DIFFERENTIAL DIAGNOSTICS FOR LITHIUM ION BATTERY CELLS CONNECTED IN SERIES

Jariullah Safi
Control Optimization Lab
Department of Mechanical and Nuclear Engineering
Pennsylvania State University
State College, Pennsylvania 16801
Email: jxs1112@psu.edu

Joel Anstrom
Sean Brennan
Hosam K. Fathy∗
Department of Mechanical and Nuclear Engineering
Pennsylvania State University
State College, Pennsylvania, 16801
Email addresses: (janstrom, snb10, hkf2)@engr.psu.edu

ABSTRACT

This paper presents a new method for estimating the capacity of a lithium ion battery cell in the presence of a reference cell - the parameters of which are well characterized - in series with it. The method assumes that both cells are cycled using the same current trajectory starting from the same state of charge (e.g. fully charged). Voltage measurements for both cells as well as current measurements for the series string constitute the input to a nonlinear least squares minimization problem. The goal of this problem is to estimate the capacity of the cell given the difference between its voltage and that of the reference cell. We refer to this as the differential estimation problem, and use Monte Carlo simulation to compare it to the more traditional approach of estimating the capacity of each cell in a battery string independently using its current/voltage measurements. Two key conclusions emerge from this simulation. Compared to traditional estimation, differential estimation results in capacity estimates whose variance is (i) twice as sensitive to voltage measurement noise but (ii) significantly less sensitive to current measurement noise. This makes differential estimation more appealing for battery packs with high current measurement noise and low voltage measurement noise.

NOMENCLATURE

$s_t$ The state of charge of a cell at time $t$
$s_0$ Initial state of charge
$Q^{(i)}$ Charge capacity of the $i^{th}$ cell
$I_t$ True value of current at time $t$
$I_t = \sum_{k=1}^{t-1} I_k$
$I_t$ Measured value of current at time $t$, $\hat{I}_t = I_t + w_t$
$w_t \sim \mathcal{N}(0, \sigma^2_w)$
$\sigma_w$ Standard deviation of current measurement noise $w_t$
$y^{(i)}_t$ True voltage response of the $i^{th}$ cell at time $t$
$\hat{y}^{(i)}_t$ Predicted voltage for the $i^{th}$ cell at time $t$, $\hat{y}^{(i)}_t = y^{(i)}_t + \nu^{(i)}_t$
$\nu^{(i)}_t \sim \mathcal{N}(0, \sigma^2_v)$
$g(s)$ The open circuit voltage map of the battery
$\hat{Q}^{(2)}$ Estimate for cell 2’s capacity
$\nu^{(i)}_t$ Predicted voltage for the $i^{th}$ cell at time $t$
$\sigma_{Q_2}$ Standard deviation of $\hat{Q}^{(2)}$

INTRODUCTION

Consider two lithium ion (Li-Ion) battery cells connected in series that differ only in their charge capacity and are cycled starting from the same state of charge (SOC). In this paper we investigate the possibility of estimating this difference in capacity by using the difference in the voltage response of the two cells.

∗Address all correspondence to this author.
Herein we refer to this idea as differential diagnostics.

The rising popularity of Li-Ion batteries for vehicle electrification has made the determination of remaining capacity more important [1] [2]. It is customary to declare a vehicle battery pack dead when the usable capacity reaches 80 percent of its initial value [3]. As such, battery cell capacity needs to be monitored for end-of-life determination. Knowledge of capacity distributions in a pack can potentially be used for health conscious control of a battery pack (e.g. switch based charging as studied by Moura et al. [4]).

Capacity determination for battery cells can be conducted either in offline laboratory tests or online, as part of onboard battery health management and diagnostics. Offline tests usually benefit from well-calibrated, high-quality sensors. Experiment duration is typically less constrained in these tests (compared to online estimation), and one can employ a variety of sensors ranging from simple voltage/current sensors to electrochemical impedance spectroscopy (EIS) instruments [5]. Specific tests that maximize the identifiability of battery parameters are relatively easy to implement [6]. Online testing/estimation, in contrast, typically takes place during operation of the battery’s host device (e.g., car, cellphone, laptop, etc.). The quality and availability of sensors has a direct impact on the cost of the host device, and estimation usually needs to be performed alongside the typical operation of the host. Vehicle battery packs, for instance, are typically limited to one voltage sensor per cell and one current sensor per series string of cells [7].

This paper focuses on online testing using input-output data (current-voltage measurements) because of its relevance to health monitoring for vehicle battery packs. As such we assume the availability of only voltage and current sensors. Following Plett [8], we assume that these voltage and current sensors measurements are corrupted with zero-mean, uncorrelated, additive Gaussian white noise. Our study provides initial insights into the characteristics of the proposed estimation algorithm compared to more traditional cell-by-cell capacity estimation.

The estimation of capacity problem is one of determining a dynamic model’s parameters. This problem can be solved using least squares methods [9]. For example:

1. Plett uses linear least squares for fitting parameters for simplified battery models [10].
2. He et al. propose simultaneous battery state of charge (SOC) and parameter estimation by combining a linear least squares estimator with an adaptive Kalman filter [11].
3. Balasingam et al. use a linear version of a total least squares estimator for online determination of battery capacity [12].

Least squares estimators are computationally efficient, and the above studies give good parameter estimation results. Battery cells are inherently nonlinear systems. As such Plett investigates parameter estimation using approximate non-linear methods like extended Kalman filters [8] and Saha et al. propose the use of particle filters which are useful when enough processing power is available [13].

The above studies focus on estimating the capacity of a single battery cell at a time. This means that they do not exploit the collective behavior of battery cells in a string to improve capacity estimation accuracy. This paper’s objective is to study least squares based estimation (owing to its simplicity and wide use) of the capacity difference between two cells by using the difference between their voltage responses. We have chosen to call this method differential diagnostics or differential estimation because it discovers differences in capacity which we believe to be the first step in characterizing deviations from a norm and diagnosing unhealthy behavior for a pack. The differential diagnostics method assumes that the internal resistance for the two cells and the capacity for one of the cells (called the reference cell) are all well known. A more elaborate study where the differential estimation approach is utilized to determine both the capacity and internal resistance of the non-reference cell is left open for future research.

The rest of the paper’s structure is: i) a discussion of the model used in this paper, ii) the mathematical formulation of the estimator, iii) proof of concept Monte Carlo simulations for the estimator, iv) a discussion of the results, v) and conclusions.

MODEL DEFINITION

This paper uses the following equivalent circuit model for Li-Ion battery cells:

\[ s_t = s_0 + \frac{1}{Q} \bar{l}_t \]

\[ y_t = g(s_t) + I_t R \]

where the model is presented in a discrete time form with a time step of 1 second. The state variable \( s_t \) represents the state of charge (SOC) of the cell at time \( t \). \( y_t \) is the terminal voltage, \( Q \) represents the useful capacity, \( I_t \) is the current applied during the \( t \)th timestep, and \( R \) is the internal resistance of the cell. The aggregate input \( \bar{l}_t \) is defined for convenience as

\[ \bar{l}_t = \sum_{m=1}^{t-1} I_m \]

and \( g(s_t) \) is a monotonically increasing nonlinear mapping from SOC to the open circuit voltage of the cell. The map is shown in Fig. 1.

The above model typically falls under the classification of equivalent circuit models in the literature. It captures two fundamental characteristics of a Li-Ion cell:
1. The cell stores charge like a current integrator.
2. The relationship between the amount of charge stored (or SOC) and the cell’s open circuit voltage is given by a non-linear map.

While the model does not capture higher order dynamics like diffusion; it simplifies the process of drawing insight from mathematical analysis. Plett [8] and Lin [14] have both shown the effectiveness of models like this for:

1. Estimation of battery states and parameters.
2. Gaining insights about the behavior of battery state and parameter estimators.

DIFFERENTIAL DIAGNOSTICS PROBLEM FORMULATION

This section presents the differential diagnostics problem as a least-squares estimation problem. Voltage and current measurements of two battery cells - connected in series with one another - are available. Conventional least squares estimation for Li-Ion cells works on voltage measurements for a single cell. Differential diagnostics, in contrast, makes use of the difference in voltage measurements for two cells. Fig. 2 shows this difference visually. The goal of both estimators in Figure 2 is to determine the charge capacity of Cell 2. The conventional estimator achieves this using the voltage and current measurements for Cell 2 only. The differential estimator, in contrast, uses the difference in voltage between Cells 1 and 2 to estimate the difference in their capacity parameters, exploiting the fact that the capacity of Cell 1 is known.

To keep bookkeeping simple the superscript \( (\cdot)^{(1)} \) denotes states and parameters for cell 1 from Fig. 2 and \( (\cdot)^{(2)} \) for cell 2. Then the model for voltage measurement for the \( i \)th cell at time \( t \)

\[
\tilde{y}_i^{(i)} = g \left( s_0 + \frac{1}{Q^{(i)}} I_t \right) + I_t R^{(i)} + v_i^{(i)}
\]

where \( \tilde{y}_i^{(i)} \) is a measurement of the true voltage \( y_i^{(i)} \) and is corrupted by \( v_i^{(i)} \sim N(0, \sigma_i^2) \). Note that the random variables \( v_i^{(i)} \) are independent in time. Using the above information a least squares formulation of the differential diagnostics problem can be given by

\[
\min_{\hat{Q}^{(2)}} \sum_{i=1}^{N} (\tilde{y}_i^{(2)} - \hat{y}_i^{(1)})^2
\]

Subject to:

\[
\delta \hat{y}_i = \hat{y}_i^{(2)} - \hat{y}_i^{(1)}
\]

\[
\delta \hat{y} = g \left( s_0 + \frac{1}{Q^{(1)}} \tilde{I}_t \right) - g \left( s_0 + \frac{1}{Q^{(2)}} \tilde{I}_t \right) + \tilde{I}_t (R^{(2)} - R^{(1)})
\]

Note that \( \tilde{I}_t \) is the measurement of current given by

\[
\tilde{I}_t = I_t + w_t
\]

where \( w_t \sim N(0, \sigma_w^2) \) and is also independent in time. The relationship between \( \tilde{I}_t \) and \( I_t \) is the same as that between \( I_t \) and \( \tilde{I}_t \). Since the assumption is that \( Q^{(1)}, R^{(1)}, \) and \( R^{(2)} \) are given the only non-random unknown in Eq. (5) (also called a cost function) is \( \hat{Q}^{(2)} \) (an estimate of the true parameter \( Q^{(2)} \)). Numerical solutions of this minimization problem are possible using a variety of methods (e.g. gradient descent) [15]. Note that the additive
noise in the $\delta \tilde{y}$ term is different from $v_{1}^{(0)}$. It is the difference of $v^{(2)}$ and $v^{(1)}$ which gives it a variance of $2\sigma_{v}^{2}$ and a mean of zero.

For the sake of comparison the mathematical formulation for a non-differential estimator for cell 2 follows. The estimation problem is

$$
\min_{Q(2)} \sum_{t=1}^{N} \left( \tilde{y}_{t}^{(2)} - \hat{y}_{t}^{(2)} \right)^{2}
$$

Subject to:

$$
\hat{y}_{t} = g \left( s_{0} + \frac{1}{Q(2)} \tilde{I}_{t} \right) + \tilde{I}_{t} R(2)
$$

**SIMULATION SETUP**

One way to study the effects of varying amounts of voltage and current noise on estimator performance is through Monte Carlo simulation. The following are true for the simulations studies in this paper:

1. Cell 1 represents a pristine reference cell. Its capacity and resistance are known quantities. Cell 2 is an aged cell with a smaller capacity and larger resistance. Table 1 shows the true parameters used for simulation of the two cells.

2. The cells are subjected to the same current trajectory. It is representative of a 300 mile range electric vehicle going through multiple US FTP-75 drive cycles. Fig. 3 shows this trajectory.

3. The cells are always fully charged at the start of a simulation. This is possible in a battery pack with top balancing capabilities [7].

4. The current trajectory discharges the cells over the course of nine hours.

Simulation of these cells produces nominal voltage trajectories, uncorrupted by noise. Zero mean Gaussian noise addition to the voltage and current trajectories simulates the measurement process. The standard deviations of these noise processes varies between simulations. This gives insight into how the estimator properties change as a function of these noise terms. The total time for a simulation can also affect estimator performance. A matrix of simulation studies is performed covering all combinations of the following parameter values:

$$
\sigma_{v} \in [0.0005, 0.005, 0.05, 0.1] \text{ Volts} \quad (11)
$$

$$
\sigma_{w} \in [0.001, 0.01, 0.1, 0.25] \text{ Amps} \quad (12)
$$

$$
t_{f} \in [1, 2, 4] \text{ Hours} \quad (13)
$$

where $\sigma_{v}$ and $\sigma_{w}$ are the standard deviations for voltage and current noise respectively and $t_{f}$ is the total simulation time. For each test the simulated measurements for the system constitute the input to the least squares terms of Eqs. (5) and (9). The simulation minimizes these terms with respect to $Q(2)$. We perform this optimization using an exhaustive search over a large set of discretized values of $Q(2)$. Furthermore, for every combination of parameters from Equations (11-13), we repeat the estimation process for 1,000 realizations of the noise processes. This furnishes a Monte Carlo assessment of the statistical properties (e.g., estimation bias and estimation variance) of the differential estimation algorithm.

**SIMULATION RESULTS**

Figures 4 and 5 show the absolute estimation errors and standard deviations for all tests on the same plot for the conventional and differential cases. These two plots show that both the conventional and differential estimators produce absolute estimation errors significantly smaller than the corresponding standard deviations.

<table>
<thead>
<tr>
<th>$Q$ (A s)</th>
<th>$R$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell 1</td>
<td>3921 0.02</td>
</tr>
<tr>
<td>Cell 2</td>
<td>3725 0.023</td>
</tr>
</tbody>
</table>

**TABLE 1.** True cell parameters for simulation studies.
Figures 6 through 8 show the same data from the last two figures but parsed out so that the effect of $\sigma_v$ and $\sigma_w$ on estimator performance is easily observable. The two horizontal axes in these figures correspond to measurement noise whereas the vertical axis represents the estimate standard deviation. These figures also superimpose results for both estimators which allows for direct visual comparison of the performance of one estimator compared to the other. The plots in Figs. 6, 7, and 8 correspond to different time durations of the estimation study.

**DISCUSSION - ESTIMATOR BIAS**

This paper compares the performance of the traditional and differential estimators by comparing the resulting standard deviations of capacity estimates (for the non-reference cell). Such a comparison assumes, implicitly, that estimation biases are small compared to standard deviations. A quick visual inspection of Figs. 4 and 5 shows that the largest estimate bias is on the order of 20 Coulombs. The cases in which this is true also have a very large standard deviation which makes the seemingly large error bias negligible. As such, the authors conclude that it is reasonable to evaluate estimator performance based solely on standard deviations.
DISCUSSION - EFFECT OF TEST DURATION

Comparing the results in Figs. 6 through 8 shows that both the traditional and differential capacity estimators achieve higher accuracy levels for longer time durations. Perhaps more importantly, the relationships between current/voltage sensor noise on the one hand and capacity estimation standard deviations, on the other hand, appear to be consistent for different experiment durations. This is important because these relationships show an advantage for differential estimation when current noise is high, as explained later.

DISCUSSION - EFFECT OF VOLTAGE NOISE

Figures 6 through 8 show that both the conventional and differential estimator degrade in performance as the voltage noise increases. The differential estimator, however, has a higher standard deviation than the conventional one for very small values of current noise. One explanation for this result is the artificial increase in measurement noise variance for the differential estimator that comes from subtracting one voltage measurement from another. More specifically, because the differential estimator uses the voltage difference \( \tilde{y}^{(2)} - \tilde{y}^{(1)} \) for estimation, the combined overall output measurement noise process affecting this estimator has twice the variance of the measurement noise process affecting the traditional estimator. For a small amount of current noise this effect appears to dominate resulting in an increase in estimator standard deviation.

DISCUSSION - EFFECT OF CURRENT NOISE

Referring to Figs. 6 through 8 again, note that the standard deviation of the conventional estimator is a strong function of both \( \sigma_w \) and \( \sigma_v \). This is not true for the differential estimator which is a strong function of \( \sigma_v \) only. This causes the differential estimator to outperform the conventional estimator as the current noise increases. This is an encouraging result given the cost and complexity of accurate current measurement in practical battery systems. Differential estimation thus provides a pathway towards improvement in cell capacity estimation accuracy for battery systems with inexpensive, noisy current sensors.

CONCLUSION

This paper presents the differential diagnostics method: a way to determine Li-ion battery capacity for a cell connected in series with a reference cell whose parameters are well known. Results suggest that the differential method is not very sensitive to high values of current sensor noise. This makes it ideal for systems where accurate current measurement might be a challenge.

The notion of a reference cell whose parameters are accurately known presents a potential challenge, as does the possibility of using the same cell for multiple strings. Perhaps future battery pack designs will incorporate an externally accessible port for a reference cell which could then be connected to different strings through a switch mechanism. This reference cell will either need to be periodically replaced or periodically tested to ensure that its own parameters are known with very high accuracy. This possibility is, of course, subject to the balance between the cost of implementing differential estimation/diagnostics and the resulting capacity estimation accuracy gains.

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