:

1. mass \( I + a = 0 \)
2. time \( -b = 0 \)
3. length \( 2 + b + c = 0 \)

Solving the equations gives \( a = -1, b = 0, \) and \( c = -2 \). Therefore, the second Pi group is \( \frac{C_{af} \cdot m^2 \cdot V^2}{L^2} \). A summary of all the Pi groups is:

\[ n_1 = L, n_2 = L, n_3 = C_{af}, n_4 = \frac{C_{af}}{m^2}, n_5 = \frac{L}{m^2} \]

The Buckingham Pi theorem states that if two dynamic systems are described by the same differential equations, then the solution to these differential equations will be the same if the Pi groups are the same. This becomes clear during nondimensionalization of the governing differential equations.

To determine the validity of the use of scaled vehicles on the IRS, originally the pole locations of the scale vehicle were compared to the full sized vehicles. These pole locations are determined by the eigenvalues of the 'A' matrix for the bicycle model in Eq. (2). Not including the double integrator, these open loop eigenvalues are the solution to the equation:

If a solution to the differential equations in Eq. (2) for the IRS vehicles exists, then the vehicle lateral position will be a function dependent on the scaled parameters:

\[
Y = \frac{[g, a_1, a_2, m, l, Y_v, L_1, L_2, C_{af}, C_{a_2}, T, r]}{[l, L_1, L_2, C_{af}, a_2, T, r]} \]

The Buckingham Pi theorem (Buckingham, 1914) states that any function that can be written in the above form can be rewritten in a dimensionless form without changing the solution to the differential equation. This rewriting is achieved by grouping the parameters into \( n - m \) independent dimensionless parameters, where \( n \) is the number of parameters and \( m \) is the dimension of the unit space occupied by the parameters. The parameters, along with their primary unit dimensions, are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Primary Unit Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>kg = [m]</td>
</tr>
<tr>
<td>V</td>
<td>m/s = [L/T]</td>
</tr>
<tr>
<td>L</td>
<td>m = [L]</td>
</tr>
<tr>
<td>a_1, a_2</td>
<td>[w = [1/a]]</td>
</tr>
<tr>
<td>c_1, c_2</td>
<td>[k = [M/L^2]]</td>
</tr>
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<td>m = [L]</td>
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<tr>
<td>a_1, a_2</td>
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To determine the validity of the use of scaled vehicles on the IRS, originally the pole locations of the scale vehicle were compared to the full sized vehicles. These pole locations are determined by the eigenvalues of the 'A' matrix for the bicycle model in Eq. (2). Not including the double integrator, these open loop eigenvalues are the solution to the equation:

\[
I_1, I_2, I_3 = \frac{ML^2}{m^2}, \frac{ML}{m}, \frac{L}{m^2}
\]
\[ l_{1 \text{m}} v^2 + 2 \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \left( \frac{1}{2} l_{1 \text{m}} v^2 + l_{1 \text{m}} v^2 \right) \frac{1}{4 \text{m}} \left( \frac{1}{2} l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) \]

\[ \Rightarrow s^2 + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

\[ + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

Note that the \( s \) term has units of (sec\(^{-2}\)), so we may make a scale transformation to non-dimensional coordinates:

\[ \frac{v^2}{l_{\text{m}}} \frac{s^2}{l_{\text{m}}} + \frac{1}{l_{\text{m}}} \left( l_{\text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \frac{1}{l_{\text{m}}} \left( l_{\text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

\[ l_{1 \text{m}} v^2 + 2 \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \frac{1}{4 \text{m}} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) \]

\[ \Rightarrow s^2 + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

\[ l_{1 \text{m}} v^2 + l_{1 \text{m}} v^2 \frac{1}{4 \text{m}} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) \]

\[ \Rightarrow s^2 + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

\[ + \frac{1}{l_{1 \text{m}} v^2} \left( l_{1 \text{c}} c_{\text{ad}} + c_{\text{ar}} \right) = 0 \]

Clearly, if the Pi groups agree between two systems governed by the bicycle model, then the normalized pole locations will be the same. This will indicate a very high degree of dynamic similitude between the two systems. To test this concept pole locations and Pi groups were compiled for full-sized and IRS-scale vehicles.

<table>
<thead>
<tr>
<th>Vehicle #2 Avg. Full Size</th>
<th>Speed (m/s / mph)</th>
<th>Poles</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.6/52</td>
<td>0.0, 0.4.4x/- 3.5</td>
<td>0.4229</td>
<td>-0.5771</td>
<td>0.2698</td>
<td>0.2698</td>
<td>0.2755</td>
</tr>
</tbody>
</table>

* These two values can be matched by varying car speed

The average values of vehicle parameters were taken for 4 different mid-size vehicles (LeBlanc et al 1996, Reid et al 1981).

To summarize the results shown in the table, analysis shows experimentally and theoretically that there can be good agreement between scale and full-sized vehicle dynamics. To get the proper match between scale and full-sized vehicles may involve adjustments of vehicle parameters. For example, in the table above, the inertia of the scaled vehicle was tuned to match the Pi groups by adding and distributing weight to the front and rear of the vehicle.

**CONCLUSION**

This paper has given a detailed exposition on a novel, scaled testbed for vehicle control studies. The testbed is intended to provide initial results on vehicle controllers in a rapid manner. Additionally, the use of a physical experimental system can provide easy visualization of actual control performance. The components of the system were given separately, including methods for measuring various system parameters. The accuracy of the system dynamics identification

was given in both the time and frequency domains. The key to the use of the IRS is the fact that the vehicles acting on it can be made to have a high degree of dynamic similitude with real vehicles. The Buckingham Pi theorem was used to develop a series of Pi groups that were then compared with the same Pi groups for the average of a sample of real vehicles. The matching of the scaled Pi groups with the full-sized Pi groups should give confidence in the IRS dynamic similitude.

The Pi group matching that was done depends on the system model used. In this case, the IRS is focused on matching the planar vehicle dynamics. The dynamics of a system with roll or pitch has not yet been investigated with this system.

**REFERENCES**


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